

GCSE Higher Mathematics

Practice Test 5: Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

(a) $7(x - 5) = 3x + 9$

(b) $\frac{x+9}{6} - \frac{x-7}{4} = 2$

(c) $\frac{6x-5}{10} = \frac{3x+4}{6} - 1$

(d) $0.7x + 1.4 = 0.5x - 1.6$

2. Solve these simultaneous equations by elimination:

(a)
$$\begin{cases} 6x + 5y = 32 \\ 4x - 3y = 2 \end{cases}$$

(b)
$$\begin{cases} 3x + 8y = 34 \\ 7x - 4y = 6 \end{cases}$$

(c)
$$\begin{cases} 6x + 3y = 24 \\ 4x - 7y = 9 \end{cases}$$

(d)
$$\begin{cases} 8x + 7y = 43 \\ 3x - 4y = 5 \end{cases}$$

3. Solve these simultaneous equations by substitution:

(a)
$$\begin{cases} y = 6x - 9 \\ 5x + 2y = 33 \end{cases}$$

(b)
$$\begin{cases} x = 5y - 6 \\ 3x - 4y = 14 \end{cases}$$

(c)
$$\begin{cases} y = 13 - 5x \\ 4x + 7y = 51 \end{cases}$$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

(a) $y = 6x - 4$ and $y = 14 - 3x$

(b) $y = x^2 - 9$ and $y = 4x - 1$

(c) $x^2 + y^2 = 17$ and $y = 4x - 2$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

(a) $x^2 + 15x + 36$

(b) $x^2 - 13x - 30$

(c) $x^2 - 17x + 72$

(d) $x^2 + 6x - 27$

(e) $x^2 - 144$

(f) $x^2 - 24x + 144$

6. Solve these quadratic equations by factorizing:

(a) $x^2 + 15x + 56 = 0$

(b) $x^2 - 14x - 15 = 0$

(c) $x^2 - 11x = 0$

(d) $x^2 - 169 = 0$

(e) $x^2 + 26x + 169 = 0$

(f) $6x^2 - 24x = 0$

7. Factorize these harder quadratics:

(a) $4x^2 + 11x + 6$

(b) $6x^2 - 17x + 5$

(c) $49x^2 - 64$

(d) $5x^2 + 16x - 21$

(e) $49x^2 - 84x + 36$

(f) $9x^2 - 25x - 6$

8. Solve by factorizing:

(a) $6x^2 + 13x - 5 = 0$

(b) $7x^2 - 15x + 2 = 0$

(c) $36x^2 - 25 = 0$

(d) $4x^2 + 13x - 12 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

(a) $x^2 + 14x + 13$

(b) $x^2 - 18x + 5$

(c) $x^2 + 10x - 7$

(d) $x^2 - 18x + 15$

(e) $6x^2 + 24x + 11$

(f) $2x^2 - 12x + 5$

10. Solve by completing the square:

(a) $x^2 + 14x + 11 = 0$

(b) $x^2 - 12x - 5 = 0$

(c) $x^2 + 8x - 3 = 0$

(d) $6x^2 + 12x - 5 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 11x - 5 = 0$

(b) $6x^2 - 13x + 5 = 0$

(c) $x^2 - 14x + 9 = 0$

(d) $5x^2 + 9x - 6 = 0$

(e) $4x^2 + 11x + 6 = 0$

(f) $8x^2 - 11x - 3 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 13x + 30 = 0$

(b) $x^2 - 12x + 36 = 0$

(c) $x^2 + 6x + 13 = 0$

(d) $5x^2 - 11x + 6 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 12x + 32$:

- (a) Find the y-intercept
- (b) Find the x-intercepts by factorizing
- (c) Complete the square to find the vertex
- (d) Sketch the graph
- (e) State the line of symmetry

14. For the quadratic $y = 5x^2 + 15x - 7$:

- (a) Complete the square
- (b) Find the coordinates of the vertex
- (c) Find the y-intercept
- (d) State the line of symmetry
- (e) Sketch the graph

15. A baseball is hit upward. Its height h (in meters) after t seconds is given by: $h = -5t^2 + 40t + 3$

- (a) What is the initial height?
- (b) At what times is the baseball at ground level?
- (c) What is the maximum height reached?
- (d) At what time does it reach maximum height?

16. The cost C (in thousands of pounds) for hosting x thousand guests is: $C = 3x^2 - 18x + 35$

- (a) How many guests should be hosted to minimize cost?
- (b) What is the minimum cost?
- (c) At what guest levels is the cost £8,000?

Section E: Linear Inequalities

17. Solve these linear inequalities:

(a) $7x + 13 > 41$

(b) $6x - 15 \leq 27$

(c) $10 - 7x < 3$

(d) $\frac{x+7}{4} \geq 5$

(e) $8 - 7x > 6x + 21$

(f) $\frac{6x-5}{7} < \frac{x+8}{4}$

18. Show these inequalities on number lines:

(a) $x > -6$

(b) $x \leq 8$

(c) $-3 < x \leq 9$

(d) $x < 5$ or $x > 10$

19. Solve these compound inequalities:

(a) $-8 < 6x + 4 < 16$

(b) $11 \leq 5x - 4 \leq 26$

(c) $-3 \leq \frac{6x+4}{4} < 10$

20. Find the integer solutions to:

(a) $6x + 9 > 21$ and $x < 9$

(b) $-4 \leq x + 1 < 8$

(c) $x^2 < 64$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

(a) $x^2 - 13x + 30 > 0$

(b) $x^2 - 14x + 33 \leq 0$

(c) $x^2 - 81 < 0$

(d) $x^2 + 6x - 16 \geq 0$

(e) $x^2 - 14x + 49 > 0$

(f) $4x^2 - 5x - 6 < 0$

22. Solve and show on number lines:

(a) $x^2 - 100 \leq 0$

(b) $x^2 + 5x - 14 > 0$

(c) $3x^2 - 10x + 8 \geq 0$

23. Find the values of x for which:

(a) $x^2 < 7x + 8$

(b) $4x^2 + 9x \geq 9$

(c) $x^2 + 12x + 36 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

- (a) $(x + 9)(6x - 4)$
- (b) $(5x - 6)(x + 10)$
- (c) $(4x + 9)^2$
- (d) $(8x - 5)^2$
- (e) $(x + 13)(x - 13)$
- (f) $(7x + 3)(7x - 3)$

25. Expand these expressions:

- (a) $(x + 6)(x^2 - 5x + 7)$
- (b) $(4x - 5)(x^2 + 4x - 3)$
- (c) $(x + 5)^3$
- (d) $(5x - 2)^3$

26. Factorize completely:

- (a) $14x^2 + 21x$
- (b) $36x^2 - 121$
- (c) $x^3 - 25x$
- (d) $6x^3 + 24x^2 + 24x$
- (e) $x^3 - 343$
- (f) $216x^3 + 125$

27. Simplify these algebraic fractions:

- (a) $\frac{x^2-36}{x+6}$
- (b) $\frac{x^2+13x+42}{x+7}$
- (c) $\frac{7x^2-28}{x^2-4}$
- (d) $\frac{x^3-343}{x^2-49}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

- (a) $\frac{7}{x} + \frac{4}{x}$
- (b) $\frac{11}{x} - \frac{6}{x}$
- (c) $\frac{4}{6x} + \frac{1}{12x}$
- (d) $\frac{6}{x+4} + \frac{3}{x-5}$
- (e) $\frac{x}{x+6} - \frac{5}{x-3}$
- (f) $\frac{6x}{x^2-25} + \frac{5}{x+5}$

29. Multiply and divide these algebraic fractions:

- (a) $\frac{x}{7} \times \frac{21}{x^2}$
- (b) $\frac{x+6}{8} \times \frac{16}{x+6}$
- (c) $\frac{x^2-36}{x+5} \div \frac{x-6}{x+5}$

(d) $\frac{6x+18}{x^2-49} \times \frac{x-7}{12}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{7} + \frac{x}{5} = 24$

(b) $\frac{6x+5}{8} = \frac{x-4}{5}$

(c) $\frac{7}{x} = \frac{6}{x-5}$

(d) $\frac{x+5}{x-6} = \frac{4x}{x+5}$

31. Make the subject of these formulae:

(a) $P = \frac{F}{A}$, make F the subject

(b) $A = \pi r^2$, make r the subject

(c) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, make f the subject

(d) $h = \frac{3V}{\pi r^2}$, make V the subject

Section I: Sequences

32. Find the n th term for these arithmetic sequences:

(a) 15, 21, 27, 33, ...

(b) 9, 16, 23, 30, ...

(c) 45, 41, 37, 33, ...

(d) $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \dots$

33. For these geometric sequences, find the n th term:

(a) 6, 18, 54, 162, ...

(b) 9, 36, 144, 576, ...

(c) 256, 128, 64, 32, ...

(d) 5, -20, 80, -320, ...

34. Find the sum of these series:

(a) First 40 terms of $10 + 14 + 18 + 22 + \dots$

(b) First 5 terms of $8 + 24 + 72 + 216 + \dots$

(c) $5 + 9 + 13 + \dots + 101$ (arithmetic series)

(d) $7 + 14 + 28 + \dots + 448$ (geometric series)

35. These are quadratic sequences. Find the n th term:

(a) 6, 24, 54, 96, 150, ...

(b) 5, 15, 31, 53, 81, ...

(c) 4, 16, 36, 64, 100, ...

(d) 8, 23, 44, 71, 104, ...

36. A sequence is defined by $u_1 = 6$ and $u_{n+1} = 4u_n - 5$.

(a) Find the first 5 terms

(b) Find a formula for u_n

(c) Calculate u_5

Section J: Problem Solving

37. The sum of two numbers is 30 and their product is 216. Find the two numbers.
38. A rectangular courtyard has perimeter 72m. If the length is 14m more than the width, find the dimensions.
39. The difference between a positive number and its reciprocal is $\frac{48}{7}$. Find the number.
40. A volleyball's height h (in meters) after t seconds is given by: $h = 140t - 5t^2$
- (a) When does it hit the ground?
 - (b) What is its maximum height?
 - (c) When is it 975m high?
41. Prove that the sum of the first n terms of the sequence 3, 6, 9, 12, ... is $\frac{3n(n+1)}{2}$.
42. The quadratic $ax^2 + bx + c = 0$ has roots r and s .
- (a) Show that $r + s = -\frac{b}{a}$
 - (b) Show that $rs = \frac{c}{a}$
 - (c) If the roots are 7 and -4, find a , b , and c when $a = 4$
43. A function is defined as $k(x) = x^2 + dx + e$. If $k(3) = 14$ and $k(5) = 30$, find d and e .
44. The sum of the first n terms of a sequence is $S_n = 6n^2 - 3n$. Find the n th term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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