A Level Pure Mathematics
Practice Test 2: Algebra and Functions

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Algebraic Manipulation

1. Simplify these expressions:
   (a) $\frac{x^2-16}{x^2-8x+16}$
   (b) $\frac{3x^2-12}{x^2-3x-10}$
   (c) $\frac{x^3+27}{x^2-9}$
   (d) $\frac{x^4-81}{x^2-3x-18}$

2. Factorize completely:
   (a) $x^3 - 8x^2 + 12x + 20$
   (b) $27x^3 + 8$
   (c) $x^4 - 81$
   (d) $x^6 - 1$
   (e) $x^4 - 6x^2 + 9$
   (f) $x^3 + 2x^2 - 9x - 18$

3. Express as single fractions in simplest form:
   (a) $\frac{3}{x+1} - \frac{2}{x-3}$
   (b) $\frac{2x}{x^2-9} + \frac{1}{x-3}$
   (c) $\frac{x-2}{x^2+2x-3} - \frac{x+1}{x^2-4}$
   (d) $\frac{2}{x} - \frac{3}{x+2} + \frac{5}{x^2+2x}$

4. Use the binomial theorem to expand:
   (a) $(3x - 2)^4$
   (b) $(x + \frac{2}{x})^6$
   (c) $(2 - x)^5$, and find the coefficient of x^3
   (d) Find the coefficient of x^6 in the expansion of $(x^3 - \frac{1}{x^2})^8$

5. Simplify using laws of indices:
   (a) $\frac{3^{2x} \cdot 9^{x+1}}{27^{x-1}}$

(b) $\frac{16^{x-2} \cdot 8^{2x+1}}{4^{3x-1}}$

(c) $\left(x^{\frac{3}{4}}\right)^{\frac{2}{3}} \cdot x^{-\frac{1}{6}}$

(d) $\frac{(3x)^2 \cdot (2x^3)^3}{18x^7}$

Section B: Linear and Quadratic Equations

6. Solve these equations:

   (a) $\frac{3x+2}{4} - \frac{2x-1}{3} = \frac{5}{6}$

   (b) $\frac{2x}{x+1} = \frac{4}{x-3}$

   (c) $\sqrt{3x-2} = x - 2$

   (d) $\frac{2}{x+2} - \frac{1}{x-1} = \frac{1}{3}$

7. Solve these quadratic equations, leaving answers in exact form where appropriate:

   (a) $3x^2 - 8x + 2 = 0$

   (b) $x^2 - 6x + 2 = 0$

   (c) $2x^2 = 3x + 5$

   (d) $(3x + 1)^2 = 2(x - 1)$

8. For the quadratic equation $2x^2 + (k-1)x + k = 0$:

   (a) Find the discriminant in terms of k

   (b) Find the values of k for which the equation has equal roots

   (c) Find the values of k for which the equation has no real roots

   (d) When $k = 3$, find the sum and product of the roots

9. The quadratic $px^2 + qx + r = 0$ has roots α and β.

   (a) Express $\alpha + \beta$ and $\alpha\beta$ in terms of p, q, and r

   (b) Find a quadratic equation with roots 3α and 3β

   (c) Find a quadratic equation with roots $\alpha - 2$ and $\beta - 2$

   (d) If $\alpha^2 + \beta^2 = 14$ and $\alpha + \beta = 6$, find $\alpha\beta$

Section C: Cubic and Higher Order Equations

10. Solve these cubic equations:

    (a) $x^3 - 4x^2 + 5x - 2 = 0$

    (b) $x^3 - 3x^2 - 6x + 8 = 0$

    (c) $3x^3 + 2x^2 - 19x - 6 = 0$

    (d) $x^3 + 3x^2 - 4 = 0$

11. Given that $x = -1$ is a root of $x^3 + ax^2 + bx + 6 = 0$:

    (a) Find a relationship between a and b

    (b) If the sum of all three roots is 2, find a and b

    (c) Hence find all three roots

    (d) Verify your answer by substitution

12. Solve these quartic equations:

(a) $x^4 - 10x^2 + 9 = 0$

(b) $x^4 - 8x^2 + 15 = 0$

(c) $(x^2 - 2x)^2 - 5(x^2 - 2x) + 6 = 0$

(d) $x^4 - 2x^3 - 13x^2 + 14x + 24 = 0$ (given that $x = -2$ is a root)

13. Use the substitution $u = x - \frac{2}{x}$ to solve:

(a) $x^2 + \frac{4}{x^2} = 6$

(b) $3x^2 - 4x + \frac{4}{x} - \frac{3}{x^2} = 0$

Section D: Functions - Definition and Notation

14. For the function $f(x) = \frac{3x-2}{x+1}$ where $x \neq -1$:

(a) Find $f(0)$, $f(2)$, and $f(-3)$

(b) Solve $f(x) = 2$

(c) Find the value of x for which $f(x)$ is undefined

(d) Find the range of $f(x)$

15. Given $g(x) = x^2 + 6x + 5$:

(a) Express $g(x)$ in the form $(x + a)^2 + b$

(b) State the minimum value of $g(x)$ and the value of x at which it occurs

(c) Solve $g(x) = 0$

(d) Find the range of $g(x)$

16. For $h(x) = \sqrt{16 - x^2}$:

(a) Find the domain of $h(x)$

(b) Find the range of $h(x)$

(c) Sketch the graph of $y = h(x)$

(d) Solve $h(x) = 3$

17. Define $k(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ 3x & \text{if } 0 < x \leq 2 \\ 7 & \text{if } x > 2 \end{cases}$

(a) Find $k(-1)$, $k(0)$, $k(1)$, and $k(3)$

(b) Is $k(x)$ continuous at $x = 0$? Justify your answer

(c) Is $k(x)$ continuous at $x = 2$? Justify your answer

(d) Sketch the graph of $y = k(x)$

Section E: Composite and Inverse Functions

18. Given $f(x) = 3x - 1$ and $g(x) = x^2 + 2$:

(a) Find $f(g(x))$ and $g(f(x))$

(b) Calculate $f(g(1))$ and $g(f(1))$

(c) Solve $f(g(x)) = 14$

(d) Find $(f \circ g)^{-1}(x)$

19. For $p(x) = \frac{2x-1}{x+3}$ where $x \neq -3$:
    (a) Find $p^{-1}(x)$
    (b) Verify that $p(p^{-1}(x)) = x$
    (c) State the domain and range of $p^{-1}(x)$
    (d) Solve $p(x) = p^{-1}(x)$

20. Given $f(x) = 2x + 5$ and $g(x) = \frac{1}{x-2}$ where $x \neq 2$:
    (a) Find $(f \circ g)(x)$ and state its domain
    (b) Find $(g \circ f)(x)$ and state its domain
    (c) Find $(f \circ g)^{-1}(x)$
    (d) Verify your answer by showing $(f \circ g)((f \circ g)^{-1}(x)) = x$

21. The function $h(x) = x^2 - 6x + 2$ is defined for $x \geq 3$.
    (a) Explain why the domain restriction is necessary for h^{-1} to exist
    (b) Find $h^{-1}(x)$
    (c) State the domain and range of $h^{-1}(x)$
    (d) Sketch $h(x)$ and $h^{-1}(x)$ on the same axes

Section F: Graphing Functions

22. Sketch the graphs of these functions, clearly showing key features:
    (a) $y = x^3 + 3x^2 - 4$
    (b) $y = \frac{3x-1}{x+2}$
    (c) $y = |x^2 - 6x + 8|$
    (d) $y = \frac{x^2+4}{x^2-1}$

23. For the rational function $f(x) = \frac{x^2-3x+2}{x^2-1}$:
    (a) Find the domain of $f(x)$
    (b) Find the x and y intercepts
    (c) Identify any vertical asymptotes
    (d) Find the horizontal asymptote
    (e) Sketch the graph of $y = f(x)$

24. Analyze the function $g(x) = \frac{3x^2-12}{x^2-2x-8}$:
    (a) Factorize the numerator and denominator
    (b) Simplify $g(x)$ and state its domain
    (c) Find any asymptotes
    (d) Find the coordinates of any stationary points
    (e) Sketch the graph of $y = g(x)$

25. For the polynomial $p(x) = x^4 - 2x^3 - 8x^2$:
    (a) Factorize $p(x)$ completely
    (b) Find the roots and their multiplicities
    (c) Determine the behavior at each root
    (d) Find $p'(x)$ and locate stationary points
    (e) Sketch the graph of $y = p(x)$

Section G: Function Transformations

26. Given the function $f(x) = x^2$, describe the transformations and sketch:

    (a) $y = f(x + 3) - 2$
    (b) $y = -3f(x - 1)$
    (c) $y = f(\frac{x}{2}) + 1$
    (d) $y = 2f(-x) - 4$

27. The graph of $y = f(x)$ has vertex at $(-2, 4)$ and passes through $(0, 8)$ and $(-4, 8)$. Find the vertex and two other points for:

    (a) $y = f(x) - 3$
    (b) $y = f(x + 1)$
    (c) $y = 2f(x)$
    (d) $y = f(3x)$
    (e) $y = -f(x)$
    (f) $y = f(-x)$

28. Given that $g(x) = |x + 2| - 1$:

    (a) Describe the transformations applied to $y = |x|$
    (b) State the vertex of the graph
    (c) Find the range of $g(x)$
    (d) Solve $g(x) = 3$
    (e) Sketch the graph of $y = g(x)$

29. The function $h(x) = \cos x$ is transformed to $k(x) = 2\cos(3x - \frac{\pi}{4}) + 1$.

    (a) Identify each transformation in the correct order
    (b) State the amplitude of $k(x)$
    (c) State the period of $k(x)$
    (d) Find the phase shift
    (e) Find the vertical shift
    (f) Sketch one complete cycle of $y = k(x)$

Section H: Special Functions and Applications

30. For the exponential function $f(x) = 2^{x+2} - 3$:

    (a) State the domain and range
    (b) Find the y-intercept
    (c) Find the horizontal asymptote
    (d) Solve $f(x) = 5$
    (e) Find $f^{-1}(x)$ and state its domain and range

31. For the logarithmic function $g(x) = \log_3(x - 1) + 2$:

    (a) State the domain and range
    (b) Find the x-intercept
    (c) Find the vertical asymptote

(d) Solve $g(x) = 4$

(e) Express $g(x)$ in terms of natural logarithms

32. A function is defined as $f(x) = \frac{px+q}{rx+s}$ where $ps - qr \neq 0$.

(a) Find the domain of $f(x)$

(b) Find $f^{-1}(x)$

(c) Show that $(f^{-1} \circ f)(x) = x$

(d) Under what condition is $f(x) = f^{-1}(x)$?

33. The modulus function $|x|$ can be written as: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

(a) Sketch $y = |3x + 2|$

(b) Solve $|3x + 2| = 7$

(c) Solve $|3x + 2| \leq 5$

(d) Find the range of values for which $|3x + 2| > 4$

Section I: Problem Solving and Applications

34. A triangular field has one side along a straight road. The farmer has 80 meters of fencing to enclose the other two sides. Let x be the length of one of the equal sides.

(a) Express the base of the triangle in terms of x

(b) Show that the area $A = x\sqrt{1600 - x^2}$

(c) Find the value of x that maximizes the area

(d) Calculate the maximum area

(e) State the domain of the function in this context

35. The revenue R (in thousands of pounds) from selling x thousand units is given by: $R(x) = -3x^2 + 24x - 36$

(a) Express $R(x)$ in completed square form

(b) Find the break-even points (where $R(x) = 0$)

(c) Determine the sales level for maximum revenue

(d) Calculate the maximum revenue

(e) For what range of sales levels is the revenue positive?

36. A ball is thrown upward and its height h (in meters) after t seconds is: $h(t) = -5t^2 + 20t + 1.5$ for $t \geq 0$

(a) Find when the ball reaches its maximum height

(b) Calculate the maximum height

(c) Determine when the ball hits the ground

(d) Find the ball's height after 3 seconds

37. A function $f(x) = \frac{x^2+1}{x^2-9}$ models a physical relationship.

(a) Find the domain and range of $f(x)$

(b) Determine the horizontal asymptote and explain its physical meaning

(c) Find any vertical asymptotes

(d) Analyze the behavior as $x \to \pm\infty$

(e) Sketch the graph and discuss any symmetry

38. Two functions are related by $g(x) = f(3x + 2) - 1$ where $f(x) = x^2$.

(a) Find an explicit expression for $g(x)$

(b) Describe the transformations that map f to g

(c) Find the vertex of the parabola $y = g(x)$

(d) If f has domain $[-2, 3]$, find the domain of g

(e) Solve $g(x) = f(x)$ and interpret geometrically

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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