

GCSE Higher Mathematics

Practice Test 8: Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

(a) $10(x - 8) = 6x + 16$

(b) $\frac{x+15}{7} - \frac{x-13}{5} = 6$

(c) $\frac{9x-8}{18} = \frac{6x+7}{12} - 4$

(d) $1.1x + 2.2 = 0.9x - 4.8$

2. Solve these simultaneous equations by elimination:

(a)
$$\begin{cases} 9x + 8y = 52 \\ 7x - 6y = 13 \end{cases}$$

(b)
$$\begin{cases} 6x + 13y = 49 \\ 10x - 7y = 28 \end{cases}$$

(c)
$$\begin{cases} 9x + 6y = 39 \\ 7x - 10y = 25 \end{cases}$$

(d)
$$\begin{cases} 11x + 10y = 71 \\ 6x - 8y = 16 \end{cases}$$

3. Solve these simultaneous equations by substitution:

(a)
$$\begin{cases} y = 9x - 15 \\ 8x + 6y = 54 \end{cases}$$

(b)
$$\begin{cases} x = 8y - 12 \\ 9x - 7y = 66 \end{cases}$$

(c)
$$\begin{cases} y = 19 - 8x \\ 7x + 10y = 91 \end{cases}$$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

(a) $y = 9x - 7$ and $y = 20 - 6x$

(b) $y = x^2 - 15$ and $y = 7x + 5$

(c) $x^2 + y^2 = 41$ and $y = 7x - 5$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

- (a) $x^2 + 21x + 54$
- (b) $x^2 - 19x - 42$
- (c) $x^2 - 23x + 112$
- (d) $x^2 + 9x - 36$
- (e) $x^2 - 324$
- (f) $x^2 - 36x + 324$

6. Solve these quadratic equations by factorizing:

- (a) $x^2 + 21x + 98 = 0$
- (b) $x^2 - 20x - 21 = 0$
- (c) $x^2 - 17x = 0$
- (d) $x^2 - 361 = 0$
- (e) $x^2 + 38x + 361 = 0$
- (f) $9x^2 - 36x = 0$

7. Factorize these harder quadratics:

- (a) $7x^2 + 15x + 8$
- (b) $9x^2 - 23x + 10$
- (c) $100x^2 - 121$
- (d) $8x^2 + 19x - 36$
- (e) $100x^2 - 180x + 81$
- (f) $12x^2 - 31x - 15$

8. Solve by factorizing:

- (a) $9x^2 + 19x - 10 = 0$
- (b) $10x^2 - 21x + 8 = 0$
- (c) $81x^2 - 64 = 0$
- (d) $7x^2 + 18x - 16 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

- (a) $x^2 + 20x + 19$
- (b) $x^2 - 24x + 11$
- (c) $x^2 + 16x - 13$
- (d) $x^2 - 24x + 21$
- (e) $9x^2 + 36x + 17$
- (f) $5x^2 - 40x + 19$

10. Solve by completing the square:

- (a) $x^2 + 20x + 17 = 0$
- (b) $x^2 - 18x - 11 = 0$

(c) $x^2 + 14x - 9 = 0$

(d) $9x^2 + 18x - 11 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 17x - 8 = 0$

(b) $9x^2 - 19x + 8 = 0$

(c) $x^2 - 20x + 15 = 0$

(d) $8x^2 + 15x - 9 = 0$

(e) $7x^2 + 15x + 6 = 0$

(f) $11x^2 - 17x - 6 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 19x + 48 = 0$

(b) $x^2 - 18x + 81 = 0$

(c) $x^2 + 9x + 21 = 0$

(d) $8x^2 - 17x + 9 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 18x + 65$:

- (a) Find the y-intercept
- (b) Find the x-intercepts by factorizing
- (c) Complete the square to find the vertex
- (d) Sketch the graph
- (e) State the line of symmetry

14. For the quadratic $y = 8x^2 + 24x - 11$:

- (a) Complete the square
- (b) Find the coordinates of the vertex
- (c) Find the y-intercept
- (d) State the line of symmetry
- (e) Sketch the graph

15. A cricket ball is hit upward. Its height h (in meters) after t seconds is given by: $h = -5t^2 + 70t + 9$

- (a) What is the initial height?
- (b) At what times is the cricket ball at ground level?
- (c) What is the maximum height reached?
- (d) At what time does it reach maximum height?

16. The cost C (in thousands of pounds) for organizing x thousand events is: $C = 6x^2 - 30x + 48$

- (a) How many events should be organized to minimize cost?
- (b) What is the minimum cost?
- (c) At what event levels is the cost £10,500?

Section E: Linear Inequalities

17. Solve these linear inequalities:

- (a) $10x + 19 > 59$
- (b) $9x - 21 \leq 42$
- (c) $13 - 10x < 3$
- (d) $\frac{x+13}{7} \geq 2$
- (e) $11 - 10x > 9x + 30$
- (f) $\frac{9x-8}{10} < \frac{x+11}{7}$

18. Show these inequalities on number lines:

- (a) $x > -9$
- (b) $x \leq 11$
- (c) $-6 < x \leq 12$
- (d) $x < 8$ or $x > 13$

19. Solve these compound inequalities:

- (a) $-11 < 9x + 7 < 25$
- (b) $17 \leq 8x - 7 \leq 41$
- (c) $-6 \leq \frac{9x+7}{7} < 13$

20. Find the integer solutions to:

- (a) $9x + 15 > 33$ and $x < 12$
- (b) $-7 \leq x + 5 < 11$
- (c) $x^2 < 121$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

- (a) $x^2 - 19x + 48 > 0$
- (b) $x^2 - 20x + 51 \leq 0$
- (c) $x^2 - 225 < 0$
- (d) $x^2 + 9x - 22 \geq 0$
- (e) $x^2 - 20x + 100 > 0$
- (f) $7x^2 - 8x - 12 < 0$

22. Solve and show on number lines:

- (a) $x^2 - 256 \leq 0$
- (b) $x^2 + 8x - 20 > 0$
- (c) $6x^2 - 13x + 5 \geq 0$

23. Find the values of x for which:

- (a) $x^2 < 10x + 11$
- (b) $7x^2 + 15x \geq 18$
- (c) $x^2 + 18x + 81 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

(a) $(x + 12)(9x - 7)$

(b) $(8x - 9)(x + 13)$

(c) $(7x + 12)^2$

(d) $(11x - 10)^2$

(e) $(x + 19)(x - 19)$

(f) $(10x + 9)(10x - 9)$

25. Expand these expressions:

(a) $(x + 9)(x^2 - 8x + 10)$

(b) $(7x - 8)(x^2 + 7x - 6)$

(c) $(x + 8)^3$

(d) $(8x - 6)^3$

26. Factorize completely:

(a) $20x^2 + 30x$

(b) $81x^2 - 289$

(c) $x^3 - 64x$

(d) $9x^3 + 36x^2 + 36x$

(e) $x^3 - 1000$

(f) $729x^3 + 512$

27. Simplify these algebraic fractions:

(a) $\frac{x^2-81}{x+9}$

(b) $\frac{x^2+19x+90}{x+10}$

(c) $\frac{10x^2-40}{x^2-4}$

(d) $\frac{x^3-1000}{x^2-100}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

(a) $\frac{10}{x} + \frac{7}{x}$

(b) $\frac{14}{x} - \frac{9}{x}$

(c) $\frac{7}{10x} + \frac{1}{20x}$

(d) $\frac{9}{x+7} + \frac{6}{x-8}$

(e) $\frac{x}{x+9} - \frac{8}{x-6}$

(f) $\frac{9x}{x^2-64} + \frac{8}{x+8}$

29. Multiply and divide these algebraic fractions:

(a) $\frac{x}{10} \times \frac{30}{x^2}$

(b) $\frac{x+9}{11} \times \frac{22}{x+9}$

(c) $\frac{x^2-81}{x+8} \div \frac{x-9}{x+8}$

(d) $\frac{9x+27}{x^2-100} \times \frac{x-10}{18}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{10} + \frac{x}{8} = 36$

(b) $\frac{9x+8}{11} = \frac{x-7}{8}$

(c) $\frac{10}{x} = \frac{9}{x-8}$

(d) $\frac{x+8}{x-9} = \frac{7x}{x+8}$

31. Make the subject of these formulae:

(a) $W = \frac{mg}{k}$, make g the subject

(b) $S = \frac{ut + \frac{1}{2}at^2}{t}$, make a the subject

(c) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, make v the subject

(d) $T = 2\pi\sqrt{\frac{L}{g}}$, make g the subject

Section I: Sequences

32. Find the n th term for these arithmetic sequences:

(a) 21, 27, 33, 39, ...

(b) 12, 19, 26, 33, ...

(c) 60, 56, 52, 48, ...

(d) $\frac{3}{7}, \frac{5}{7}, \frac{7}{7}, \frac{9}{7}, \dots$

33. For these geometric sequences, find the n th term:

(a) 9, 27, 81, 243, ...

(b) 12, 48, 192, 768, ...

(c) 729, 243, 81, 27, ...

(d) 8, -32, 128, -512, ...

34. Find the sum of these series:

(a) First 55 terms of $13 + 17 + 21 + 25 + \dots$

(b) First 2 terms of $11 + 33 + 99 + 297 + \dots$

(c) $8 + 12 + 16 + \dots + 104$ (arithmetic series)

(d) $10 + 20 + 40 + \dots + 640$ (geometric series)

35. These are quadratic sequences. Find the n th term:

(a) 9, 36, 81, 144, 225, ...

(b) 8, 21, 40, 65, 96, ...

(c) 7, 28, 63, 112, 175, ...

(d) 11, 32, 59, 92, 131, ...

36. A sequence is defined by $u_1 = 9$ and $u_{n+1} = 7u_n - 8$.

(a) Find the first 5 terms

(b) Find a formula for u_n

(c) Calculate u_2

Section J: Problem Solving

37. The sum of two numbers is 42 and their product is 432. Find the two numbers.
38. A rectangular hall has perimeter 96m. If the length is 20m more than the width, find the dimensions.
39. The difference between a positive number and its reciprocal is $\frac{99}{10}$. Find the number.
40. A badminton shuttlecock's height h (in meters) after t seconds is given by: $h = 200t - 5t^2$
- (a) When does it hit the ground?
 - (b) What is its maximum height?
 - (c) When is it 1875m high?
41. Prove that the sum of the first n terms of the sequence 6, 12, 18, 24, ... is $3n(n+1)$.
42. The quadratic $px^2 + qx + r = 0$ has roots λ and κ .
- (a) Show that $\lambda + \kappa = -\frac{q}{p}$
 - (b) Show that $\lambda\kappa = \frac{r}{p}$
 - (c) If the roots are 10 and -7, find p , q , and r when $p = 7$
43. A function is defined as $p(x) = x^2 + kx + m$. If $p(6) = 35$ and $p(8) = 57$, find k and m .
44. The sum of the first n terms of a sequence is $S_n = 9n^2 - 6n$. Find the n th term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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