

A Level Mechanics

Practice Test 5: Work, Energy, and Power

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Draw diagrams where appropriate to illustrate your solutions.

Time allowed: 3 hours

Section A: Work and Energy Fundamentals [29 marks]

Question 1 [10 marks]

- (a) Define work done by a force and explain why it depends on both the magnitude and direction of the force. [3 marks]
- (b) Explain the physical significance of positive, negative, and zero work with practical examples. [4 marks]
- (c) Distinguish between work and energy, explaining how they are related through the work-energy theorem. [3 marks]

Question 2 [12 marks] Five forces act on an object: $\vec{F}_1 = (18\hat{i} + 12\hat{j})$ N, $\vec{F}_2 = (-8\hat{i} - 15\hat{j})$ N, $\vec{F}_3 = (14\hat{i} - 6\hat{j})$ N, $\vec{F}_4 = (-12\hat{i} + 20\hat{j})$ N, and $\vec{F}_5 = (6\hat{i} - 8\hat{j})$ N. The object moves from $(3, -1)$ m to $(11, 7)$ m.

- (a) Calculate the resultant force vector. [2 marks]
- (b) Determine the displacement vector and its magnitude. [2 marks]
- (c) Calculate the work done by each individual force. [4 marks]
- (d) Find the total work done using the individual force method. [2 marks]
- (e) Verify your result using the resultant force approach. [2 marks]

Question 3 [7 marks] A 12 kg object initially at rest experiences a constant force of 72 N at 60° to its eventual direction of motion for 4.5 seconds.

- (a) Calculate the component of force causing acceleration and find the acceleration. [3 marks]
- (b) Determine the distance traveled and the work done by the applied force. [3 marks]
- (c) Calculate the final kinetic energy and verify the work-energy theorem. [1 mark]

Section B: Work Done by Constant Forces [26 marks]

Question 4 [14 marks] A heavy box of mass 42 kg is hauled 35 m up a rough inclined plane at 22° to the horizontal by a cable exerting 280 N force parallel to the incline. The coefficient of kinetic friction is 0.18.

- (a) Draw a comprehensive force diagram showing all forces. [2 marks]
- (b) Calculate the normal force and the friction force. [3 marks]
- (c) Find the work done by: (i) the cable force, (ii) gravity, (iii) friction force. [5 marks]
- (d) Determine the net work done on the box. [2 marks]
- (e) Calculate the final speed if the box starts from rest. [2 marks]

Question 5 [12 marks] A high-speed train of mass 180,000 kg accelerates from 25 m/s to 65 m/s over 1500 m while climbing a 1.5° gradient. Total resistance forces amount to 28,000 N.

- (a) Calculate the change in kinetic energy of the train. [3 marks]
- (b) Find the work done against gravity during the climb. [3 marks]
- (c) Calculate the work done against resistance forces. [2 marks]
- (d) Determine the total work done by the traction motors. [2 marks]
- (e) Find the average traction force during acceleration. [2 marks]

Section C: Variable Force and Work-Energy Theorem [32 marks]

Question 6 [18 marks] A particle of mass 3.5 kg moves along the x-axis under a variable force $F = 25x - 5x^3$ N, where x is measured in metres.

- (a) Calculate the work done as the particle moves from $x = 0$ to $x = 2.5$ m. [4 marks]
- (b) Find the work done as the particle moves from $x = 1$ m to $x = 3$ m. [4 marks]
- (c) If the particle has speed 2.5 m/s at $x = 0.8$ m, calculate its speed at $x = 2.2$ m. [4 marks]
- (d) Determine all positions where the force is zero in the interval $[0, 3]$. [3 marks]
- (e) Find the position where the force reaches its maximum value. [3 marks]

Question 7 [14 marks]

- (a) State and prove the work-energy theorem for a particle subject to multiple variable forces. [4 marks]
- (b) A projectile of mass 2.2 kg is launched at 55° above horizontal with initial speed 35 m/s. Calculate the speed when it reaches 22 m above launch level. [4 marks]
- (c) A 28 kg block slides down a rough inclined plane (angle 45°) for 15 m. With friction coefficient 0.32, find the final speed starting from rest. [4 marks]
- (d) A 15 kg object experiences a sinusoidal force $F(t) = 60\sin(3t)$ N for $\frac{2\pi}{3}$ seconds from rest. Calculate the final kinetic energy. [2 marks]

Section D: Gravitational Potential Energy [27 marks]**Question 8 [9 marks]**

- (a) Define gravitational potential energy and explain the concept of a conservative force field. [3 marks]
- (b) Prove rigorously that gravitational work is path-independent using vector calculus. [4 marks]
- (c) Discuss how different reference levels affect energy calculations and when this matters. [2 marks]

Question 9 [18 marks] A cannonball of mass 4.5 kg is fired from the top of a 35 m tower with initial velocity 45 m/s at 52° above horizontal.

- (a) Calculate the initial kinetic and potential energies (ground reference). [3 marks]
- (b) Find the total mechanical energy of the projectile. [1 mark]
- (c) Use energy conservation to determine the maximum height above ground reached. [4 marks]
- (d) Calculate the speed when the cannonball is 50 m above ground. [3 marks]
- (e) Find the impact speed when it hits the ground. [3 marks]
- (f) At what height is the kinetic energy exactly double the potential energy? [2 marks]
- (g) Determine the horizontal and vertical velocity components at the peak of trajectory. [2 marks]

Section E: Elastic Potential Energy [25 marks]

Question 10 [15 marks] A spring system has spring constant $k = 2000 \text{ N/m}$ and is compressed 0.35 m from its natural length.

- (a) Calculate the elastic potential energy stored in the spring. [2 marks]
- (b) A 3.2 kg mass is attached and released. Find the maximum speed achieved. [3 marks]
- (c) At what compression does the mass reach speed 6 m/s? [3 marks]
- (d) Calculate the acceleration when the spring is compressed by 0.28 m. [2 marks]
- (e) Find the position where kinetic energy equals twice the potential energy. [3 marks]
- (f) Determine the spring force at maximum compression and at equilibrium. [2 marks]

Question 11 [10 marks] A 6 kg block is suspended from a vertical spring with spring constant 1200 N/m. The block is pulled down 0.40 m below equilibrium and released.

- (a) Calculate the equilibrium extension when the block hangs at rest. [2 marks]
- (b) Find the total mechanical energy of the oscillating system. [2 marks]
- (c) Determine the maximum speed during the oscillation. [2 marks]
- (d) Calculate the speed when the block is 0.30 m below equilibrium. [2 marks]
- (e) Find the maximum height the block reaches above its release point. [2 marks]

Section F: Conservation of Mechanical Energy [30 marks]

Question 12 [18 marks] A pendulum bob of mass 2.5 kg hangs from a 3.5 m string and is released from rest when the string makes 52° with the vertical.

- (a) Calculate the initial height of the bob above its lowest position. [3 marks]
- (b) Find the speed at the bottom of the swing using energy conservation. [3 marks]
- (c) Calculate the tension in the string at the lowest point. [3 marks]
- (d) Determine the speed when the string makes 28° with the vertical. [3 marks]
- (e) Find the minimum release angle required for the bob to complete a full vertical circle. [3 marks]
- (f) Calculate the tension at the top and bottom of the circle for this critical case. [3 marks]

Question 13 [12 marks] A 1400 kg roller coaster car starts from rest at 48 m height and travels on a track with significant energy losses.

- (a) Calculate the speed at ground level with no friction. [2 marks]
- (b) The car then rises to 32 m height. Find the speed there assuming no friction. [3 marks]
- (c) If friction dissipates 55,000 J total energy, calculate the actual speed at 32 m height. [3 marks]
- (d) For a vertical circular loop of radius 8.5 m at ground level, find the minimum starting height with this friction level. [4 marks]

Section G: Power Calculations [27 marks]

Question 14 [11 marks]

- (a) Define average and instantaneous power, giving examples where each is most appropriate. [3 marks]
- (b) Derive the fundamental power equation $P = \vec{F} \cdot \vec{v}$ from the definition of work. [4 marks]
- (c) Explain why power limitations are crucial in engineering applications, giving specific examples. [4 marks]

Question 15 [16 marks] A bridge construction crane operates under demanding conditions with heavy loads.

- (a) Calculate the power required to lift 2500 kg at constant speed 2.8 m/s. [2 marks]
- (b) With motor efficiency 84%, find the electrical power input needed. [2 marks]
- (c) The crane accelerates 2200 kg upward at 1.3 m/s^2 from rest. Calculate the motor force required. [3 marks]
- (d) Find the power output after 8 seconds of this acceleration. [3 marks]
- (e) Calculate the total work done during the first 12 seconds of acceleration. [3 marks]
- (f) If the crane then operates at constant speed for 25 seconds, determine the total energy consumed. [3 marks]

Section H: Power in Vehicle Motion [23 marks]

Question 16 [15 marks] A Formula 1 racing car engine delivers constant power 180 kW. The car has mass 740 kg and experiences air resistance $F_r = 280 + 3.2v^2$ N, where v is speed in m/s.

- (a) Find the theoretical maximum speed by equating driving force to air resistance. [3 marks]
- (b) Calculate the acceleration when the car travels at 35 m/s. [4 marks]
- (c) Determine the speed when acceleration equals 2.5 m/s^2 . [3 marks]
- (d) Estimate the time needed to accelerate from 30 m/s to 50 m/s. [5 marks]

Question 17 [8 marks] A loaded freight truck (total mass 42,000 kg) operates with constant engine power 220 kW.

- (a) On level highway with 1800 N resistance, find the maximum speed. [2 marks]
- (b) When climbing a 3.5° gradient at steady speed, calculate the maximum speed achievable. [3 marks]
- (c) What power would be required to maintain 18 m/s up this gradient? [3 marks]

Section I: Energy in Collisions and Springs [25 marks]

Question 18 [15 marks] An 8 kg object moving at 16 m/s undergoes an elastic collision with a 12 kg object initially at rest.

- (a) Apply conservation of momentum and energy to find both final velocities. [5 marks]
- (b) Calculate kinetic energies before and after collision to verify energy conservation. [3 marks]
- (c) Find the velocity change experienced by each object. [2 marks]
- (d) Calculate the impulse magnitude for each object during the collision. [2 marks]
- (e) Determine the fraction of kinetic energy transferred between the objects. [3 marks]

Question 19 [10 marks] A 0.35 kg ball moving horizontally at 24 m/s collides with a wall and rebounds at 19 m/s.

- (a) Calculate the momentum change and impulse delivered by the wall. [3 marks]
- (b) Find the kinetic energies before and after the collision. [2 marks]
- (c) Determine the energy dissipated and the coefficient of restitution. [3 marks]
- (d) With contact time 0.018 seconds, calculate the average impact force magnitude. [2 marks]

Section J: Comprehensive Applications [31 marks]

Question 20 [19 marks] A 9 kg block slides down a frictionless curved track from height 8.5 m, then compresses a spring ($k = 3500 \text{ N/m}$) on a horizontal surface with friction coefficient 0.14.

- (a) Calculate the speed just before the block contacts the spring. [2 marks]
- (b) If the horizontal surface were frictionless, find the maximum spring compression. [3 marks]
- (c) With friction present, set up the energy conservation equation for maximum compression. [3 marks]

- (d) Calculate the actual maximum compression including friction effects. [4 marks]
- (e) Find the speed when the block leaves the spring on its return journey. [3 marks]
- (f) Determine how high the block rises when it returns up the curved track. [2 marks]
- (g) Calculate the total mechanical energy dissipated by friction during the complete cycle. [2 marks]

Question 21 [12 marks] A geothermal power plant harnesses energy from underground steam. Superheated steam at 180°C drives turbines, with $280 \text{ m}^3/\text{s}$ steam flow and density 0.85 kg/m^3 . The steam velocity entering turbines is 45 m/s .

- (a) Calculate the mass flow rate of steam through the turbines. [2 marks]
- (b) Find the kinetic energy per second in the steam flow. [3 marks]
- (c) If turbine efficiency is 91%, calculate the electrical power output. [2 marks]
- (d) The plant supplies 320 MW to the electrical grid. What steam velocity would be needed to meet this demand? [3 marks]
- (e) Discuss the environmental advantages of geothermal power over fossil fuel alternatives. [2 marks]

Physics Data and Formulae

Work and Energy:

$$\text{Work done: } W = \vec{F} \cdot \vec{s} = Fs \cos \theta$$

$$\text{Variable force: } W = \int_{s_1}^{s_2} F(s) ds$$

$$\text{Kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{Gravitational potential energy: } PE_g = mgh$$

$$\text{Elastic potential energy: } PE_e = \frac{1}{2}kx^2$$

Work-Energy Theorem:

$$W_{net} = \Delta KE = KE_f - KE_i$$

Conservation of Energy:

$$E_{total} = KE + PE = \text{constant (conservative forces only)}$$

$$KE_i + PE_i = KE_f + PE_f + \text{energy lost to friction/inelastic processes}$$

Power:

$$P = \frac{W}{t} \text{ (average), } P = \frac{dW}{dt} \text{ (instantaneous)}$$

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

$$\text{Efficiency: } \eta = \frac{P_{useful}}{P_{input}} \times 100\%$$

Springs and Simple Harmonic Motion:

$$\text{Hooke's Law: } F = -kx$$

$$\text{Equilibrium position: } x_0 = \frac{mg}{k} \text{ (vertical spring)}$$

$$\text{Maximum speed: } v_{max} = A\sqrt{\frac{k}{m}} \text{ (amplitude } A)$$

$$\text{Total energy: } E = \frac{1}{2}kA^2$$

Circular Motion and Loops:

$$\text{At bottom of vertical loop: } T - mg = \frac{mv^2}{r}$$

$$\text{At top of vertical loop: } mg + T = \frac{mv^2}{r}$$

$$\text{Minimum speed at top: } v_{min} = \sqrt{gr}$$

Collisions:

Coefficient of restitution: $e = \frac{\text{relative speed of separation}}{\text{relative speed of approach}}$

Elastic collision: $e = 1$, kinetic energy conserved

Perfectly inelastic: $e = 0$, objects stick together

Impulse-momentum theorem: $J = \Delta p = F_{avg} \Delta t$

Constants:

Acceleration due to gravity: $g = 9.8 \text{ m/s}^2$

Density of water: $\rho = 1000 \text{ kg/m}^3$

Trigonometric Values:

$\sin 1.5 = 0.026$, $\cos 1.5 = 1.000$, $\tan 1.5 = 0.026$

$\sin 3.5 = 0.061$, $\cos 3.5 = 0.998$, $\tan 3.5 = 0.061$

$\sin 22 = 0.375$, $\cos 22 = 0.927$, $\tan 22 = 0.404$

$\sin 28 = 0.469$, $\cos 28 = 0.883$, $\tan 28 = 0.532$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

$\sin 52 = 0.788$, $\cos 52 = 0.616$, $\tan 52 = 1.280$

$\sin 55 = 0.819$, $\cos 55 = 0.574$, $\tan 55 = 1.428$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 295

Grade boundaries: A* 266, A 236, B 207, C 177, D 148, E 118

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