A Level Mechanics
Practice Test 6: Work, Energy, and Power

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Work and Energy Fundamentals [30 marks]

Question 1 [11 marks]

(a) Define work done by a force and explain how the scalar nature of work emerges from vector quantities. **[3 marks]**

(b) Analyze the conditions for maximum, minimum, and zero work done, providing mathematical justification. **[4 marks]**

(c) Explain the fundamental relationship between work and energy, stating the principle that connects them. **[4 marks]**

Question 2 [13 marks] Six forces act simultaneously on an object: $\vec{F_1} = (22\hat{i} - 14\hat{j})$ N, $\vec{F_2} = (-16\hat{i} + 28\hat{j})$ N, $\vec{F_3} = (18\hat{i} + 8\hat{j})$ N, $\vec{F_4} = (-12\hat{i} - 18\hat{j})$ N, $\vec{F_5} = (8\hat{i} + 15\hat{j})$ N, and $\vec{F_6} = (-14\hat{i} - 6\hat{j})$ N. The object moves from $(2, -3)$ m to $(14, 9)$ m.

(a) Calculate the resultant force vector and its magnitude. **[3 marks]**

(b) Determine the displacement vector and calculate its magnitude and direction. **[3 marks]**

(c) Calculate the work done by each individual force. **[4 marks]**

(d) Find the total work done using the summation of individual works. **[2 marks]**

(e) Verify your result using the resultant force method. **[1 mark]**

Question 3 [6 marks] A 15 kg object moving at 3 m/s experiences a constant force of 90 N at 53° to its direction of motion for 5 seconds.

(a) Calculate the effective force component and resulting acceleration. **[2 marks]**

(b) Determine the distance traveled and work done by the applied force. **[3 marks]**

(c) Calculate the final kinetic energy and verify the work-energy relationship. **[1 mark]**

Section B: Work Done by Constant Forces [27 marks]

Question 4 [**15 marks**] A massive shipping container of mass 8000 kg is dragged 45 m up a loading ramp inclined at 25° to the horizontal. The pulling force is 35,000 N acting parallel to the ramp surface, and the coefficient of kinetic friction is 0.22.

(a) Draw a complete free body diagram showing all forces acting on the container. [**2 marks**]

(b) Calculate the normal force between the container and ramp surface. [**2 marks**]

(c) Determine the friction force opposing the motion. [**2 marks**]

(d) Calculate the work done by: (i) the pulling force, (ii) gravitational force, (iii) friction force. [**6 marks**]

(e) Find the net work done on the container. [**2 marks**]

(f) Calculate the final speed if the container starts from rest. [**1 mark**]

Question 5 [**12 marks**] A supersonic jet aircraft of mass 85,000 kg accelerates from 180 m/s to 280 m/s over a distance of 2500 m while climbing at a 3° angle. The aircraft faces air resistance and drag forces totaling 45,000 N.

(a) Calculate the change in kinetic energy of the aircraft. [**3 marks**]

(b) Find the work done against gravitational force during the climb. [**3 marks**]

(c) Calculate the work done against air resistance and drag. [**2 marks**]

(d) Determine the total work done by the jet engines. [**2 marks**]

(e) Find the average engine thrust force during this maneuver. [**2 marks**]

Section C: Variable Force and Work-Energy Theorem [33 marks]

Question 6 [**19 marks**] A particle of mass 4.2 kg moves along the x-axis under a variable force $F = 30x - 6x^3$ N, where x is measured in metres.

(a) Calculate the work done as the particle moves from $x = 0$ to $x = 2.8$ m. [**5 marks**]

(b) Find the work done as the particle moves from $x = 1.2$ m to $x = 2.5$ m. [**4 marks**]

(c) If the particle has speed 3.2 m/s at $x = 0.6$ m, calculate its speed at $x = 2.1$ m. [**4 marks**]

(d) Determine all equilibrium positions where the force equals zero. [**3 marks**]

(e) Find the position where the force reaches its maximum value in the interval [0, 3]. [**3 marks**]

Question 7 [**14 marks**]

(a) State and rigorously prove the work-energy theorem for a particle under multiple time-varying forces. [**5 marks**]

(b) A rocket projectile of mass 2.8 kg is launched at 62° above horizontal with initial speed 42 m/s. Calculate the speed when it reaches 28 m above the launch point. [**4 marks**]

(c) A 35 kg crate slides down a rough inclined plane (angle 38°) for 18 m. With friction coefficient 0.26, find the final speed starting from rest. [**4 marks**]

(d) An object of mass 18 kg experiences an exponential force $F(t) = 80e^{-0.5t}$ N for 6 seconds from rest. Calculate the final kinetic energy. [**1 mark**]

Section D: Gravitational Potential Energy [28 marks]

Question 8 [10 marks]

(a) Define gravitational potential energy and establish the conditions under which gravitational force is conservative. **[3 marks]**

(b) Prove mathematically that gravitational work is independent of the path taken between two points. **[4 marks]**

(c) Analyze how the choice of reference level affects energy calculations and when absolute values matter. **[3 marks]**

Question 9 [18 marks] A military projectile of mass 6.5 kg is launched from the top of a 48 m cliff with initial velocity 52 m/s at 58° above horizontal.

(a) Calculate the initial kinetic and potential energies using ground level as reference. **[3 marks]**

(b) Find the total mechanical energy of the projectile system. **[1 mark]**

(c) Use energy conservation to determine the maximum height above ground level reached. **[4 marks]**

(d) Calculate the speed when the projectile is 65 m above ground level. **[3 marks]**

(e) Find the impact speed when the projectile strikes the ground. **[3 marks]**

(f) At what height above ground is the kinetic energy exactly 2.5 times the potential energy? **[2 marks]**

(g) Determine the velocity components (horizontal and vertical) at the trajectory peak. **[2 marks]**

Section E: Elastic Potential Energy [26 marks]

Question 10 [16 marks] A sophisticated spring system has spring constant $k = 2800$ N/m and is compressed 0.42 m from its equilibrium position.

(a) Calculate the elastic potential energy stored in the compressed spring. **[2 marks]**

(b) A 4.5 kg mass is attached to the spring and released. Find the maximum speed achieved. **[3 marks]**

(c) At what compression distance does the mass achieve speed 7.5 m/s? **[4 marks]**

(d) Calculate the acceleration when the spring compression is 0.32 m. **[2 marks]**

(e) Find the position where kinetic energy equals 2.5 times the potential energy. **[3 marks]**

(f) Determine the spring force magnitude at maximum compression and at the equilibrium position. **[2 marks]**

Question 11 [10 marks] A 7.5 kg block hangs from a vertical spring with spring constant 1800 N/m. The block is displaced 0.45 m below its equilibrium position and released.

(a) Calculate the equilibrium extension when the block hangs at rest. **[2 marks]**

(b) Find the total mechanical energy of the oscillating system. **[2 marks]**

(c) Determine the maximum speed achieved during oscillation. **[2 marks]**

(d) Calculate the speed when the block is 0.35 m below equilibrium. **[2 marks]**

(e) Find the maximum height the block reaches above its initial release point. **[2 marks]**

Section F: Conservation of Mechanical Energy [31 marks]

Question 12 [**19 marks**] A pendulum bob of mass 3.2 kg is suspended by a 4 m string and released from rest when the string makes 56° with the vertical.

(a) Calculate the initial height of the bob above its lowest position. [**3 marks**]

(b) Find the speed at the bottom of the swing using conservation of energy. [**3 marks**]

(c) Calculate the tension in the string at the lowest point of the swing. [**3 marks**]

(d) Determine the speed when the string makes 32° with the vertical. [**3 marks**]

(e) Find the minimum release angle required for the bob to just complete a full vertical circle. [**4 marks**]

(f) Calculate the tension forces at the top and bottom of the circle for this critical case. [**3 marks**]

Question 13 [**12 marks**] A 1600 kg roller coaster car begins from rest at 52 m height and travels on a complex track with substantial energy losses due to friction and air resistance.

(a) Calculate the theoretical speed at ground level assuming no energy losses. [**2 marks**]

(b) The car then climbs to 35 m height. Find the speed there with no friction. [**3 marks**]

(c) If friction and air resistance dissipate 68,000 J total energy, calculate the actual speed at 35 m height. [**3 marks**]

(d) For a vertical circular loop of radius 9.5 m at ground level, determine the minimum starting height required with these energy losses. [**4 marks**]

Section G: Power Calculations [28 marks]

Question 14 [**12 marks**]

(a) Define average and instantaneous power, explaining their practical applications in engineering systems. [**4 marks**]

(b) Derive the fundamental power equation $P = \vec{F} \cdot \vec{v}$ starting from the definition of work. [**4 marks**]

(c) Explain why power considerations are critical in mechanical system design, providing specific engineering examples. [**4 marks**]

Question 15 [**16 marks**] A massive shipyard crane handles extremely heavy loads in demanding marine construction projects.

(a) Calculate the power required to lift 3500 kg at constant speed 3.2 m/s. [**2 marks**]

(b) With motor efficiency 86%, find the electrical power input needed for this operation. [**2 marks**]

(c) The crane accelerates 3200 kg upward at 1.5 m/s^2 from rest. Calculate the required motor force. [**3 marks**]

(d) Find the power output after 10 seconds of this acceleration phase. [**3 marks**]

(e) Calculate the total work done during the first 15 seconds of acceleration. [**3 marks**]

(f) If the crane then operates at constant speed for 30 seconds, determine the total energy consumed during the complete cycle. [**3 marks**]

Section H: Power in Vehicle Motion [24 marks]

Question 16 [16 marks] A high-performance electric sports car has motors delivering constant power 250 kW. The car has mass 1800 kg and experiences combined air and rolling resistance $F_r = 420 + 2.8v^2$ N, where v is speed in m/s.

(a) Find the theoretical maximum speed by equating motor driving force to total resistance. **[4 marks]**

(b) Calculate the acceleration when the car travels at 42 m/s. **[4 marks]**

(c) Determine the speed when acceleration equals 3.2 m/s². **[4 marks]**

(d) Estimate the time required to accelerate from 35 m/s to 55 m/s using appropriate approximations. **[4 marks]**

Question 17 [8 marks] A fully loaded articulated truck (total mass 65,000 kg) operates with constant engine power 280 kW on varied terrain.

(a) On level motorway with 2200 N total resistance, find the maximum sustainable speed. **[2 marks]**

(b) When climbing a 2.8° gradient at steady speed, calculate the maximum speed achievable. **[3 marks]**

(c) What engine power would be required to maintain 22 m/s up this gradient? **[3 marks]**

Section I: Energy in Collisions and Springs [26 marks]

Question 18 [16 marks] A 10 kg object moving at 18 m/s undergoes a perfectly elastic collision with a 15 kg object initially at rest.

(a) Apply conservation of momentum and kinetic energy to determine both final velocities. **[6 marks]**

(b) Calculate kinetic energies before and after collision to verify energy conservation. **[3 marks]**

(c) Find the velocity change experienced by each object during the collision. **[2 marks]**

(d) Calculate the impulse magnitude delivered to each object. **[2 marks]**

(e) Determine what fraction of the initial kinetic energy is transferred between the objects. **[3 marks]**

Question 19 [10 marks] A 0.42 kg ball moving horizontally at 28 m/s strikes a rigid wall perpendicularly and rebounds at 22 m/s.

(a) Calculate the momentum change and the impulse delivered by the wall. **[3 marks]**

(b) Find the kinetic energies before and after the collision. **[2 marks]**

(c) Determine the energy dissipated and calculate the coefficient of restitution. **[3 marks]**

(d) With contact time 0.022 seconds, calculate the average impact force magnitude. **[2 marks]**

Section J: Comprehensive Applications [32 marks]

Question 20 [20 marks] A 12 kg block slides down a frictionless curved track from height 9.5 m, then encounters a spring system (k = 4200 N/m) on a horizontal surface with friction coefficient 0.12.

(a) Calculate the speed just before the block makes contact with the spring. **[2 marks]**

(b) If the horizontal surface were completely frictionless, find the maximum spring compression. **[3 marks]**

(c) With friction present, establish the complete energy conservation equation for maximum compression. **[4 marks]**

(d) Calculate the actual maximum compression accounting for friction effects. **[4 marks]**

(e) Find the speed when the block separates from the spring on its return journey. **[3 marks]**

(f) Determine the maximum height the block reaches when it returns up the curved track. **[2 marks]**

(g) Calculate the total mechanical energy dissipated by friction during the entire motion cycle. **[2 marks]**

Question 21 [12 marks] A cutting-edge wave energy converter harnesses ocean wave power. Ocean waves with 2.5 m amplitude move at 8 m/s, with water density 1025 kg/m^3. The converter captures energy from a 150 m^2 surface area.

(a) Calculate the mass of water involved per wave cycle over the capture area. **[3 marks]**

(b) Find the kinetic energy per unit volume in the wave motion. **[2 marks]**

(c) Calculate the theoretical power available from the wave motion over the capture area. **[3 marks]**

(d) If the converter efficiency is 28%, determine the electrical power output. **[2 marks]**

(e) Discuss the advantages and challenges of wave energy compared to other renewable sources. **[2 marks]**

Physics Data and Formulae

Work and Energy:

Work done: $W = \vec{F} \cdot \vec{s} = Fs\cos\theta$

Variable force: $W = \int_{s_1}^{s_2} F(s)\, ds$

Kinetic energy: $KE = \frac{1}{2}mv^2$

Gravitational potential energy: $PE_g = mgh$

Elastic potential energy: $PE_e = \frac{1}{2}kx^2$

Work-Energy Theorem:

$W_{net} = \Delta KE = KE_f - KE_i$

Conservation of Energy:

$E_{total} = KE + PE = \text{constant}$ (conservative forces only)

$KE_i + PE_i = KE_f + PE_f +$ energy dissipated by non-conservative forces

Power:

$P = \frac{W}{t}$ (average), $P = \frac{dW}{dt}$ (instantaneous)

$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$

Efficiency: $\eta = \frac{P_{useful}}{P_{input}} \times 100\%$

Springs and Oscillations:

Hooke's Law: $F = -kx$

Equilibrium position: $x_0 = \frac{mg}{k}$ (vertical spring)

Maximum speed in SHM: $v_{max} = A\sqrt{\frac{k}{m}}$ (amplitude A)

Total energy in SHM: $E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$

Circular Motion:

At bottom of vertical loop: $T - mg = \frac{mv^2}{r}$

At top of vertical loop: $mg + T = \frac{mv^2}{r}$

Critical speed at top: $v_{critical} = \sqrt{gr}$

Collisions:

Coefficient of restitution: $e = \frac{\text{relative speed of separation}}{\text{relative speed of approach}}$

Elastic collision: $e = 1$, kinetic energy fully conserved

Perfectly inelastic: $e = 0$, objects stick together after collision

Impulse-momentum theorem: $J = \Delta p = F_{average} \times \Delta t$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s²

Density of water: $\rho = 1000$ kg/m³

Density of seawater: $\rho = 1025$ kg/m³

Trigonometric Values:

$\sin 2.8 = 0.049$, $\cos 2.8 = 0.999$, $\tan 2.8 = 0.049$

$\sin 3 = 0.052$, $\cos 3 = 0.999$, $\tan 3 = 0.052$

$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$

$\sin 32 = 0.530$, $\cos 32 = 0.848$, $\tan 32 = 0.625$

$\sin 38 = 0.616$, $\cos 38 = 0.788$, $\tan 38 = 0.781$

$\sin 53 = 0.799$, $\cos 53 = 0.602$, $\tan 53 = 1.327$

$\sin 56 = 0.829$, $\cos 56 = 0.559$, $\tan 56 = 1.483$

$\sin 58 = 0.848$, $\cos 58 = 0.530$, $\tan 58 = 1.600$

$\sin 62 = 0.883$, $\cos 62 = 0.469$, $\tan 62 = 1.881$

END OF TEST

Total marks: 305

Grade boundaries: A* 275, A 244, B 214, C 183, D 153, E 122

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