A Level Mechanics
Practice Test 2: Rotational Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Rotational Motion Fundamentals [26 marks]

Question 1 [**8 marks**]

(a) Explain the concepts of angular displacement, angular velocity, and angular acceleration, giving their symbols and SI units. [**3 marks**]

(b) Describe the relationship between angular and linear quantities for a particle moving in a circle. [**3 marks**]

(c) State the rotational kinematic equations and compare them with their linear counterparts. [**2 marks**]

Question 2 [**14 marks**] A centrifuge rotor has angular velocity that varies according to $\omega(t) = 6t^2 - 10t + 4$ rad/s.

(a) Determine the angular acceleration as a function of time. [**2 marks**]

(b) Calculate the angular velocity at $t = 1.5$ seconds. [**2 marks**]

(c) Find the angular acceleration at $t = 2$ seconds. [**2 marks**]

(d) Determine when the rotor momentarily stops and changes direction. [**4 marks**]

(e) Calculate the total angular displacement between $t = 0$ and $t = 3$ seconds. [**4 marks**]

Question 3 [**4 marks**] A particle moves in a circle of radius 0.8 m with tangential speed 15 m/s and tangential acceleration 4 m/s^2.

(a) Calculate the angular velocity and angular acceleration. [**2 marks**]

(b) Find the magnitude of the total acceleration of the particle. [**2 marks**]

Section B: Moment of Inertia - Point Masses [30 marks]

Question 4 [**6 marks**]

(a) Define moment of inertia and explain its physical significance in rotational motion. [**3 marks**]

(b) State the parallel axis theorem and explain when it is most useful. [**3 marks**]

Question 5 [24 marks] Calculate moments of inertia for the following systems:

(a) Six point masses arranged as follows: 2 kg at (3,2), 4 kg at (-1,3), 3 kg at (2,-2), 1.5 kg at (-3,-1), 2.5 kg at (0,4), and 3.5 kg at (4,0) meters. Find the moment of inertia about the z-axis. **[6 marks]**

(b) Three masses of 8 kg each form an equilateral triangle with side length 1.8 m. Calculate the moment of inertia about an axis through one vertex perpendicular to the plane. **[5 marks]**

(c) Two point masses of 12 kg and 18 kg are connected by a massless rigid rod of length 2.4 m. Find the moment of inertia about an axis perpendicular to the rod passing through: (i) the center of mass, (ii) a point 0.8 m from the 12 kg mass. **[8 marks]**

(d) A thin ring of radius 0.5 m has four point masses of 3 kg each attached at positions separated by 90°. Calculate the moment of inertia about the central axis. **[3 marks]**

(e) Verify one of your calculations using the parallel axis theorem. **[2 marks]**

Section C: Moment of Inertia - Extended Bodies [34 marks]

Question 6 [10 marks]

(a) List the standard moment of inertia formulas for: thin rod about center, thin rod about end, solid disc, solid sphere, and thin ring. **[5 marks]**

(b) Derive the moment of inertia of a thin uniform ring of mass M and radius R about its central axis. **[3 marks]**

(c) Explain why different mass distributions with the same total mass and outer radius have different moments of inertia. **[2 marks]**

Question 7 [24 marks] Calculate moments of inertia for composite objects:

(a) A solid disc of mass 8 kg and radius 0.35 m has a concentric hole of radius 0.12 m removed. Find its moment of inertia about the central axis. **[5 marks]**

(b) A T-shaped object consists of a horizontal rod (mass 4 kg, length 2.4 m) with a vertical rod (mass 3 kg, length 1.6 m) attached perpendicularly at its center. Calculate the moment of inertia about the center of the horizontal rod perpendicular to both rods. **[6 marks]**

(c) A system consists of two solid spheres (each mass 6 kg, radius 0.18 m) connected by a uniform rod (mass 2.5 kg, length 1.4 m) with the sphere centers at the rod ends. Find the moment of inertia about an axis through the rod's center perpendicular to its length. **[7 marks]**

(d) A flywheel consists of a solid disc (mass 15 kg, radius 0.6 m) with eight uniform spokes extending radially. Each spoke has mass 1.2 kg and length 0.6 m. Calculate the total moment of inertia about the central axis. **[6 marks]**

Section D: Torque and Rotational Dynamics [28 marks]

Question 8 [8 marks]

(a) Define torque mathematically and physically, stating its SI unit. **[3 marks]**

(b) Explain the vector nature of torque and how its direction is determined. **[3 marks]**

(c) State and explain the rotational equivalent of Newton's second law. **[2 marks]**

Question 9 [20 marks] A solid disc of mass 10 kg and radius 0.5 m is mounted on a frictionless horizontal axle through its center.

(a) Calculate the moment of inertia of the disc. **[2 marks]**

(b) Two forces are applied: a 25 N force tangentially at the rim and a 15 N force at 30° to the radius at distance 0.3 m from the center. Find the net torque. **[5 marks]**

(c) Calculate the angular acceleration of the disc. **[2 marks]**

(d) Starting from rest, find the angular velocity after 6 seconds. **[2 marks]**

(e) Determine the number of revolutions completed in 6 seconds. **[3 marks]**

(f) Calculate the work done by the net torque during this time. **[3 marks]**

(g) Find the instantaneous power at t = 6 seconds. **[3 marks]**

Section E: Rotational Energy [32 marks]

Question 10 [10 marks]

(a) Derive the expression for rotational kinetic energy from first principles. **[4 marks]**

(b) Define work done by a torque and derive the work-energy theorem for rotational motion. **[4 marks]**

(c) Write expressions for power in rotational systems. **[2 marks]**

Question 11 [22 marks] A solid sphere of mass 20 kg and radius 0.4 m rolls without slipping down an inclined plane of angle 30°.

(a) Calculate the moment of inertia of the sphere about its center. **[2 marks]**

(b) State the rolling constraint and explain its significance. **[3 marks]**

(c) Use energy methods to find the linear acceleration of the sphere down the plane. **[6 marks]**

(d) Calculate the angular acceleration and compare with the linear acceleration. **[3 marks]**

(e) If the sphere starts from rest and rolls 5 m down the incline, find its final linear and angular velocities. **[4 marks]**

(f) Calculate the total kinetic energy at the bottom and verify using gravitational potential energy. **[2 marks]**

(g) What percentage of the total kinetic energy is rotational? **[2 marks]**

Section F: Angular Momentum [24 marks]

Question 12 [6 marks]

(a) Define angular momentum for a rigid body and state its SI unit. **[2 marks]**

(b) State the conservation of angular momentum principle and the conditions for its validity. **[4 marks]**

Question 13 [18 marks] A gymnast performs on a high bar, initially hanging with arms extended (moment of inertia 8.5 kg·m^2) and rotating at 1.5 rad/s.

(a) Calculate the initial angular momentum. **[2 marks]**

(b) During the routine, the gymnast pulls into a tucked position, reducing the moment of inertia to 2.8 kg·m^2. Find the new angular velocity. **[3 marks]**

(c) Calculate the change in rotational kinetic energy and explain its source. **[5 marks]**

(d) If the gymnast takes 0.8 seconds to change position, find the average torque exerted by internal muscular forces. **[4 marks]**

(e) When a 1.2 kg weight is thrown tangentially at 6 m/s and catches on the bar at 1.5 m from the axis while the gymnast is in the extended position, find the new angular velocity. **[4 marks]**

Section G: Conservation in Rotational Systems [26 marks]

Question 14 **[16 marks]** A playground roundabout (uniform disc, mass 250 kg, radius 2.2 m) rotates at 0.6 rad/s when two children jump on simultaneously: a 40 kg child at the edge and a 35 kg child at 1.5 m from the center.

(a) Calculate the initial angular momentum of the roundabout. **[3 marks]**

(b) Find the moment of inertia of the system after both children jump on. **[4 marks]**

(c) Determine the final angular velocity using conservation of angular momentum. **[3 marks]**

(d) Calculate the kinetic energy before and after, and explain the energy change. **[4 marks]**

(e) If the 40 kg child runs onto the roundabout tangentially at 4.5 m/s while the other child is stationary on it, find the final angular velocity. **[2 marks]**

Question 15 **[10 marks]** Two rotating discs on the same axis can be coupled together. Disc X (moment of inertia 1.2 kg·m^2) rotates at 8 rad/s, while disc Y (moment of inertia 0.9 kg·m^2) rotates at 3 rad/s in the same direction.

(a) Calculate the total angular momentum before coupling. **[2 marks]**

(b) Find the common angular velocity after coupling. **[3 marks]**

(c) Calculate the kinetic energy loss during coupling. **[3 marks]**

(d) Explain what happens to the lost energy. **[2 marks]**

Section H: Combined Motion [22 marks]

Question 16 **[14 marks]** A uniform solid cylinder of mass 5 kg and radius 0.12 m is used as a yo-yo with string wound around it.

(a) Calculate the moment of inertia about the central axis. **[2 marks]**

(b) Derive the linear acceleration of the yo-yo using Newton's laws and rotational dynamics. **[6 marks]**

(c) Calculate the tension in the string. **[3 marks]**

(d) Find the ratio of the yo-yo's acceleration to free fall acceleration. **[2 marks]**

(e) If the yo-yo is released from rest and falls 1.5 m, calculate its final linear and angular velocities. **[1 mark]**

Question 17 **[8 marks]** A hollow sphere of mass 3 kg and radius 0.2 m rolls up an inclined plane with initial speed 10 m/s.

(a) Calculate the initial total kinetic energy. **[3 marks]**

(b) Find the maximum height reached. **[3 marks]**

(c) Determine the speed when it returns to the starting point. **[2 marks]**

Section I: Complex Systems [26 marks]

Question 18 [**16 marks**] A compound pulley system has two wheels of different radii fixed together on the same axle. The larger wheel has radius 0.6 m and the smaller wheel has radius 0.35 m. The total moment of inertia is 4.5 kg·m^2. A 30 kg mass hangs from the larger wheel and a 15 kg mass hangs from the smaller wheel.

(a) Draw a clear free body diagram for the system. [**3 marks**]

(b) Write the equations of motion for the masses and the wheel system. [**5 marks**]

(c) Calculate the angular acceleration of the wheels. [**4 marks**]

(d) Find the tensions in both strings. [**4 marks**]

Question 19 [**10 marks**] A uniform rod of mass 5 kg and length 2.5 m is pivoted at a point 0.8 m from one end and released from rest when horizontal.

(a) Calculate the moment of inertia about the pivot point. [**3 marks**]

(b) Find the initial angular acceleration. [**4 marks**]

(c) Calculate the angular velocity when the rod passes through the vertical position. [**3 marks**]

Section J: Advanced Applications [24 marks]

Question 20 [**16 marks**] A communication satellite consists of a main body (solid cylinder, mass 600 kg, radius 1.2 m, height 2 m) with two solar panel arrays. Each panel (mass 80 kg, dimensions 4 m $\times$ 1.5 m) is attached 3 m from the satellite's central axis.

(a) Calculate the moment of inertia of the main body about its central axis. [**2 marks**]

(b) Find the moment of inertia of both solar panels about the satellite's central axis. [**4 marks**]

(c) Calculate the total moment of inertia of the satellite. [**2 marks**]

(d) If the satellite spins at 0.08 rad/s, find its angular momentum. [**2 marks**]

(e) Reaction wheels provide a total torque of 75 N·m for 40 seconds. Calculate the change in angular velocity. [**3 marks**]

(f) If one solar panel detaches completely, find the new angular velocity of the remaining system. [**3 marks**]

Question 21 [**8 marks**] A grinding wheel (solid disc, mass 25 kg, radius 0.4 m) rotates at 180 rpm. A 1.5 kg tool is pressed against the rim with a friction force of 80 N.

(a) Calculate the initial angular momentum of the wheel. [**2 marks**]

(b) Find the angular deceleration due to friction. [**3 marks**]

(c) Calculate how long it takes for the wheel to stop. [**3 marks**]

Physics Data and Formulae

Rotational Kinematics:

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $v = r\omega$, $a_t = r\alpha$, $a_c = \frac{v^2}{r} = \omega^2 r$

Moment of Inertia:

Point mass: $I = mr^2$

Parallel axis theorem: $I = I_{cm} + md^2$

Uniform rod (center): $I = \frac{1}{12}ML^2$, (end): $I = \frac{1}{3}ML^2$

Solid disc/cylinder: $I = \frac{1}{2}MR^2$, Thin ring: $I = MR^2$

Solid sphere: $I = \frac{2}{5}MR^2$, Hollow sphere: $I = \frac{2}{3}MR^2$

Rotational Dynamics:

$\tau = I\alpha$ (equation of rotational motion)

$\tau = \vec{r} \times \vec{F}$ (torque from force)

Work: $W = \tau\theta$, Power: $P = \tau\omega$

Rotational Energy:

$KE_{rot} = \frac{1}{2}I\omega^2$

Rolling motion: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling constraint: $v = r\omega$ (no slipping)

Angular Momentum:

$L = I\omega$ (rigid body), $L = \vec{r} \times \vec{p}$ (point particle)

Conservation: $L_i = L_f$ (when $\sum \tau_{ext} = 0$)

Constants:

Acceleration due to gravity: $g = 9.8$ m/s²

Trigonometric Values:

$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 272

Grade boundaries: A* 245, A 218, B 190, C 163, D 136, E 109

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