

GCSE Higher Mathematics

Practice Test 6: Geometry and Measures

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Pythagoras' Theorem

- Use Pythagoras' theorem to find the missing lengths:
 - A right-angled triangle with legs 15 cm and 20 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 53 cm and one leg 28 cm. Find the other leg.
 - A right-angled triangle with legs 16.5 cm and 22 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 58 cm and one leg 42 cm. Find the other leg.
- Find the distance between these pairs of coordinate points:
 - A(6, 9) and B(18, 24)
 - C(-4, 3) and D(8, -9)
 - E(-9, -5) and F(3, 7)
 - G(0, 13) and H(-15, 8)
- A rectangle has length 33 cm and diagonal 39 cm. Find:
 - The width of the rectangle
 - The area of the rectangle
 - The perimeter of the rectangle
- A ladder 14.5 m long leans against a vertical wall. The foot of the ladder is 8.7 m from the wall.
 - How high up the wall does the ladder reach?
 - If the ladder slips so its foot is now 10.5 m from the wall, how far does the top slide down?
- Determine whether these triangles are right-angled:
 - Sides 14 cm, 48 cm, 50 cm
 - Sides 15 cm, 112 cm, 113 cm
 - Sides 20 cm, 26 cm, 32 cm
 - Sides 27 cm, 36 cm, 45 cm

Section B: Basic Trigonometry

6. In a right-angled triangle with hypotenuse 34 cm and one leg 30 cm:
- (a) Find the other leg using Pythagoras
 - (b) Calculate $\sin A$, $\cos A$, and $\tan A$ where A is the angle opposite the 30 cm side
 - (c) Find angle A to 1 decimal place
7. Use trigonometry to find the missing sides (give answers to 3 significant figures):
- (a) Right-angled triangle: angle 55° , adjacent side 15 cm, find opposite side
 - (b) Right-angled triangle: angle 33° , hypotenuse 22 cm, find opposite side
 - (c) Right-angled triangle: angle 69° , opposite side 14 cm, find adjacent side
 - (d) Right-angled triangle: angle 44° , adjacent side 12 cm, find hypotenuse
8. Find the missing angles (to 1 decimal place):
- (a) Right-angled triangle: opposite 11 cm, adjacent 18 cm
 - (b) Right-angled triangle: opposite 15.8 cm, hypotenuse 21.3 cm
 - (c) Right-angled triangle: adjacent 11.2 cm, hypotenuse 15.7 cm
 - (d) Right-angled triangle: opposite 28 cm, adjacent 17 cm
9. A 27 m ladder makes an angle of 82° with the ground.
- (a) How high up the wall does it reach?
 - (b) How far is the foot of the ladder from the wall?
 - (c) What angle would the ladder make if its foot was 15 m from the wall?
10. From the top of a 54 m cliff, the angle of depression to a boat is 33° .
- (a) Draw a diagram
 - (b) Find the horizontal distance from the cliff to the boat
 - (c) Find the direct distance from the top of the cliff to the boat

Section C: Advanced Trigonometry - Sine and Cosine Rules

11. Use the sine rule to find the missing parts:
- (a) Triangle ABC: $a = 18$ cm, $A = 57^\circ$, $B = 66^\circ$. Find b and c .
 - (b) Triangle PQR: $p = 24$ cm, $q = 28$ cm, $P = 52^\circ$. Find angle Q .
 - (c) Triangle STU: $s = 14.8$ cm, $t = 18.6$ cm, $S = 47^\circ$. Find angle T .
12. Use the cosine rule to find the missing parts:
- (a) Triangle ABC: $a = 19$ cm, $b = 16$ cm, $c = 11$ cm. Find angle C .
 - (b) Triangle DEF: $d = 21$ cm, $e = 17$ cm, $F = 64^\circ$. Find side f .
 - (c) Triangle GHI: $g = 13.7$ cm, $h = 16.8$ cm, $i = 10.4$ cm. Find the largest angle.
13. A triangle has sides 21 cm, 23 cm, and 25 cm.
- (a) Use the cosine rule to find the largest angle
 - (b) Use the sine rule to find another angle
 - (c) Find the area using the formula $\text{Area} = \frac{1}{2}ab \sin C$

14. Two ships leave port at the same time. Ship A travels 54 km on a bearing of 095° . Ship B travels 42 km on a bearing of 165° .
- (a) Find the angle between their paths
 - (b) Calculate the distance between the ships
 - (c) Find the bearing of ship B from ship A
15. A parallelogram has sides 18 cm and 24 cm, with an included angle of 78° .
- (a) Find the length of the diagonals
 - (b) Find the area of the parallelogram
 - (c) Find the other angles of the parallelogram

Section D: Circle Theorems

16. Apply circle theorems to find the missing angles:
- (a) Angle at center is 148° . Find the angle at circumference subtending the same arc.
 - (b) Angle at circumference is 41° . Find the angle at center subtending the same arc.
 - (c) Two angles at circumference subtend the same arc. One is 53° . Find the other.
 - (d) Angle in semicircle PQR where PQ is diameter. Find angle PRQ.
17. In a circle with center O:
- (a) Chord MN subtends angle 77° at circumference point P. Find angle MON.
 - (b) Tangent YZ touches circle at Z. If angle $YZO = 47^\circ$, find angle ZYO.
 - (c) Two chords AB and CD intersect at Y inside the circle. If angles $AYC = 95^\circ$, find angle AYD.
 - (d) Tangents from external point Y touch circle at M and N. If angle $MYN = 68^\circ$, find angle MON.
18. Prove these circle theorems (provide written proof):
- (a) Angle in semicircle is 90°
 - (b) Angles in same segment are equal
 - (c) Angle at center is twice angle at circumference
 - (d) Tangent is perpendicular to radius at point of contact
19. A circle has center O and radius 18 cm. Chord MN has length 28 cm.
- (a) Find the perpendicular distance from O to chord MN
 - (b) Find angle MON
 - (c) Find the area of sector MON
 - (d) Find the area of triangle MON

Section E: Geometrical Proof and Reasoning

20. Prove that the sum of angles in any triangle is 180° .
21. Prove that the exterior angle of a triangle equals the sum of the two interior opposite angles.
22. In triangle PQR, S is the midpoint of QR. Prove that PS is a median and that the three medians of a triangle meet at a single point.

23. EFGH is a parallelogram. Prove that:
- (a) Opposite sides are equal
 - (b) Opposite angles are equal
 - (c) Diagonals bisect each other
24. Prove that if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.
25. In triangle PQR, the perpendicular bisectors of sides PQ and QR meet at point Y. Prove that Y is equidistant from P, Q, and R.
26. EFGH is a cyclic quadrilateral (vertices lie on a circle). Prove that opposite angles sum to 180° .
27. Prove that the angle bisectors of a triangle meet at a single point (the incenter).

Section F: Surface Area and Volume

28. Calculate the surface area and volume of these shapes:
- (a) Cuboid: length 20 cm, width 15 cm, height 11 cm
 - (b) Cylinder: radius 9 cm, height 22 cm
 - (c) Sphere: radius 13 cm
 - (d) Cone: radius 11 cm, height 27 cm
29. A triangular prism has:
- Triangular cross-section with base 16 cm and height 9 cm
 - Length 28 cm

Calculate:

- (a) The volume
 - (b) The surface area
 - (c) The total edge length
30. A square-based pyramid has:
- Base edge 18 cm
 - Vertical height 14 cm
- Find:
- (a) The volume
 - (b) The slant height
 - (c) The surface area
31. A hemisphere sits on top of a cylinder. Both have radius 8 cm and the cylinder has height 21 cm.
- (a) Find the total volume
 - (b) Find the total surface area
 - (c) If this shape is a water tank, how much water does it hold?
32. A cone has base radius 12 cm and slant height 21 cm.
- (a) Find the vertical height
 - (b) Calculate the volume
 - (c) Calculate the curved surface area
 - (d) Calculate the total surface area

Section G: Complex 3D Problems

33. A rectangular swimming pool is 50 m long, 25 m wide, and 4.5 m deep.
- (a) Calculate the volume of water needed to fill it
 - (b) Find the surface area of the pool (5 faces - no top)
 - (c) How much does it cost to tile the pool at £68 per m^2 ?
 - (d) If water is added at 1800 liters per minute, how long to fill the pool?
34. A solid consists of a cylinder with radius 14 cm and height 20 cm, with a cone of radius 14 cm and height 18 cm on top.
- (a) Calculate the total volume
 - (b) Calculate the total surface area
 - (c) If the solid is made of metal with density 9.4 g/cm^3 , find its mass
35. A spherical ball has radius 28 cm. A cylindrical hole of radius 9 cm is drilled through the center.
- (a) Calculate the volume of material removed
 - (b) Calculate the remaining volume
 - (c) Calculate the new surface area (including the cylindrical hole)
36. A water tank is a horizontal cylinder with length 11 m and radius 2.7 m.
- (a) Calculate its capacity in liters
 - (b) If it's half full, find the depth of water
 - (c) Calculate the surface area of water

Section H: Problem Solving and Applications

37. A regular heptagon is inscribed in a circle of radius 24 cm.
- (a) Find the side length of the heptagon
 - (b) Calculate the area of the heptagon
 - (c) Find the area between the circle and heptagon
38. From point V, the angle of elevation to the top of a tower is 27° . From point W, 120 m closer to the tower, the angle of elevation is 49° .
- (a) Draw a diagram
 - (b) Find the height of the tower
 - (c) Find the distances from V and W to the base of the tower
39. A field is in the shape of a triangle with sides 220 m, 260 m, and 300 m.
- (a) Find the area of the field using Heron's formula
 - (b) Find the area using trigonometry
 - (c) A path 4.5 m wide runs around the perimeter. Find the area of the path
40. A church spire consists of a square-based pyramid on top of a cuboid. The cuboid is $18 \text{ m} \times 18 \text{ m} \times 28 \text{ m}$ high. The pyramid has height 22 m.
- (a) Calculate the total volume
 - (b) Calculate the total surface area

- (c) Find the angle the pyramid faces make with the horizontal
41. Two circles with radii 21 cm and 15 cm have centers 42 cm apart.
- (a) Do the circles intersect? Justify your answer.
 - (b) Find the length of their common external tangent
 - (c) Calculate the area of overlap if they moved closer together
42. A satellite orbits Earth at height 800 km above the surface. Earth's radius is 6400 km.
- (a) Calculate the satellite's orbital radius
 - (b) Find the maximum distance the satellite can see on Earth's surface
 - (c) Calculate the area of Earth visible from the satellite
43. Prove that in any triangle, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.
44. A frustum (truncated cone) has top radius 8 cm, bottom radius 21 cm, and height 16 cm.
- (a) Find the height of the complete cone
 - (b) Calculate the volume of the frustum
 - (c) Calculate the curved surface area of the frustum

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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