

A Level Mechanics

Practice Test 4: Elasticity and Springs

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Draw clear, labelled diagrams where appropriate.

Time allowed: 2 hours 15 minutes

Section A: Hooke's Law and Elastic Behavior [50 marks]

Question 1 [15 marks] A spring has a natural length of 30.0 cm. The table below shows how the total length varies with applied force:

Force (N)	Total Length (cm)
0	30.0
2	32.5
4	35.0
6	37.5
8	40.0
10	42.5
12	45.0

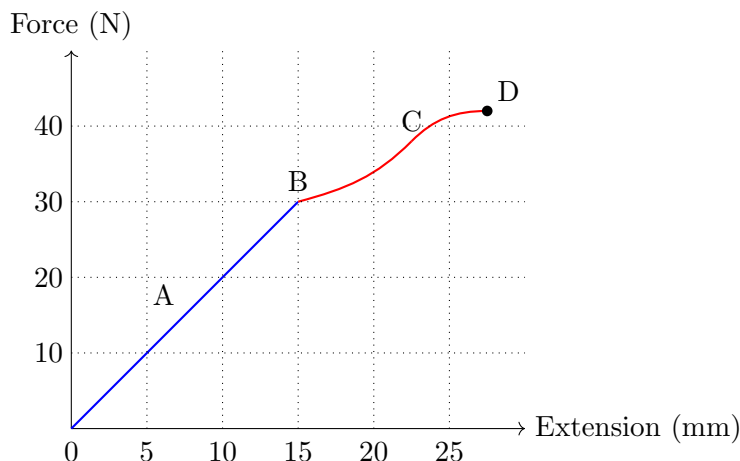
- (a) Calculate the extension for each applied force and complete a force-extension table. [3 marks]
- (b) Plot a graph of force against extension. [4 marks]
- (c) Use your graph to determine the spring constant. [3 marks]
- (d) Write the equation relating force and extension for this spring. [2 marks]
- (e) Calculate the force needed to extend the spring to a total length of 48.0 cm. [3 marks]

Question 2 [20 marks] A physics teacher demonstrates elastic behavior using two springs, P and Q, with spring constants 80 N/m and 120 N/m respectively.

- (a) Define the term "spring constant" and state its SI unit. [3 marks]
- (b) The springs are connected in series and a force of 24 N is applied. Calculate:
 - (i) The extension of spring P. [3 marks]
 - (ii) The extension of spring Q. [3 marks]
 - (iii) The total extension of the series combination. [2 marks]
 - (iv) The effective spring constant of the series combination. [3 marks]
- (c) The same springs are now connected in parallel. When the same 24 N force is applied:

- (i) Calculate the force in each spring. [2 marks]
- (ii) Find the extension of the parallel combination. [2 marks]
- (iii) Calculate the effective spring constant of the parallel combination. [2 marks]

Question 3 [15 marks] The force-extension graph below shows the behavior of an elastic material up to its breaking point:



- (a) Describe what is happening to the material in each region: AB, BC, CD. [6 marks]
- (b) Calculate the spring constant in the elastic region. [3 marks]
- (c) At which point does the material cease to obey Hooke's Law? [2 marks]
- (d) Calculate the elastic potential energy stored at point B. [4 marks]

Section B: Elastic Potential Energy and Applications [55 marks]

Question 4 [18 marks]

- (a) Define elastic potential energy. [2 marks]
- (b) Starting from the definition of work done, derive the expression for elastic potential energy stored in a spring: $E_p = \frac{1}{2}kx^2$. [6 marks]
- (c) Explain what is meant by elastic potential energy in terms of:
 - (i) The work done on the spring. [2 marks]
 - (ii) The energy that can be recovered from the spring. [2 marks]
- (d) A spring with spring constant 200 N/m is compressed by 8.0 cm. Calculate the elastic potential energy stored. [3 marks]
- (e) If this energy is completely converted to kinetic energy of a 0.5 kg mass, calculate the velocity of the mass. [3 marks]

Question 5 [22 marks] A catapult consists of a spring with spring constant 500 N/m. The spring is compressed by 15 cm and used to launch a 50 g projectile.

- (a) Calculate the elastic potential energy stored in the compressed spring. [3 marks]
- (b) Assuming 90% energy efficiency, calculate:
 - (i) The kinetic energy of the projectile as it leaves the catapult. [2 marks]

- (ii) The launch velocity of the projectile. [3 marks]
- (iii) The energy lost due to friction and other factors. [2 marks]
- (c) The projectile is launched horizontally from a height of 2.0 m above the ground. Calculate:
 - (i) The time of flight. [3 marks]
 - (ii) The horizontal distance traveled. [3 marks]
 - (iii) The velocity components just before impact with the ground. [4 marks]
 - (iv) The total kinetic energy just before impact. [2 marks]

Question 6 [15 marks] A trampolinist of mass 65 kg lands on a trampoline and compresses the springs by 0.80 m before bouncing back up.

- (a) If the trampoline can be modeled as a single spring, calculate the effective spring constant. [4 marks]
- (b) Calculate the elastic potential energy stored in the compressed trampoline. [3 marks]
- (c) Assuming 85% of this energy is returned to the trampolinist, calculate:
 - (i) The kinetic energy of the trampolinist as they leave the trampoline. [2 marks]
 - (ii) Their velocity as they leave the trampoline surface. [3 marks]
 - (iii) The maximum height they reach above the trampoline surface. [3 marks]

Section C: Stress, Strain and Material Properties [60 marks]

Question 7 [20 marks]

- (a) Define the following terms:
 - (i) Tensile stress, including its SI unit. [3 marks]
 - (ii) Tensile strain, explaining why it has no units. [3 marks]
 - (iii) Young's modulus, including its SI unit. [3 marks]
- (b) Write the equation that defines Young's modulus in terms of:
 - (i) Stress and strain. [2 marks]
 - (ii) Force, area, length and extension. [3 marks]
- (c) Explain what a large value of Young's modulus tells us about the mechanical properties of a material. [3 marks]
- (d) State two factors that might affect the Young's modulus of a material. [3 marks]

Question 8 [25 marks] A crane uses a steel cable with the following specifications:

- Length: 60.0 m
- Cross-sectional area: $2.5 \times 10 \text{ m}^2$
- Young's modulus for steel: $2.1 \times 10^{11} \text{ Pa}$
- Density of steel: 7800 kg/m^3

- (a) Calculate the mass of the cable itself. [3 marks]
- (b) When the cable supports a load of 15,000 N (in addition to its own weight), calculate:

- (i) The total force in the cable at the top. **[3 marks]**
 - (ii) The tensile stress in the cable. **[3 marks]**
 - (iii) The tensile strain in the cable. **[3 marks]**
 - (iv) The extension of the cable. **[4 marks]**
- (c) Calculate the elastic potential energy stored in the stretched cable. **[4 marks]**
- (d) If the cable diameter were doubled while keeping the length constant, calculate the new extension under the same load. **[5 marks]**

Question 9 [15 marks] A student investigates the Young's modulus of different materials by testing wire samples. For a copper wire of length 1.80 m and diameter 0.80 mm, the following stress-strain data is obtained:

Stress ($\times 10$ Pa)	Strain ($\times 10^3$)
0	0
25	0.20
50	0.40
75	0.60
100	0.80
125	1.00

- (a) Plot a graph of stress against strain. **[4 marks]**
- (b) Use your graph to determine Young's modulus for copper. **[4 marks]**
- (c) Calculate the extension of the wire when the stress is 100×10 Pa. **[3 marks]**
- (d) If the accepted value for Young's modulus of copper is 1.3×10^{11} Pa, calculate the percentage error in the student's result. **[2 marks]**
- (e) Suggest one improvement to the experimental method to reduce uncertainty. **[2 marks]**

Section D: Simple Harmonic Motion and Springs [45 marks]

Question 10 [25 marks] A mass of 1.2 kg is attached to a horizontal spring with spring constant 480 N/m and undergoes simple harmonic motion.

- (a) Explain why the motion is simple harmonic. **[3 marks]**
- (b) The mass is displaced 6.0 cm from equilibrium and released. Calculate:
 - (i) The period of oscillation. **[3 marks]**
 - (ii) The frequency of oscillation. **[2 marks]**
 - (iii) The angular frequency. **[2 marks]**
 - (iv) The amplitude of oscillation. **[1 mark]**
- (c) Calculate:
 - (i) The maximum velocity during the motion. **[3 marks]**
 - (ii) The maximum acceleration during the motion. **[3 marks]**
 - (iii) The velocity when the displacement is 3.0 cm from equilibrium. **[4 marks]**
- (d) Find the total mechanical energy of the oscillating system and show that energy is conserved at any point during the motion. **[4 marks]**

Question 11 [20 marks] A vertical spring-mass system consists of a spring with spring constant 240 N/m supporting a mass of 2.0 kg.

- (a) Calculate the extension of the spring when the mass hangs in equilibrium. **[3 marks]**
- (b) The mass is pulled down an additional 8.0 cm from its equilibrium position and released. For the resulting simple harmonic motion, calculate:
 - (i) The amplitude of oscillation. **[1 mark]**
 - (ii) The period of oscillation. **[3 marks]**
 - (iii) The maximum speed during oscillation. **[3 marks]**
 - (iv) The maximum acceleration during oscillation. **[3 marks]**
- (c) Calculate the elastic potential energy stored in the spring when the mass is:
 - (i) At its lowest point during oscillation. **[3 marks]**
 - (ii) At its highest point during oscillation. **[4 marks]**

Physics Data Sheet

Elasticity and Springs:

Hooke's Law: $F = kx$

Series springs: $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots$

Parallel springs: $k_{eff} = k_1 + k_2 + k_3 + \dots$

Elastic potential energy: $E_p = \frac{1}{2}kx^2 = \frac{1}{2}Fx$

Material Properties:

Stress: $\sigma = \frac{F}{A}$

Strain: $\epsilon = \frac{\Delta L}{L}$

Young's modulus: $E = \frac{\sigma}{\epsilon} = \frac{F \cdot L}{A \cdot \Delta L}$

Energy:

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Gravitational potential energy: $E_p = mgh$

Work done: $W = Fs \cos \theta$

Simple Harmonic Motion:

Restoring force: $F = -kx$

Period: $T = 2\pi\sqrt{\frac{m}{k}}$

Frequency: $f = \frac{1}{T}$

Angular frequency: $\omega = 2\pi f = \sqrt{\frac{k}{m}}$

Maximum velocity: $v_{max} = \omega A$

Maximum acceleration: $a_{max} = \omega^2 A$

Velocity: $v = \omega\sqrt{A^2 - x^2}$

Kinematics:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{u+v}{2}t$

Constants:

Acceleration due to gravity: $g = 9.81 \text{ m s}^{-2}$

$\pi = 3.14159\dots$

END OF TEST

Total marks: 210

Grade boundaries: A* 189, A 168, B 147, C 126, D 105, E 84

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