A Level Pure Mathematics
Practice Test 3: Trigonometry

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Radian Measure and Basic Functions

1. Convert these angles from degrees to radians (leave answers in terms of π):
   (a) $18°$
   (b) $54°$
   (c) $108°$
   (d) $162°$
   (e) $216°$
   (f) $270°$
2. Convert these angles from radians to degrees:
   (a) $\frac{\pi}{12}$
   (b) $\frac{\pi}{8}$
   (c) $\frac{5\pi}{6}$
   (d) $\frac{7\pi}{4}$
   (e) $\frac{8\pi}{3}$
   (f) $\frac{13\pi}{6}$
3. Find the exact values of these trigonometric ratios (without calculator):
   (a) $\sin(-\frac{\pi}{6})$, $\cos(-\frac{\pi}{6})$, $\tan(-\frac{\pi}{6})$
   (b) $\sin(-\frac{\pi}{4})$, $\cos(-\frac{\pi}{4})$, $\tan(-\frac{\pi}{4})$
   (c) $\sin(-\frac{\pi}{3})$, $\cos(-\frac{\pi}{3})$, $\tan(-\frac{\pi}{3})$
   (d) $\sin(2\pi)$, $\cos(2\pi)$, $\tan(2\pi)$
4. A circle has radius 15 cm. Find:
   (a) The arc length subtended by an angle of $\frac{4\pi}{5}$ radians
   (b) The area of the sector with angle $\frac{7\pi}{12}$ radians
   (c) The angle (in radians) that subtends an arc of length 25 cm
   (d) The radius of a circle where an angle of $\frac{2\pi}{3}$ radians subtends an arc of length 30 cm
5. Find the exact values:

(a) $\sin \frac{4\pi}{3}$
(b) $\cos \frac{5\pi}{4}$
(c) $\tan \frac{2\pi}{3}$
(d) $\sin \frac{5\pi}{3}$
(e) $\cos \frac{7\pi}{6}$
(f) $\tan \frac{11\pi}{6}$

Section B: Graphs of Trigonometric Functions

6. For the function $f(x) = \frac{1}{2}\sin x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$

7. For the function $g(x) = 2\cos x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$

8. For the function $h(x) = \tan \frac{x}{2}$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the asymptotes in the interval $[0, 4\pi]$
   (d) Find the zeros in the interval $[0, 2\pi]$
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$

9. Sketch the graphs of these transformed functions for $x \in [0, 4\pi]$:
   (a) $y = \frac{1}{2}\cos x$
   (b) $y = \sin \frac{x}{2}$
   (c) $y = \sin(x - \frac{\pi}{3})$
   (d) $y = \sin x + 2$
   (e) $y = -\tan x$
   (f) $y = \cos(x + \frac{\pi}{6})$

10. For the function $y = 4\sin(\frac{x}{2} + \frac{\pi}{6}) - 2$:
   (a) Identify the amplitude
   (b) Find the period
   (c) Determine the phase shift
   (d) Find the vertical shift
   (e) State the range
   (f) Sketch the graph for $x \in [0, 4\pi]$

Section C: Fundamental Trigonometric Identities

11. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to find:
    (a) $\cos\theta$ if $\sin\theta = \frac{7}{25}$ and θ is acute
    (b) $\sin\theta$ if $\cos\theta = -\frac{3}{5}$ and θ is in the second quadrant
    (c) $\tan\theta$ if $\sin\theta = \frac{8}{17}$ and $\cos\theta > 0$
    (d) $\cos\theta$ if $\tan\theta = -\frac{12}{5}$ and $\sin\theta > 0$
12. Prove these reciprocal identities:
    (a) $\sin\theta \cdot \csc\theta = 1$
    (b) $\cos\theta \cdot \sec\theta = 1$
    (c) $\tan\theta \cdot \cot\theta = 1$
    (d) $\csc^2\theta = 1 + \cot^2\theta$
13. Simplify these expressions:
    (a) $\cos^2\theta(1 + \tan^2\theta)$
    (b) $\frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta}$
    (c) $(\cos\theta - \sin\theta)^2$
    (d) $\frac{1+\sin^2\theta}{\cos^2\theta}$
14. Express in terms of $\tan\theta$ only:
    (a) $\sin^2\theta$
    (b) $\cos^2\theta$
    (c) $\sec^2\theta$
    (d) $\sin^2\theta + \cos^2\theta\sec^2\theta$
15. Prove that:
    (a) $\frac{1+\tan^2\theta}{\sec\theta} = \sec\theta$
    (b) $\tan^4\theta - \sec^4\theta = -1 - 2\tan^2\theta$
    (c) $\frac{\csc\theta+\sec\theta}{\csc\theta-\sec\theta} = \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$
    (d) $(\tan\theta + \cot\theta)^2 = \sec^2\theta + \csc^2\theta$

Section D: Addition and Double Angle Formulas

16. Use the addition formulas to find the exact values:
    (a) $\sin 105°$ (using $\sin(60° + 45°)$)
    (b) $\cos 105°$ (using $\cos(60° + 45°)$)
    (c) $\tan 75°$ (using $\tan(30° + 45°)$)
    (d) $\sin\frac{\pi}{12}$ (using $\sin(\frac{\pi}{3} - \frac{\pi}{4})$)
17. Given $\sin A = \frac{5}{13}$ with A acute and $\cos B = \frac{7}{25}$ with B acute:
    (a) Find $\cos A$ and $\sin B$
    (b) Calculate $\sin(A - B)$
    (c) Calculate $\cos(A - B)$
    (d) Find $\tan(A + B)$

18. Use double angle formulas to find:
   (a) $\sin 2\theta$ if $\cos\theta = \frac{5}{13}$ and θ is acute
   (b) $\cos 2\theta$ if $\sin\theta = \frac{9}{41}$ and θ is acute
   (c) $\tan 2\theta$ if $\tan\theta = \frac{5}{12}$
   (d) $\cos 2\theta$ if $\sin\theta = -\frac{7}{25}$ and θ is in the fourth quadrant

19. Derive these alternative double angle formulas:
   (a) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
   (b) $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$
   (c) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$
   (d) $\cot 2\theta = \frac{\cot^2\theta-1}{2\cot\theta}$

20. Express in terms of multiple angles:
   (a) $2\sin^2\theta$ in terms of $\cos 2\theta$
   (b) $2\cos^2\theta$ in terms of $\cos 2\theta$
   (c) $\cos^2\theta\sin^2\theta$ in terms of $\sin 4\theta$
   (d) $\tan^2\theta$ in terms of $\cos 2\theta$

Section E: Solving Trigonometric Equations

21. Solve these equations for $0 \le x \le 2\pi$:
   (a) $\sin x = \frac{\sqrt{3}}{2}$
   (b) $\cos x = -\frac{1}{2}$
   (c) $\tan x = -\sqrt{3}$
   (d) $\sin x = -\frac{\sqrt{3}}{2}$

22. Solve these equations for $0° \le x \le 360°$:
   (a) $2\sin x + \sqrt{3} = 0$
   (b) $4\cos x - 3 = 0$
   (c) $\tan x + 1 = 0$
   (d) $3\sin^2 x = 1$

23. Solve these equations for $0 \le x \le 2\pi$:
   (a) $\sin 2x = -\frac{1}{2}$
   (b) $\cos 4x = \frac{\sqrt{3}}{2}$
   (c) $\tan 3x = 1$
   (d) $\sin(x - \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

24. Solve these quadratic trigonometric equations for $0 \le x \le 2\pi$:
   (a) $3\sin^2 x - \sin x - 2 = 0$
   (b) $2\cos^2 x - \cos x - 1 = 0$
   (c) $\tan^2 x + 2\tan x - 3 = 0$
   (d) $4\sin^2 x - 4\sin x + 1 = 0$

25. Solve these equations involving multiple angles for $0 \leq x \leq 2\pi$:

    (a) $\tan x = \cot x$

    (b) $\sin 2x = -\cos x$

    (c) $\sin 2x = 2\cos x$

    (d) $\cos 4x = \cos 2x$

Section F: Advanced Trigonometric Identities

26. Prove these sum-to-product identities using complex methods:

    (a) $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

    (b) $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

    (c) $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

    (d) $\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

27. Use sum-to-product formulas to evaluate:

    (a) $\sin 7x - \sin 3x$

    (b) $\cos 5x + \cos x$

    (c) $\sin 67.5^\circ - \sin 22.5^\circ$

    (d) $\cos 112.5^\circ + \cos 67.5^\circ$

28. Prove these t-substitution formulas where $t = \tan\frac{\theta}{2}$:

    (a) $\sin\theta = \frac{2t}{1+t^2}$

    (b) $\cos\theta = \frac{1-t^2}{1+t^2}$

    (c) $\tan\theta = \frac{2t}{1-t^2}$

    (d) $\cot\theta = \frac{1-t^2}{2t}$

29. Express using the t-substitution:

    (a) $3\sin\theta + 4\cos\theta$

    (b) $\sin\theta - 2\cos\theta$

    (c) $\frac{\sin\theta}{1+\cos\theta}$

    (d) $\frac{1}{2+\cos\theta}$

30. Prove the quadruple angle formulas:

    (a) $\sin 4\theta = 4\sin\theta\cos\theta(1 - 2\sin^2\theta)$

    (b) $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

    (c) $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

Section G: Complex Trigonometric Problems

31. Solve these equations for $0 \leq x < 2\pi$:

    (a) $\sin x - \cos x = \sqrt{2}$

    (b) $\sin x - \sin 2x = 0$

    (c) $\sin x + \sin 2x + \sin 4x = 0$

    (d) $\tan x - \tan 2x = 0$

32. Prove these advanced identities:

    (a) $\frac{\sin 4\theta}{\sin\theta} + \frac{\cos 4\theta}{\cos\theta} = 4\cos 3\theta$

    (b) $\sin^2\theta + \sin^2(\theta - \frac{2\pi}{3}) + \sin^2(\theta + \frac{2\pi}{3}) = \frac{3}{2}$

    (c) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

    (d) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ when $A + B + C = \pi$

33. Find the general solution to these equations:

    (a) $\sin x = \frac{3}{4}$

    (b) $\cos 4x = -0.6$

    (c) $\tan\frac{x}{2} = 2$

    (d) $\cos(4x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2}$

34. Express these in the form $R\sin(x + \alpha)$ or $R\cos(x + \alpha)$:

    (a) $8\sin x + 15\cos x$

    (b) $7\sin x - 24\cos x$

    (c) $2\sin x + 2\cos x$

    (d) $4\cos x + 4\sqrt{3}\sin x$

35. Find the range of these functions:

    (a) $f(x) = 8\sin x + 15\cos x$

    (b) $g(x) = 7\sin 3x - 24\cos 3x - 5$

    (c) $h(x) = \sin^2 x - 4\cos x$

    (d) $k(x) = 6\sin x\cos x + 1$

Section H: Applications of Trigonometry

36. A particle moves in simple harmonic motion with displacement $s = 8\sin(4t + \frac{\pi}{6})$ centimeters, where t is time in seconds.

    (a) Find the amplitude of the motion

    (b) Determine the period of oscillation

    (c) Find the phase shift

    (d) Calculate the displacement when $t = 0$

    (e) Find when the particle first reaches minimum displacement

37. The depth of water at a harbor is modeled by $d(t) = 2\sin(\frac{\pi t}{6} - \frac{\pi}{3}) + 5$ meters, where t is hours after midnight.

    (a) Find the maximum and minimum depths

    (b) Determine the period of the tide cycle

    (c) Find the depth at 3 AM

    (d) Calculate when low tide occurs

    (e) Find when the depth is exactly 6 meters

38. An electromagnetic wave is given by $E = 15\sin(2\pi \times 10^6 t + \frac{\pi}{4})$ volts per meter.

    (a) Find the maximum electric field strength

    (b) Determine the frequency of the wave

(c) Calculate the electric field when $t = 2.5 \times 10^{-7}$ seconds
(d) Find when the field first equals 10 V/m
(e) Determine the wavelength if the speed is 3×10^8 m/s

39. A satellite dish rotates with angular position $\theta = \frac{\pi}{6}\cos(3t - \frac{\pi}{2})$ radians, where t is time in minutes.

(a) Find the maximum angular displacement
(b) Determine the period of rotation
(c) Calculate the angular position at $t = 0$
(d) Find when the dish first reaches maximum displacement
(e) Determine the angular velocity at $t = \frac{\pi}{6}$ minutes

40. Two radio waves with equations $y_1 = 4\sin 5x$ and $y_2 = 3\cos 5x$ combine.

(a) Find the equation of the combined wave
(b) Express the result in the form $R\sin(5x + \alpha)$
(c) Determine the amplitude of the combined wave
(d) Find the phase relationship between the original waves
(e) Calculate where the waves have maximum constructive interference

Section I: Advanced Problem Solving

41. In triangle XYZ, $x = 11$, $z = 15$, and $\angle Y = 120°$.

(a) Use the cosine rule to find side y
(b) Use the sine rule to find $\angle X$
(c) Calculate the area of the triangle
(d) Find the radius of the circumcircle
(e) Determine the length of the median from Y to side XZ

42. Prove that in any triangle XYZ:

(a) $\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z} = 2R$ (sine rule)
(b) $x^2 = y^2 + z^2 - 2yz\cos X$ (cosine rule)
(c) $\tan\frac{X}{2} = \frac{r}{s-x}$ where r is inradius and s is semiperimeter
(d) Area $= rs$ where r is inradius and s is semiperimeter

43. A regular octagon is inscribed in a circle of radius r.

(a) Find the central angle for each sector
(b) Calculate the side length of the octagon
(c) Find the area of the octagon
(d) Determine the apothem (distance from center to side)
(e) Calculate the perimeter of the octagon

44. The function $h(x) = m\sin 2x + n\cos 2x$ has maximum value 15 and minimum value -15.

(a) Express $h(x)$ in the form $R\sin(2x + \gamma)$
(b) Find the relationship between m and n
(c) If $h(\frac{\pi}{8}) = 12$, find the values of m and n

(d) Solve $h(x) = 9$ for $0 \leq x \leq \pi$

(e) Find the values of x where $h'(x) = 0$

45. Consider the Chebyshev polynomial $T_4(x) = 8x^4 - 8x^2 + 1$ where $T_4(\cos\theta) = \cos 4\theta$.

(a) Verify this identity for $\theta = \frac{\pi}{4}$

(b) Use this to solve $\cos 4\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$

(c) Find the exact values of $\cos\frac{\pi}{8}$ and $\cos\frac{3\pi}{8}$

(d) Express $\sin 4\theta$ using Chebyshev polynomials

(e) Use these results to find the vertices of a regular octagon

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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