A Level Pure Mathematics
Practice Test 1: Sequences and Series

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Arithmetic Sequences

1. For the arithmetic sequence $5, 8, 11, 14, 17, \ldots$:
   (a) Find the first term a and common difference d
   (b) Find the general term u_n
   (c) Calculate u_{20}
   (d) Find which term equals 71
   (e) Determine if 100 is a term in the sequence

2. An arithmetic sequence has $u_3 = 17$ and $u_8 = 37$.
   (a) Find the first term and common difference
   (b) Write the general term u_n
   (c) Calculate u_{15}
   (d) Find the first negative term
   (e) Determine the largest value of n for which $u_n < 100$

3. The nth term of an arithmetic sequence is $u_n = 3n - 2$.
   (a) Write down the first five terms
   (b) Find the common difference
   (c) Calculate u_{50}
   (d) Find the sum of the first 20 terms
   (e) For what value of n is $u_n = 97$?

4. Three numbers $x - d$, x, and $x + d$ are in arithmetic progression with sum 21 and product 231.
   (a) Find the value of x
   (b) Set up an equation for d
   (c) Solve to find the values of d
   (d) Write down the three numbers for each case

5. An arithmetic sequence has first term a and common difference d.
   (a) Prove that $u_n = a + (n - 1)d$
   (b) If $u_p = q$ and $u_q = p$ where $p \neq q$, find u_{p+q}
   (c) Show that $u_1 + u_2 + \ldots + u_n = \frac{n}{2}[2a + (n - 1)d]$
   (d) Prove that $u_1 + u_n = u_2 + u_{n-1} = u_3 + u_{n-2} = \ldots$

Section B: Arithmetic Series

6. Calculate the sum of these arithmetic series:
   (a) $2 + 5 + 8 + 11 + \ldots$ (first 15 terms)
   (b) $20 + 17 + 14 + 11 + \ldots$ (first 12 terms)
   (c) $\frac{1}{2} + 1 + \frac{3}{2} + 2 + \ldots$ (first 25 terms)
   (d) The series with first term 7, last term 84, and 12 terms
7. An arithmetic series has first term 8 and common difference 3.
   (a) Find the sum of the first 20 terms
   (b) Find the smallest value of n for which $S_n > 1000$
   (c) If the sum of the first n terms is 440, find n
   (d) Express S_n in terms of n
8. The sum of the first n terms of an arithmetic series is $S_n = 2n^2 + 3n$.
   (a) Find the first term u_1
   (b) Find u_2 and u_3
   (c) Determine the common difference
   (d) Find the general term u_n
   (e) Verify using the formula $u_n = S_n - S_{n-1}$ for $n \geq 2$
9. Find the sum of:
   (a) All multiples of 7 between 50 and 500
   (b) All even integers from 2 to 100
   (c) All odd integers from 1 to 199
   (d) The integers from 1 to 100 that are divisible by 3 or 5
10. An arithmetic series has $S_{10} = 185$ and $S_{20} = 710$.
    (a) Find the first term and common difference
    (b) Calculate S_{30}
    (c) Find the 15th term
    (d) Determine when the sum first exceeds 2000

Section C: Geometric Sequences

11. For the geometric sequence $3, 6, 12, 24, 48, \ldots$:
    (a) Find the first term a and common ratio r
    (b) Find the general term u_n
    (c) Calculate u_{10}
    (d) Find which term equals 768
    (e) Determine if 1500 is a term in the sequence
12. A geometric sequence has $u_2 = 6$ and $u_5 = 162$.
    (a) Find the common ratio r
    (b) Find the first term a

(c) Write the general term u_n
(d) Calculate u_8
(e) Find the first term to exceed 10000

13. The nth term of a geometric sequence is $u_n = 5 \times 2^{n-1}$.
    (a) Write down the first five terms
    (b) Find the common ratio
    (c) Calculate u_{12}
    (d) Find the sum of the first 8 terms
    (e) For what value of n is $u_n = 1280$?

14. Three numbers $\frac{x}{r}$, x, and xr are in geometric progression with sum 26 and product 216.
    (a) Find the value of x
    (b) Set up an equation for r
    (c) Solve to find the values of r
    (d) Write down the three numbers for each case

15. A geometric sequence has first term a and common ratio r.
    (a) Prove that $u_n = ar^{n-1}$
    (b) If $u_p = x$ and $u_q = y$ where $p < q$, find u_{p+q} in terms of x and y
    (c) Show that $u_1 \cdot u_n = u_2 \cdot u_{n-1} = u_3 \cdot u_{n-2} = \dots$
    (d) Prove that if u_p, u_q, and u_s are in geometric progression, then p, q, and s are in arithmetic progression

Section D: Geometric Series

16. Calculate the sum of these geometric series:
    (a) $2 + 6 + 18 + 54 + \dots$ (first 8 terms)
    (b) $1 - 3 + 9 - 27 + \dots$ (first 10 terms)
    (c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ (first 12 terms)
    (d) $100 + 90 + 81 + 72.9 + \dots$ (first 15 terms)

17. A geometric series has first term 5 and common ratio $\frac{2}{3}$.
    (a) Find the sum of the first 10 terms
    (b) Find the smallest value of n for which $S_n > 14$
    (c) Calculate the sum to infinity
    (d) Find how many terms are needed for the sum to be within 0.01 of the sum to infinity

18. The sum of the first n terms of a geometric series is $S_n = 3(2^n - 1)$.
    (a) Find the first term u_1
    (b) Find u_2 and u_3
    (c) Determine the common ratio
    (d) Find the general term u_n
    (e) Verify using the formula $u_n = S_n - S_{n-1}$ for $n \geq 2$

19. Evaluate these infinite geometric series:

(a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

(b) $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \dots$

(c) $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \dots$

(d) $0.7 + 0.07 + 0.007 + 0.0007 + \dots$

20. A geometric series has $S_3 = 13$ and $S_6 = 364$.

    (a) Set up equations for the first term and common ratio
    (b) Solve to find a and r
    (c) Calculate S_{10}
    (d) Find the sum to infinity (if it exists)
    (e) Determine the first term to exceed 1000

Section E: Sigma Notation

21. Evaluate these sums:

    (a) $\sum_{r=1}^{10}(2r+3)$
    (b) $\sum_{r=1}^{20}(3r-1)$
    (c) $\sum_{r=1}^{15} r^2$
    (d) $\sum_{r=1}^{12}(r^2+2r)$

22. Express these series using sigma notation:

    (a) $5 + 8 + 11 + 14 + \dots + 32$
    (b) $2 + 6 + 18 + 54 + \dots + 4374$
    (c) $1^3 + 2^3 + 3^3 + \dots + 10^3$
    (d) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{90}$

23. Use the standard formulae to evaluate:

    (a) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$: Find $\sum_{r=1}^{50} r$
    (b) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$: Find $\sum_{r=1}^{20} r^2$
    (c) $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$: Find $\sum_{r=1}^{15} r^3$
    (d) $\sum_{r=1}^{30}(2r^2 - r + 3)$

24. Simplify these expressions:

    (a) $\sum_{r=1}^{n}(ar+b)$ in terms of a, b, and n
    (b) $\sum_{r=1}^{n}(r^2 - 2r + 1)$
    (c) $\sum_{r=1}^{n}(3r+1)^2$
    (d) $\sum_{r=1}^{n} r(r+1)$

25. Prove these results:

    (a) $\sum_{r=1}^{n}(2r-1) = n^2$
    (b) $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$
    (c) $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$
    (d) $\sum_{r=1}^{n}(r^3 - (r-1)^3) = n^3$

Section F: Binomial Expansion - Integer Powers

26. Expand using the binomial theorem:
    (a) $(x+2)^4$
    (b) $(2x-1)^5$
    (c) $(1-3x)^6$
    (d) $(x+\frac{1}{x})^4$
27. Find the specified terms in these expansions:
    (a) The coefficient of x^3 in $(2x+3)^7$
    (b) The coefficient of x^5 in $(x-2)^8$
    (c) The constant term in $(x^2+\frac{2}{x})^9$
    (d) The coefficient of x^{-2} in $(2x-\frac{1}{x^2})^{10}$
28. Use the binomial theorem to evaluate:
    (a) $(1.01)^5$ to 6 decimal places
    (b) $(0.99)^6$ to 5 decimal places
    (c) $(1.02)^4$ exactly
    (d) 101^4 by writing it as $(100+1)^4$
29. In the expansion of $(1+ax)^n$:
    (a) The coefficient of x is 12 and the coefficient of x^2 is 60. Find a and n.
    (b) Find the coefficient of x^3
    (c) Write out the first four terms of the expansion
    (d) For what values of x does the expansion converge?
30. The coefficient of x^r in the expansion of $(1+x)^n$ is $\binom{n}{r}$.
    (a) Show that $\binom{n}{r}=\binom{n}{n-r}$
    (b) Prove that $\binom{n}{r}+\binom{n}{r+1}=\binom{n+1}{r+1}$
    (c) Use this to construct Pascal's triangle up to row 6
    (d) Find the sum $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n}$

Section G: Binomial Expansion - Non-Integer Powers

31. Expand these expressions up to and including the term in x^3:
    (a) $(1+x)^{1/2}$
    (b) $(1-x)^{-1}$
    (c) $(1+2x)^{-2}$
    (d) $(1-3x)^{1/3}$
32. Find the first four terms in the expansion of:
    (a) $(4+x)^{1/2}$
    (b) $(9-x)^{-1/2}$
    (c) $\frac{1}{(1+x)^2}$
    (d) $\sqrt{1-4x}$

33. State the range of values of x for which these expansions are valid:
    (a) $(1+3x)^{-1} = 1 - 3x + 9x^2 - 27x^3 + \dots$
    (b) $(1-2x)^{1/2} = 1 - x - \frac{x^2}{2} - \frac{x^3}{2} - \dots$
    (c) $(2+x)^{-1} = \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$
    (d) $\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \dots$
34. Use binomial expansions to find approximations:
    (a) $\sqrt{1.01}$ to 5 decimal places
    (b) $\frac{1}{\sqrt{0.98}}$ to 4 decimal places
    (c) $(1.02)^{-2}$ to 6 decimal places
    (d) $\sqrt[3]{1.03}$ to 5 decimal places
35. Find the coefficient of x^2 in the expansion of:
    (a) $(1+x)^{1/2}(1-x)^{1/2}$
    (b) $(1+2x)^{-1}(1+x)^2$
    (c) $\frac{1+x}{\sqrt{1-x}}$
    (d) $(1+x+x^2)(1-x)^{-2}$

Section H: Mixed Series and Advanced Topics

36. A sequence is defined by $u_1 = 2$ and $u_{n+1} = 3u_n + 1$ for $n \geq 1$.
    (a) Find the first five terms
    (b) Prove by induction that $u_n = \frac{5\times3^{n-1}-1}{2}$
    (c) Calculate u_{10}
    (d) Find the sum of the first 8 terms
37. The sequence $\{v_n\}$ satisfies $v_n = 2v_{n-1} + 3v_{n-2}$ with $v_1 = 1$ and $v_2 = 5$.
    (a) Find the first six terms
    (b) Show that the characteristic equation is $r^2 - 2r - 3 = 0$
    (c) Solve to find $r = 3$ and $r = -1$
    (d) Use the general solution $v_n = A \cdot 3^n + B \cdot (-1)^n$ to find A and B
    (e) Write the explicit formula for v_n
38. Consider the series $\sum_{r=1}^{\infty} \frac{1}{r(r+1)}$.
    (a) Use partial fractions to show that $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$
    (b) Write out the first few terms and observe the telescoping pattern
    (c) Find the sum of the first n terms
    (d) Determine the sum to infinity
39. The Fibonacci sequence is defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$.
    (a) Write down the first 10 terms
    (b) Calculate the ratios $\frac{F_{n+1}}{F_n}$ for $n = 1, 2, 3, \dots, 9$
    (c) Show that these ratios approach the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$

(d) Prove that $F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}$ (Binet's formula)

40. A ball is dropped from a height of 10 meters. After each bounce, it reaches $\frac{3}{4}$ of its previous height.

    (a) Find the height after the 5th bounce
    (b) Calculate the total distance traveled when the ball comes to rest
    (c) Find the time taken for the ball to come to rest (use $t = \sqrt{\frac{2h}{g}}$ with $g = 10$ m/s^2)
    (d) Model the heights as a geometric sequence and find its sum

Section I: Applications and Problem Solving

41. A loan of £10,000 is taken out at 8% annual compound interest. Monthly payments of £150 are made.

    (a) Set up a recurrence relation for the amount owed after n months
    (b) Find the amount owed after 12 months
    (c) Determine how many months it takes to pay off the loan
    (d) Calculate the total amount paid

42. A bacteria culture doubles every 3 hours. Initially, there are 500 bacteria.

    (a) Model the population as a geometric sequence
    (b) Find the population after 24 hours
    (c) After how many hours will the population exceed 1 million?
    (d) If the growth rate changes to tripling every 4 hours after 12 hours, find the population after 20 hours

43. An infinite checkerboard pattern uses squares with side lengths forming a geometric sequence: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ cm.

    (a) Find the total area of all the squares
    (b) Find the total perimeter of all the squares
    (c) If each square costs £2 per cm^2 to make, find the total cost
    (d) What fraction of the total area is occupied by the first 5 squares?

44. A pharmaceutical company models drug concentration in the bloodstream. Initially, 100 mg is present. Every hour, 20% is eliminated, and 10 mg is added.

    (a) Set up a recurrence relation for the concentration after n hours
    (b) Find the concentration after 5 hours
    (c) Determine the long-term equilibrium concentration
    (d) After how many hours is the concentration within 1% of the equilibrium?

45. An investment strategy involves investing £1000 in the first year, £1100 in the second year, £1210 in the third year, and so on (increasing by 10% each year).

    (a) Model the annual investments as a geometric sequence
    (b) Find the total amount invested over 20 years
    (c) If each investment earns 5% annual compound interest from the time it's made, find the total value after 20 years
    (d) Compare this with investing £1000 annually at 5% compound interest

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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