GCSE Higher Mathematics
Practice Test 2: Probability

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Conditional Probability Fundamentals

1. A survey of 180 students shows:

   - 110 study Chemistry
   - 85 study Biology
   - 55 study both Chemistry and Biology

   (a) Draw a Venn diagram
   (b) Find P(studies Chemistry — studies Biology)
   (c) Find P(studies Biology — studies Chemistry)
   (d) Find P(studies exactly one subject)
   (e) Are studying Chemistry and Biology independent? Justify your answer

2. A box contains 6 green balls and 14 yellow balls. Two balls are drawn without replacement.

   (a) Find P(second ball is green — first ball is green)
   (b) Find P(second ball is yellow — first ball is green)
   (c) Find P(both balls are the same color)
   (d) Find P(balls are different colors)
   (e) Verify that your probabilities sum to 1

3. Events C and D are such that:

   - P(C) = 0.7
   - P(D) = 0.3
   - P(C D) = 0.18

   Calculate:

   (a) P(C D)
   (b) P(C')
   (c) P(C — D)
   (d) P(D — C)

(e) P(C' — D')
(f) P(C — D')

4. A card is drawn from a standard 52-card deck. Let C = ”card is black” and D = ”card is an Ace”.
   (a) Find P(C), P(D), and P(C D)
   (b) Calculate P(C — D)
   (c) Calculate P(D — C)
   (d) Are events C and D independent? Show your working
   (e) Find P(C' D')

Section B: Tree Diagrams and Sequential Events

5. A bag contains 7 green counters and 4 purple counters. A counter is drawn, its color noted, and then replaced. This process is repeated twice more.
   (a) Draw a tree diagram for all three draws
   (b) Find P(all three counters are green)
   (c) Find P(exactly two counters are purple)
   (d) Find P(at least one counter is green)
   (e) Find P(first counter is purple — exactly two are purple)
6. Box X contains 3 red balls and 7 white balls. Box Y contains 8 red balls and 2 white balls. A fair coin is flipped to choose a box, then a ball is drawn.
   (a) Draw a tree diagram
   (b) Find P(red ball)
   (c) Find P(white ball)
   (d) Given a red ball was drawn, find P(it came from Box X)
   (e) Given a white ball was drawn, find P(it came from Box Y)
7. Three machines produce components with different defect rates:
   - Machine P: produces 40% of components, 4% defective
   - Machine Q: produces 35% of components, 6% defective
   - Machine R: produces 25% of components, 9% defective

   (a) Find the overall probability of a defective component
   (b) If a component is defective, find the probability it came from Machine P
   (c) If a component is defective, find the probability it came from Machine R
   (d) If a component is not defective, which machine most likely produced it?
8. A student takes three tests. The probability of passing each test is 0.75, and the tests are independent.
   (a) Find P(passes all three tests)
   (b) Find P(fails all three tests)
   (c) Find P(passes exactly two tests)
   (d) Find P(passes at least one test)
   (e) Given the student passed at least two tests, find P(passed all three)

Section C: Bayes' Theorem Applications

9. A medical test for a condition has the following characteristics:
   - If a person has the condition, the test is positive 92% of the time
   - If a person doesn't have the condition, the test is negative 96% of the time
   - 0.8% of the population has the condition

   (a) Find P(positive test)
   (b) If someone tests positive, find P(they have the condition)
   (c) If someone tests negative, find P(they don't have the condition)
   (d) Comment on the reliability of a positive test result
   (e) How would the results change if 8% of the population had the condition?

10. A security system has three sensors. The probability each sensor detects an intruder is:
    - Sensor X: 0.88
    - Sensor Y: 0.92
    - Sensor Z: 0.87

    The sensors operate independently.

    (a) Find P(all three sensors detect an intruder)
    (b) Find P(at least one sensor detects an intruder)
    (c) Find P(exactly two sensors detect an intruder)
    (d) If exactly two sensors detect an intruder, find P(Sensor X failed)
    (e) Which single sensor is most reliable for detection?

11. A factory produces items using two processes. Process X is used 65% of the time and produces 3% defective items. Process Y is used 35% of the time and produces 8% defective items.

    (a) A random item is selected and found to be defective. Use Bayes' theorem to find P(produced by Process X)
    (b) If 1000 items are produced, how many would you expect to be defective?
    (c) How many of the defective items would come from each process?
    (d) To reduce overall defect rate to 2.5%, what should Process Y's defect rate be?

12. Three weather forecasting models predict rain independently:
    - Model X: 78% accurate when it will rain, 88% accurate when it won't rain
    - Model Y: 82% accurate when it will rain, 85% accurate when it won't rain
    - Model Z: 77% accurate when it will rain, 91% accurate when it won't rain

    Historically, it rains 25% of days.

    (a) If all three models predict rain, find P(it actually rains)
    (b) If Model X predicts rain but Models Y and Z predict no rain, find P(it rains)
    (c) Which model would you trust most for a "rain" prediction?
    (d) Which model would you trust most for a "no rain" prediction?

Section D: Introduction to Binomial Distribution

13. A fair coin is flipped 10 times.
    (a) Find P(exactly 6 heads)
    (b) Find P(at most 3 heads)
    (c) Find P(at least 7 heads)
    (d) Find the expected number of heads
    (e) Find the most likely number of heads
    (f) Calculate the variance of the number of heads

14. A multiple choice test has 15 questions, each with 4 possible answers. A student guesses randomly on all questions.
    (a) State the distribution of the number of correct answers
    (b) Find P(exactly 4 correct answers)
    (c) Find P(more than 5 correct answers)
    (d) Find the expected number of correct answers
    (e) Find P(passes the test) if the pass mark is 60%
    (f) Calculate the standard deviation of correct answers

15. The probability that a seed germinates is 0.85. A packet contains 12 seeds.
    (a) Find P(all seeds germinate)
    (b) Find P(exactly 10 seeds germinate)
    (c) Find P(fewer than 8 seeds germinate)
    (d) How many seeds would you expect to germinate?
    (e) Find P(at least 75% of seeds germinate)
    (f) What's the most likely number of seeds to germinate?

16. A manufacturing process produces 6% defective items. Quality control samples 18 items.
    (a) Find P(no defective items in the sample)
    (b) Find P(exactly 1 defective item)
    (c) Find P(more than 2 defective items)
    (d) Calculate the expected number of defective items
    (e) Find P(defect rate in sample exceeds 15%)
    (f) Calculate the probability that the sample defect rate is between 3% and 9%

Section E: Advanced Binomial Applications

17. A basketball player has a 70% free throw success rate. In a game, they attempt 20 free throws.
    (a) Model this situation and state any assumptions
    (b) Find P(makes at least 15 free throws)
    (c) Find P(makes between 12 and 16 free throws inclusive)
    (d) Calculate the expected number of successful free throws
    (e) Find the probability their success rate in this game is above 85%
    (f) What's the minimum number of attempts needed for P(at least 1 success) 0.999?

18. A quality control inspector checks 30 items per hour. The probability any item is defective is 0.07.
    (a) Find P(finds exactly 2 defective items in one hour)
    (b) Find P(finds no defective items in one hour)
    (c) Over a 10-hour shift, find the expected number of defective items found
    (d) In what percentage of hours would you expect to find more than 3 defective items?
    (e) If the inspector finds 5 defective items in one hour, comment on whether this is unusual

19. A pharmaceutical company claims their drug is effective for 85% of patients. A trial involves 40 patients.
    (a) If the claim is true, find P(drug works for exactly 35 patients)
    (b) Find P(drug works for at least 30 patients)
    (c) Calculate the expected number of patients for whom the drug works
    (d) If the drug works for only 28 patients, test whether this supports the company's claim
    (e) What's the minimum number of successes that would support the 85% claim at 5% significance?

20. A survey shows 42% of people support a proposal. A random sample of 25 people is surveyed.
    (a) Find P(exactly 12 people support the proposal)
    (b) Find P(fewer than 8 people support the proposal)
    (c) Calculate the expected number of supporters
    (d) Find P(between 30% and 50% of the sample support the proposal)
    (e) If 16 people in the sample support the proposal, is this significantly different from expected?

Section F: Combined Probability Scenarios

21. An online retailer has two suppliers. Supplier X provides 60% of goods with 3% defect rate. Supplier Y provides 40% of goods with 7% defect rate.
    (a) A customer receives 12 items. Find P(exactly 1 is defective)
    (b) If a customer complains about a defective item, find P(it came from Supplier Y)
    (c) A batch of 150 items arrives. Find the expected number from each supplier
    (d) Calculate the overall defect rate
    (e) If the company wants to reduce defects to 2.5%, what should Supplier Y's rate be?

22. A casino game involves drawing 4 cards from a standard deck without replacement. The player wins if all 4 cards are hearts.
    (a) Calculate P(all 4 cards are hearts)
    (b) Calculate P(all 4 cards are the same suit)
    (c) If 1200 people play this game, how many would you expect to win?
    (d) What should be the payout ratio for this to be a fair game?
    (e) How does the probability change if cards are replaced after each draw?

23. A communication system sends signals through 4 independent channels. Each channel has probability 0.85 of successful transmission.
    (a) Find P(message received successfully through all channels)

(b) Find P(message fails on exactly one channel)

(c) The system works if at least 3 channels succeed. Find P(system works)

(d) If the system sends 60 messages, find P(fewer than 50 are received successfully)

(e) What should be the individual channel reliability for 99.5% system reliability?

24. A hospital emergency department sees an average of 12% critical cases. On a particular shift, 20 patients arrive.

    (a) Model the number of critical cases and state assumptions

    (b) Find P(exactly 3 critical cases)

    (c) Find P(no critical cases)

    (d) Find P(more than 5 critical cases)

    (e) Calculate the expected number of critical cases

    (f) If there are 7 critical cases in one shift, is this unusually high?

Section G: Advanced Problem Solving

25. A genetic disorder affects 1 in 800 births. A screening test is 96% accurate for positive cases and 99.2% accurate for negative cases.

    (a) Calculate the probability of testing positive

    (b) If a baby tests positive, what's the probability they have the disorder?

    (c) How many false positives occur per 80,000 births?

    (d) Design a two-stage testing procedure to reduce false positives

    (e) Comment on the ethical implications of these probabilities

26. A software company releases updates with bugs 18% of the time. They use a testing protocol that catches 85% of buggy updates but also flags 6% of good updates as potentially buggy.

    (a) If an update is flagged, find P(it actually has bugs)

    (b) If an update passes testing, find P(it's actually bug-free)

    (c) In 200 updates, how many false alarms would you expect?

    (d) Suggest improvements to the testing protocol

    (e) Calculate the overall accuracy of the testing system

27. A lottery has the following structure: pick 5 numbers from 1-45. You win the jackpot if all 5 match.

    (a) Calculate P(winning the jackpot)

    (b) Find P(matching exactly 4 numbers)

    (c) Find P(matching exactly 3 numbers)

    (d) If 8 million tickets are sold, find P(no one wins the jackpot)

    (e) Model the number of jackpot winners as a binomial distribution

28. A cybersecurity system monitors network traffic. It correctly identifies 93% of malicious attacks and incorrectly flags 3% of normal traffic. On average, 0.2% of traffic is malicious.

    (a) Find the probability of an alert

    (b) If there's an alert, find P(it's a real attack)

    (c) In monitoring 500,000 data packets, how many false alarms occur?

(d) Design a cost-benefit analysis for this system

(e) How would increasing the detection rate to 97% affect false alarms?

29. Design and analyze a probability model for a real-world scenario of your choice:

(a) Define the scenario and identify random variables

(b) State all assumptions clearly

(c) Choose appropriate probability distributions

(d) Calculate relevant probabilities

(e) Discuss limitations and potential improvements

(f) Consider practical applications of your analysis

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

For more resources and practice materials, visit:
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