A Level Mechanics
Practice Test 1: Circular Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Circular Motion Fundamentals [25 marks]

Question 1 [**8 marks**]

(a) Define angular velocity and state its SI unit. [**2 marks**]

(b) Explain the relationship between linear velocity and angular velocity for circular motion. [**2 marks**]

(c) Define centripetal acceleration and derive the formula $a_c = \frac{v^2}{r}$. [**4 marks**]

Question 2 [**10 marks**] A particle moves in a circle of radius 2.5 m with constant angular velocity 4 rad/s.

(a) Calculate the linear speed of the particle. [**2 marks**]

(b) Find the centripetal acceleration. [**2 marks**]

(c) Calculate the period of one complete revolution. [**2 marks**]

(d) If the particle has mass 0.8 kg, find the centripetal force required. [**2 marks**]

(e) How many revolutions does the particle complete in 30 seconds? [**2 marks**]

Question 3 [**7 marks**] A car travels around a circular track of radius 150 m at constant speed 25 m/s.

(a) Calculate the centripetal acceleration of the car. [**2 marks**]

(b) Find the angular velocity. [**2 marks**]

(c) Calculate the time taken for one complete lap. [**3 marks**]

Section B: Centripetal Force [30 marks]

Question 4 [**12 marks**]

(a) Define centripetal force and explain why it always acts toward the center of the circle. [**3 marks**]

(b) State Newton's second law as it applies to circular motion. [**2 marks**]

(c) Explain the difference between centripetal and centrifugal force. **[3 marks]**

(d) Give three examples of forces that can provide centripetal force. **[3 marks]**

(e) Derive the relationship $F_c = m\omega^2 r$. **[1 mark]**

Question 5 [18 marks] A mass of 1.2 kg is attached to a string of length 0.8 m and whirled in a horizontal circle.

(a) If the angular velocity is 5 rad/s, calculate the tension in the string. **[3 marks]**

(b) Find the linear speed of the mass. **[2 marks]**

(c) If the string can withstand a maximum tension of 50 N, find the maximum angular velocity. **[4 marks]**

(d) Calculate the maximum linear speed. **[2 marks]**

(e) If the mass is whirled in a vertical circle with the same string length, find the minimum speed at the top for the string to remain taut. **[4 marks]**

(f) Calculate the tension in the string at the bottom of the vertical circle when moving at this minimum speed. **[3 marks]**

Section C: Horizontal Circular Motion [35 marks]

Question 6 [15 marks] A car of mass 1200 kg travels around a flat circular track of radius 200 m.

(a) If the coefficient of friction between the tires and track is 0.6, find the maximum speed the car can travel without slipping. **[4 marks]**

(b) Calculate the centripetal acceleration at this maximum speed. **[2 marks]**

(c) Find the friction force acting on the car at maximum speed. **[2 marks]**

(d) If the car travels at 20 m/s, calculate the required centripetal force. **[2 marks]**

(e) Determine whether the car will slip at this speed. **[3 marks]**

(f) Calculate the minimum coefficient of friction needed for the car to travel at 25 m/s. **[2 marks]**

Question 7 [20 marks] A cyclist leans at an angle when going around a corner. A cyclist of total mass 80 kg (including bike) travels around a circular path of radius 25 m at speed 12 m/s.

(a) Calculate the centripetal acceleration. **[2 marks]**

(b) Find the centripetal force required. **[2 marks]**

(c) Draw a force diagram showing the forces acting on the cyclist. **[3 marks]**

(d) If the coefficient of friction is 0.8, calculate the angle at which the cyclist must lean from the vertical. **[6 marks]**

(e) Verify that the cyclist will not slip at this angle. **[4 marks]**

(f) Calculate the normal force from the ground. **[3 marks]**

Section D: Banked Curves [25 marks]

Question 8 [**10 marks**]

(a) Explain why roads are banked on curves and how this helps vehicles navigate turns. [**3 marks**]

(b) Derive the formula for the optimum banking angle: $\tan\theta = \frac{v^2}{rg}$. [**4 marks**]

(c) Explain what happens if a vehicle travels faster or slower than the design speed on a banked curve. [**3 marks**]

Question 9 [**15 marks**] A race track curve has radius 300 m and is banked at 15°.

(a) Calculate the design speed for which no friction is needed. [**3 marks**]

(b) If a car travels at this design speed, what are the forces acting on it? [**3 marks**]

(c) A car of mass 1000 kg travels at 35 m/s on this banked curve. Calculate the centripetal force required. [**2 marks**]

(d) Find the normal force from the track surface. [**4 marks**]

(e) Determine the friction force needed and its direction. [**3 marks**]

Section E: Vertical Circular Motion [30 marks]

Question 10 [**12 marks**]

(a) Explain why the speed varies in vertical circular motion even when no external work is done. [**3 marks**]

(b) Derive the condition for an object to just complete a vertical circle: $v_{top} = \sqrt{gr}$. [**4 marks**]

(c) Explain what happens if the speed at the top is less than $\sqrt{gr}$. [**2 marks**]

(d) State the relationship between speeds at the top and bottom of a vertical circle using energy conservation. [**3 marks**]

Question 11 [**18 marks**] A ball of mass 0.5 kg is attached to a string of length 1.2 m and moves in a vertical circle.

(a) Calculate the minimum speed at the top for the ball to complete the circle. [**2 marks**]

(b) Find the minimum speed at the bottom using energy conservation. [**4 marks**]

(c) Calculate the tension in the string at the top when moving at minimum speed. [**2 marks**]

(d) Find the tension at the bottom when moving at minimum speed. [**3 marks**]

(e) If the speed at the bottom is 8 m/s, calculate the speed at the top. [**3 marks**]

(f) Calculate the tension in the string at the top and bottom for this case. [**4 marks**]

Section F: Conical Pendulums [20 marks]

Question 12 [**8 marks**]

(a) Define a conical pendulum and explain the forces acting on the bob. [**3 marks**]

(b) Derive the relationship between the angle of the string and the angular velocity: $\cos\theta = \frac{g}{\omega^2 l}$. [**5 marks**]

Question 13 [**12 marks**] A conical pendulum consists of a mass of 0.6 kg suspended by a string of length 1.5 m. The mass moves in a horizontal circle of radius 0.9 m.

(a) Calculate the angle the string makes with the vertical. [**2 marks**]

(b) Find the tension in the string. [**3 marks**]

(c) Calculate the angular velocity of the mass. [**3 marks**]

(d) Find the linear speed of the mass. [**2 marks**]

(e) Calculate the period of one complete revolution. [**2 marks**]

Section G: Motion in a Vertical Circle - Loops [25 marks]

Question 14 [**15 marks**] A roller coaster car of mass 800 kg enters a vertical loop of radius 12 m.

(a) Calculate the minimum speed at the top of the loop for the car to maintain contact with the track. [**3 marks**]

(b) Find the minimum speed at the bottom of the loop using energy conservation. [**4 marks**]

(c) If the car enters the loop at 25 m/s, calculate its speed at the top. [**3 marks**]

(d) Find the normal force from the track at the top of the loop in this case. [**3 marks**]

(e) Calculate the normal force at the bottom of the loop. [**2 marks**]

Question 15 [**10 marks**] A ball is thrown horizontally from the top of a vertical circle track of radius 2 m with just enough speed to complete the loop.

(a) Calculate the speed at the top of the loop. [**2 marks**]

(b) Find the horizontal distance traveled before hitting the ground if the loop is 5 m above ground level. [**4 marks**]

(c) Calculate the speed just before hitting the ground. [**4 marks**]

Section H: Applications and Problem Solving [30 marks]

Question 16 [**15 marks**] A satellite orbits Earth in a circular orbit at altitude 400 km above Earth's surface. Take Earth's radius as 6400 km and $g = 9.8$ m/s^2 at Earth's surface.

(a) Calculate the orbital radius from Earth's center. [**1 mark**]

(b) Find the gravitational acceleration at the satellite's altitude. [**3 marks**]

(c) Calculate the orbital speed of the satellite. [**3 marks**]

(d) Find the period of the orbit. [**3 marks**]

(e) Calculate the centripetal acceleration of the satellite. [**2 marks**]

(f) If the satellite has mass 1000 kg, find the gravitational force acting on it. **[3 marks]**

Question 17 **[15 marks]** A fairground ride consists of chairs suspended by chains from a rotating horizontal platform. The chains are 3 m long and make an angle of 30° with the vertical when rotating.

(a) Calculate the radius of the circular path of the chairs. **[2 marks]**

(b) Find the angular velocity of the platform. **[4 marks]**

(c) Calculate the linear speed of the chairs. **[2 marks]**

(d) If a chair and passenger have combined mass 120 kg, find the tension in the chain. **[3 marks]**

(e) Calculate the centripetal force acting on the chair and passenger. **[2 marks]**

(f) Find the period of rotation. **[2 marks]**

Section I: Advanced Circular Motion [25 marks]

Question 18 **[12 marks]** A car travels over a hump-backed bridge that can be modeled as part of a circle of radius 80 m.

(a) Calculate the maximum speed the car can travel over the bridge without losing contact with the road surface. **[4 marks]**

(b) If the car has mass 1500 kg and travels at 20 m/s, find the normal force from the road at the top of the bridge. **[4 marks]**

(c) Calculate the apparent weight of a 70 kg passenger at this speed. **[4 marks]**

Question 19 **[13 marks]** A particle slides on the inside of a smooth vertical circular track of radius 1.8 m. It is released from rest at a height equal to the radius above the bottom of the track.

(a) Calculate the speed at the bottom of the track using energy conservation. **[3 marks]**

(b) Find the normal force at the bottom of the track if the particle has mass 2 kg. **[3 marks]**

(c) Calculate the speed when the particle is at the same height as the center of the circle. **[3 marks]**

(d) Determine whether the particle will complete the full circle. **[4 marks]**

Section J: Comprehensive Applications [30 marks]

Question 20 **[18 marks]** A bead slides on a smooth horizontal circular wire of radius 0.5 m. The wire rotates about a vertical axis through its center with angular velocity 6 rad/s.

(a) Explain why the bead experiences a centrifugal force in the rotating reference frame. **[3 marks]**

(b) Calculate the centrifugal force if the bead has mass 0.1 kg. **[2 marks]**

(c) Find the normal force from the wire required to keep the bead moving in a circle. **[2 marks]**

(d) If the wire suddenly stops rotating, describe the motion of the bead. **[3 marks]**

(e) Calculate the coefficient of friction required between bead and wire to prevent the bead sliding outward when the rotation stops. **[4 marks]**

(f) If the angular velocity increases to 10 rad/s, calculate the new centrifugal force. **[2 marks]**

(g) Find the minimum coefficient of friction needed to prevent slipping at this higher angular velocity. **[2 marks]**

Question 21 [12 marks] Two masses, 2 kg and 3 kg, are connected by a light inextensible string of total length 1.2 m. They are placed on a smooth horizontal table and made to rotate about their center of mass.

(a) Find the position of the center of mass. **[3 marks]**

(b) Calculate the distance of each mass from the center of mass. **[2 marks]**

(c) If the system rotates with angular velocity 4 rad/s, find the tension in the string. **[4 marks]**

(d) Calculate the linear speed of each mass. **[2 marks]**

(e) Find the total kinetic energy of the system. **[1 mark]**

Physics Data and Formulae

Circular Motion:

Angular velocity: $\omega = \frac{v}{r} = \frac{2\pi}{T}$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Vertical Circular Motion:

At top: $T + mg = \frac{mv^2}{r}$

At bottom: $T - mg = \frac{mv^2}{r}$

Minimum speed at top: $v_{min} = \sqrt{gr}$

Banking:

No friction: $\tan\theta = \frac{v^2}{rg}$

With friction: $\tan\theta = \frac{v^2/rg \pm \mu}{1 \mp \mu v^2/rg}$

Conical Pendulum:

$\cos\theta = \frac{g}{\omega^2 l}$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 r$

Energy Conservation:

$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

Friction:

Maximum static friction: $f_s \leq \mu_s N$

Kinetic friction: $f_k = \mu_k N$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s^2

Earth's radius: $R_E = 6.4 \times 10^6$ m

Trigonometric Values:

$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$

$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 275

Grade boundaries: A* 248, A 220, B 193, C 165, D 138, E 110

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