A Level Mechanics
Practice Test 5: Statics and Dynamics of Particles

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw clear diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Complex Equilibrium Problems [45 marks]

Question 1 [20 marks] A particle is held in equilibrium by six forces: 35 N due north, 28 N at 30° east of north, 42 N due east, 25 N at 45° south of east, 38 N due south, and an unknown force F.

(a) Set up a suitable coordinate system and resolve each of the five known forces into components. **[8 marks]**

(b) Calculate the resultant of all known forces in component form. **[3 marks]**

(c) Determine the magnitude and direction of force F required for equilibrium. **[6 marks]**

(d) Verify that the sum of all six forces equals zero. **[2 marks]**

(e) Draw a vector polygon to represent this equilibrium graphically. **[1 mark]**

Question 2 [25 marks] A heavy chandelier of mass 120 kg is suspended from the ceiling by four cables. Cable A is vertical, cables B and C each make an angle of 25° with the vertical in opposite directions, and cable D makes an angle of 40° with the vertical. All cables meet at a single point below the chandelier.

(a) Draw a detailed force diagram showing all forces at the junction point. **[4 marks]**

(b) If the tension in cable A is 500 N, write equilibrium equations for the horizontal and vertical directions. **[4 marks]**

(c) Given that cables B and C have equal tensions due to symmetry, calculate:

  (i) The tension in cables B and C. **[6 marks]**

  (ii) The tension in cable D. **[5 marks]**

(d) Calculate the total downward force supported by all cables and verify this equals the chandelier's weight. **[3 marks]**

(e) Analyze what happens if cable D fails - will the remaining cables support the chandelier? **[3 marks]**

Section B: Advanced Friction and Motion [50 marks]

Question 3 [25 marks] A crate of mass 35 kg rests on a loading ramp inclined at 18° to the horizontal. The coefficient of static friction is 0.65 and the coefficient of kinetic friction is 0.50. A winch applies a force P parallel to the ramp.

(a) Calculate the component of the crate's weight parallel to the ramp. **[3 marks]**

(b) Find the normal reaction force between the crate and ramp. **[3 marks]**

(c) Without the winch force, determine whether the crate will slide down the ramp. **[4 marks]**

(d) Calculate the minimum winch force P (up the ramp) required to start the crate moving up. **[5 marks]**

(e) If P = 200 N up the ramp, calculate the acceleration of the crate. **[5 marks]**

(f) Find the range of winch force P that will keep the crate stationary on the ramp. **[5 marks]**

Question 4 [25 marks] A race car of mass 800 kg travels around a banked circular track. The track is banked at an angle of 15° to the horizontal, and the coefficient of friction between tires and track is 0.8.

(a) Draw a force diagram for the car showing all forces when traveling at constant speed. **[4 marks]**

(b) For motion at constant speed without relying on friction, derive an expression for the speed in terms of the banking angle and track radius. **[6 marks]**

(c) If the track radius is 120 m, calculate the speed at which the car can travel without friction. **[3 marks]**

(d) At this critical speed, what is the normal reaction force from the track? **[4 marks]**

(e) If the car travels at 25 m/s, determine:

(i) Whether friction acts up or down the banking. **[4 marks]**

(ii) The magnitude of the friction force required. **[4 marks]**

Section C: Multi-Particle Dynamic Systems [55 marks]

Question 5 [30 marks] Four particles A, B, C, and D have masses 6 kg, 8 kg, 10 kg, and 4 kg respectively. Particle A rests on a smooth horizontal table. It is connected by a light string over a smooth pulley to particle B, which hangs freely. Particle A is also connected by another string over a second pulley to particle C, which rests on a rough inclined plane at 30° (coefficient of friction 0.3). A third string connects particle C over a third pulley to particle D, which hangs freely.

(a) Draw separate force diagrams for all four particles. **[8 marks]**

(b) For particle C on the inclined plane, calculate:

(i) The component of weight down the plane. **[2 marks]**

(ii) The normal reaction force. **[2 marks]**

(iii) The maximum friction force available. **[2 marks]**

(c) Analyze the forces to determine the direction of motion of the system. **[4 marks]**

(d) Write equations of motion for all four particles. **[6 marks]**

(e) Solve these equations to find the acceleration of the system. **[4 marks]**

(f) Calculate the tensions in all three connecting strings. **[2 marks]**

Question 6 [25 marks] Three particles P, Q, and R are connected by light inextensible strings. Particle P (mass 9 kg) rests on a rough horizontal surface with coefficient of kinetic friction 0.4. It is connected to particle Q (mass 5 kg), which rests on a smooth inclined plane at 25° to the horizontal. Particle Q is also connected to particle R (mass 7 kg), which hangs vertically. The strings pass over smooth pulleys.

(a) Draw force diagrams for all three particles. **[6 marks]**

(b) Calculate the component of Q's weight acting down the inclined plane. **[3 marks]**

(c) Write the equation of motion for each particle. **[6 marks]**

(d) Determine which direction the system moves and solve for the acceleration. **[8 marks]**

(e) Find the tensions in both connecting strings. **[2 marks]**

Section D: Variable Forces and Resistance [35 marks]

Question 7 [20 marks] A particle of mass 2.5 kg moves along a straight line under the action of a variable driving force $F = (30 + 8t - t^2)$ N, where t is time in seconds. The particle also experiences a resistance force of magnitude 10 N opposing its motion.

(a) Write an expression for the net force acting on the particle. **[2 marks]**

(b) Calculate the acceleration at $t = 0$, $t = 4$ s, and $t = 8$ s. **[6 marks]**

(c) If the particle starts from rest, derive expressions for velocity and displacement as functions of time. **[8 marks]**

(d) Find the velocity and displacement when $t = 6$ s. **[2 marks]**

(e) Determine when the particle comes to instantaneous rest. **[2 marks]**

Question 8 [15 marks] A parachutist of mass 75 kg falls through air. The air resistance force is given by $R = 0.5v^2$ N, where v is the velocity in m/s.

(a) Write the equation of motion for the parachutist during free fall. **[3 marks]**

(b) Calculate the terminal velocity of the parachutist. **[4 marks]**

(c) When the parachutist has reached 75

(i) The velocity at this instant. **[2 marks]**

(ii) The acceleration at this instant. **[3 marks]**

(iii) The air resistance force at this instant. **[3 marks]**

Section E: Advanced Projectile Motion [40 marks]

Question 9 [25 marks] A projectile is launched from the top of a 100 m tall building at an angle of 35° above the horizontal with initial speed 40 m/s. The target is located 180 m horizontally from the base of the building.

(a) Calculate the initial horizontal and vertical velocity components. **[3 marks]**

(b) Find the maximum height above the ground reached by the projectile. **[5 marks]**

(c) Calculate the time taken for the projectile to travel 180 m horizontally. **[4 marks]**

(d) Determine the height of the projectile when it reaches the target's horizontal position. **[4 marks]**

(e) Calculate the velocity components when the projectile is at the target location. **[4 marks]**

(f) Find the angle at which the projectile approaches the target. **[3 marks]**

(g) Determine whether the projectile hits the target (at ground level) or passes above it. **[2 marks]**

Question 10 [15 marks] A golf ball is struck from an elevated tee 25 m above the fairway. The ball is hit at an angle of 28° above the horizontal with initial speed 35 m/s toward a flag 200 m away horizontally.

(a) Calculate the time of flight until the ball reaches fairway level. **[5 marks]**

(b) Find the horizontal distance traveled when the ball hits the fairway. **[3 marks]**

(c) Determine whether the ball reaches the flag or falls short. **[2 marks]**

(d) Calculate the velocity components just before the ball hits the fairway. **[3 marks]**

(e) Find the speed and angle of impact with the fairway. **[2 marks]**

Section F: Equilibrium Analysis and Optimization [25 marks]

Question 11 [15 marks] A particle of mass 8 kg is held in equilibrium on a rough inclined plane (angle 35° to horizontal, coefficient of friction 0.6) by a force P acting at angle to the horizontal.

(a) Draw a force diagram showing all forces. **[3 marks]**

(b) Derive general expressions for the normal reaction and friction force in terms of P, , and other given quantities. **[6 marks]**

(c) If = 15°, calculate the force P required for equilibrium when the particle is on the verge of sliding up the plane. **[4 marks]**

(d) Determine the minimum value of P required for any angle , and find the corresponding angle. **[2 marks]**

Question 12 [10 marks] A load of mass 50 kg is supported by three cables meeting at a point. Two cables make angles of 30° and 45° with the vertical, while the third cable makes an angle with the vertical.

(a) If all three cables have equal tension T, find the value of angle . **[6 marks]**

(b) For this configuration, calculate the tension T in each cable. **[2 marks]**

(c) Verify that the equilibrium conditions are satisfied. **[2 marks]**

Physics Data and Formulae

Forces and Equilibrium:
Equilibrium condition: $\sum \vec{F} = 0$
Component form: $\sum F_x = 0$, $\sum F_y = 0$
Force resolution: $F_x = F\cos\theta$, $F_y = F\sin\theta$

Resultant magnitude: $|\vec{R}| = \sqrt{F_x^2 + F_y^2}$
Resultant direction: $\tan\alpha = \frac{F_y}{F_x}$

Newton's Laws:
First Law: $\sum \vec{F} = 0$ (equilibrium)
Second Law: $\sum \vec{F} = m\vec{a}$
Third Law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Friction:
Static friction: $f_s \leq \mu_s N$, maximum $f_{s,max} = \mu_s N$
Kinetic friction: $f_k = \mu_k N$
Limiting equilibrium: $f = \mu_s N$

Inclined Planes:
Weight parallel to plane: $mg\sin\alpha$ (down plane)
Weight perpendicular to plane: $mg\cos\alpha$ (into plane)
Normal reaction: $N = mg\cos\alpha + F\sin\theta$ (where F is applied force)

Circular Motion:
Centripetal force: $F_c = \frac{mv^2}{r}$
Banking without friction: $\tan\theta = \frac{v^2}{rg}$

Kinematics:
$v = u + at$
$s = ut + \frac{1}{2}at^2$
$v^2 = u^2 + 2as$
$s = \frac{u+v}{2}t$

Projectile Motion:
Horizontal: $x = u_x t$, $v_x = u_x$ (constant)
Vertical: $y = y_0 + u_y t - \frac{1}{2}gt^2$, $v_y = u_y - gt$
Maximum height: $h_{max} = y_0 + \frac{u_y^2}{2g}$
Time to max height: $t_{max} = \frac{u_y}{g}$

Calculus:
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
$v = \int a\,dt + C$
$s = \int v\,dt + C$

Constants:
Acceleration due to gravity: $g = 9.81$ m/s²

Trigonometric Values:
$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$
$\sin 18 = 0.309$, $\cos 18 = 0.951$, $\tan 18 = 0.325$
$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$
$\sin 28 = 0.469$, $\cos 28 = 0.883$, $\tan 28 = 0.532$
$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$
$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$
$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

END OF TEST

Total marks: 250

Grade boundaries: A* 225, A 200, B 175, C 150, D 125, E 100

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