

GCSE Higher Mathematics

Practice Test 9: Geometry and Measures

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Pythagoras' Theorem

- Use Pythagoras' theorem to find the missing lengths:
 - A right-angled triangle with legs 24 cm and 32 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 85 cm and one leg 36 cm. Find the other leg.
 - A right-angled triangle with legs 21.5 cm and 28.5 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 89 cm and one leg 80 cm. Find the other leg.
- Find the distance between these pairs of coordinate points:
 - A(9, 15) and B(21, 30)
 - C(-7, 12) and D(5, 0)
 - E(-15, -11) and F(-3, 1)
 - G(0, 23) and H(-16, 16)
- A rectangle has length 42 cm and diagonal 58 cm. Find:
 - The width of the rectangle
 - The area of the rectangle
 - The perimeter of the rectangle
- A ladder 20.5 m long leans against a vertical wall. The foot of the ladder is 12.3 m from the wall.
 - How high up the wall does the ladder reach?
 - If the ladder slips so its foot is now 15 m from the wall, how far does the top slide down?
- Determine whether these triangles are right-angled:
 - Sides 20 cm, 99 cm, 101 cm
 - Sides 21 cm, 220 cm, 221 cm
 - Sides 26 cm, 35 cm, 43 cm
 - Sides 36 cm, 48 cm, 60 cm

Section B: Basic Trigonometry

6. In a right-angled triangle with hypotenuse 45 cm and one leg 27 cm:
 - (a) Find the other leg using Pythagoras
 - (b) Calculate $\sin A$, $\cos A$, and $\tan A$ where A is the angle opposite the 27 cm side
 - (c) Find angle A to 1 decimal place
7. Use trigonometry to find the missing sides (give answers to 3 significant figures):
 - (a) Right-angled triangle: angle 65° , adjacent side 21 cm, find opposite side
 - (b) Right-angled triangle: angle 42° , hypotenuse 28 cm, find opposite side
 - (c) Right-angled triangle: angle 78° , opposite side 20 cm, find adjacent side
 - (d) Right-angled triangle: angle 54° , adjacent side 18 cm, find hypotenuse
8. Find the missing angles (to 1 decimal place):
 - (a) Right-angled triangle: opposite 17 cm, adjacent 25 cm
 - (b) Right-angled triangle: opposite 21.8 cm, hypotenuse 30.4 cm
 - (c) Right-angled triangle: adjacent 16.7 cm, hypotenuse 23.1 cm
 - (d) Right-angled triangle: opposite 38 cm, adjacent 23 cm
9. A 36 m ladder makes an angle of 88° with the ground.
 - (a) How high up the wall does it reach?
 - (b) How far is the foot of the ladder from the wall?
 - (c) What angle would the ladder make if its foot was 21 m from the wall?
10. From the top of a 72 m cliff, the angle of depression to a boat is 42° .
 - (a) Draw a diagram
 - (b) Find the horizontal distance from the cliff to the boat
 - (c) Find the direct distance from the top of the cliff to the boat

Section C: Advanced Trigonometry - Sine and Cosine Rules

11. Use the sine rule to find the missing parts:
 - (a) Triangle ABC: $a = 24$ cm, $A = 63^\circ$, $B = 57^\circ$. Find b and c .
 - (b) Triangle PQR: $p = 33$ cm, $q = 37$ cm, $P = 62^\circ$. Find angle Q .
 - (c) Triangle UVW: $u = 21.4$ cm, $v = 27.8$ cm, $U = 56^\circ$. Find angle V .
12. Use the cosine rule to find the missing parts:
 - (a) Triangle ABC: $a = 25$ cm, $b = 22$ cm, $c = 17$ cm. Find angle C .
 - (b) Triangle XYZ: $x = 27$ cm, $y = 23$ cm, $Z = 72^\circ$. Find side z .
 - (c) Triangle RST: $r = 19.6$ cm, $s = 24.2$ cm, $t = 16.8$ cm. Find the largest angle.
13. A triangle has sides 27 cm, 29 cm, and 31 cm.
 - (a) Use the cosine rule to find the largest angle
 - (b) Use the sine rule to find another angle
 - (c) Find the area using the formula $\text{Area} = \frac{1}{2}ab \sin C$

14. Two ships leave port at the same time. Ship A travels 72 km on a bearing of 125° . Ship B travels 57 km on a bearing of 195° .
- (a) Find the angle between their paths
 - (b) Calculate the distance between the ships
 - (c) Find the bearing of ship B from ship A
15. A parallelogram has sides 24 cm and 30 cm, with an included angle of 86° .
- (a) Find the length of the diagonals
 - (b) Find the area of the parallelogram
 - (c) Find the other angles of the parallelogram

Section D: Circle Theorems

16. Apply circle theorems to find the missing angles:
- (a) Angle at center is 172° . Find the angle at circumference subtending the same arc.
 - (b) Angle at circumference is 48° . Find the angle at center subtending the same arc.
 - (c) Two angles at circumference subtend the same arc. One is 61° . Find the other.
 - (d) Angle in semicircle GHI where GH is diameter. Find angle GIH.
17. In a circle with center O:
- (a) Chord TU subtends angle 92° at circumference point V. Find angle TOU.
 - (b) Tangent XY touches circle at Y. If angle $XYO = 55^\circ$, find angle YXO.
 - (c) Two chords PQ and RS intersect at X inside the circle. If angles $PXR = 108^\circ$, find angle PXS.
 - (d) Tangents from external point X touch circle at T and U. If angle $TXU = 82^\circ$, find angle TOU.
18. Prove these circle theorems (provide written proof):
- (a) Angle in semicircle is 90°
 - (b) Angles in same segment are equal
 - (c) Angle at center is twice angle at circumference
 - (d) Tangent is perpendicular to radius at point of contact
19. A circle has center O and radius 24 cm. Chord TU has length 40 cm.
- (a) Find the perpendicular distance from O to chord TU
 - (b) Find angle TOU
 - (c) Find the area of sector TOU
 - (d) Find the area of triangle TOU

Section E: Geometrical Proof and Reasoning

20. Prove that the sum of angles in any triangle is 180° .
21. Prove that the exterior angle of a triangle equals the sum of the two interior opposite angles.
22. In triangle GHI, J is the midpoint of HI. Prove that GJ is a median and that the three medians of a triangle meet at a single point.

23. QRST is a parallelogram. Prove that:
- (a) Opposite sides are equal
 - (b) Opposite angles are equal
 - (c) Diagonals bisect each other
24. Prove that if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.
25. In triangle GHI, the perpendicular bisectors of sides GH and HI meet at point X. Prove that X is equidistant from G, H, and I.
26. QRST is a cyclic quadrilateral (vertices lie on a circle). Prove that opposite angles sum to 180° .
27. Prove that the angle bisectors of a triangle meet at a single point (the incenter).

Section F: Surface Area and Volume

28. Calculate the surface area and volume of these shapes:
- (a) Cuboid: length 26 cm, width 21 cm, height 16 cm
 - (b) Cylinder: radius 12 cm, height 28 cm
 - (c) Sphere: radius 16 cm
 - (d) Cone: radius 14 cm, height 36 cm
29. A triangular prism has:
- Triangular cross-section with base 22 cm and height 13 cm
 - Length 36 cm

Calculate:

- (a) The volume
 - (b) The surface area
 - (c) The total edge length
30. A square-based pyramid has:
- Base edge 24 cm
 - Vertical height 19 cm
- Find:
- (a) The volume
 - (b) The slant height
 - (c) The surface area
31. A hemisphere sits on top of a cylinder. Both have radius 11 cm and the cylinder has height 28 cm.
- (a) Find the total volume
 - (b) Find the total surface area
 - (c) If this shape is a water tank, how much water does it hold?
32. A cone has base radius 15 cm and slant height 27 cm.
- (a) Find the vertical height
 - (b) Calculate the volume
 - (c) Calculate the curved surface area
 - (d) Calculate the total surface area

Section G: Complex 3D Problems

33. A rectangular swimming pool is 65 m long, 32 m wide, and 6 m deep.
- (a) Calculate the volume of water needed to fill it
 - (b) Find the surface area of the pool (5 faces - no top)
 - (c) How much does it cost to tile the pool at £88 per m^2 ?
 - (d) If water is added at 2700 liters per minute, how long to fill the pool?
34. A solid consists of a cylinder with radius 17 cm and height 26 cm, with a cone of radius 17 cm and height 24 cm on top.
- (a) Calculate the total volume
 - (b) Calculate the total surface area
 - (c) If the solid is made of metal with density 10.5 g/cm^3 , find its mass
35. A spherical ball has radius 34 cm. A cylindrical hole of radius 13 cm is drilled through the center.
- (a) Calculate the volume of material removed
 - (b) Calculate the remaining volume
 - (c) Calculate the new surface area (including the cylindrical hole)
36. A water tank is a horizontal cylinder with length 15 m and radius 3.6 m.
- (a) Calculate its capacity in liters
 - (b) If it's half full, find the depth of water
 - (c) Calculate the surface area of water

Section H: Problem Solving and Applications

37. A regular dodecagon is inscribed in a circle of radius 33 cm.
- (a) Find the side length of the dodecagon
 - (b) Calculate the area of the dodecagon
 - (c) Find the area between the circle and dodecagon
38. From point B, the angle of elevation to the top of a tower is 37° . From point C, 150 m closer to the tower, the angle of elevation is 64° .
- (a) Draw a diagram
 - (b) Find the height of the tower
 - (c) Find the distances from B and C to the base of the tower
39. A field is in the shape of a triangle with sides 280 m, 320 m, and 360 m.
- (a) Find the area of the field using Heron's formula
 - (b) Find the area using trigonometry
 - (c) A path 6 m wide runs around the perimeter. Find the area of the path
40. A church spire consists of a square-based pyramid on top of a cuboid. The cuboid is $24 \text{ m} \times 24 \text{ m} \times 36 \text{ m}$ high. The pyramid has height 28 m.
- (a) Calculate the total volume

- (b) Calculate the total surface area
 - (c) Find the angle the pyramid faces make with the horizontal
41. Two circles with radii 30 cm and 21 cm have centers 56 cm apart.
- (a) Do the circles intersect? Justify your answer.
 - (b) Find the length of their common external tangent
 - (c) Calculate the area of overlap if they moved closer together
42. A satellite orbits Earth at height 1100 km above the surface. Earth's radius is 6400 km.
- (a) Calculate the satellite's orbital radius
 - (b) Find the maximum distance the satellite can see on Earth's surface
 - (c) Calculate the area of Earth visible from the satellite
43. Prove that in any triangle, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.
44. A frustum (truncated cone) has top radius 11 cm, bottom radius 30 cm, and height 22 cm.
- (a) Find the height of the complete cone
 - (b) Calculate the volume of the frustum
 - (c) Calculate the curved surface area of the frustum

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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