A Level Mechanics
Practice Test 2: Gravitational Fields

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 50 minutes

Section A: Gravitational Forces and Field Strength [35 marks]

Question 1 [**15 marks**] Two spherical masses of 600 kg and 900 kg have their centers separated by 8.0 m.

(a) Calculate the gravitational force between the two masses. [**4 marks**]

(b) Find the gravitational field strength at a point 3.0 m from the 600 kg mass along the line joining the centers. [**5 marks**]

(c) Calculate the gravitational field strength at the same point due to the 900 kg mass. [**3 marks**]

(d) Determine the resultant gravitational field strength at this point. [**3 marks**]

Question 2 [**20 marks**] Mars has a mass of 6.39×10^{23} kg and a radius of 3.39×10 m.

(a) Calculate the gravitational field strength on the surface of Mars. [**4 marks**]

(b) Find the gravitational field strength at a height of 1000 km above Mars' surface. [**4 marks**]

(c) A Mars rover has a mass of 900 kg. Calculate its weight on Mars' surface. [**3 marks**]

(d) Compare this weight with the rover's weight on Earth. [**3 marks**]

(e) At what height above Mars' surface would the gravitational field strength be 25% of its surface value? [**6 marks**]

Section B: Gravitational Potential and Energy [40 marks]

Question 3 [**18 marks**]

(a) Explain what is meant by gravitational potential at a point in a gravitational field. [**3 marks**]

(b) State the relationship between gravitational potential V and gravitational field strength g. [**2 marks**]

(c) Show that for a uniform spherical mass M, the gravitational potential at distance r from its center is V = -GM/r. [**5 marks**]

(d) Calculate the gravitational potential at Earth's surface. [**4 marks**]

(e) Find the gravitational potential at a height of 2000 km above Earth's surface. [**4 marks**]

Question 4 [**22 marks**] A spacecraft of mass 3500 kg needs to travel from Earth's surface to a space station orbiting at 400 km altitude.

(a) Calculate the gravitational potential energy of the spacecraft at Earth's surface. [**4 marks**]

(b) Find the gravitational potential energy of the spacecraft at the space station's altitude. [**4 marks**]

(c) Calculate the change in potential energy required for this journey. [**3 marks**]

(d) Determine the minimum work that must be done to move the spacecraft to this altitude. [**2 marks**]

(e) If the spacecraft is launched with just enough energy to reach the space station with zero velocity, calculate its launch velocity. [**5 marks**]

(f) What would be the spacecraft's velocity when it reaches an altitude of 200 km during this journey? [**4 marks**]

Section C: Orbital Mechanics [45 marks]

Question 5 [**20 marks**]

(a) For a satellite in circular orbit around Earth, explain what provides the centripetal force. [**2 marks**]

(b) Starting from Newton's second law and the law of universal gravitation, derive the formula for orbital velocity $v = (GM/r)$. [**6 marks**]

(c) Show that the orbital period is given by $T = 2(r^3/GM)$. [**4 marks**]

(d) Explain why satellites in higher orbits move more slowly than those in lower orbits. [**3 marks**]

(e) For a satellite in circular orbit, show that the kinetic energy is half the magnitude of the potential energy. [**5 marks**]

Question 6 [**25 marks**] The International Space Station (ISS) orbits Earth at an average altitude of 408 km above the surface.

(a) Calculate the orbital radius of the ISS from Earth's center. [**2 marks**]

(b) Find the orbital velocity of the ISS. [**4 marks**]

(c) Calculate the orbital period of the ISS in minutes. [**4 marks**]

(d) Determine the centripetal acceleration experienced by the ISS. [**3 marks**]

(e) For a section of the ISS with mass 50,000 kg, calculate:

(i) Its kinetic energy in orbit. [**3 marks**]

(ii) Its gravitational potential energy in orbit. [**3 marks**]

(iii) Its total mechanical energy in orbit. [**3 marks**]

(f) Calculate the energy required to boost this section from Earth's surface to the ISS orbit. [**3 marks**]

Section D: Satellite Applications and Kepler's Laws [35 marks]

Question 7 [20 marks] A telecommunications company wants to place a geostationary satellite in orbit.

(a) State the three conditions that must be satisfied for a geostationary orbit. [**3 marks**]

(b) Calculate the orbital radius required for a geostationary satellite. [**6 marks**]

(c) Find the height of this orbit above Earth's surface. [**2 marks**]

(d) Calculate the orbital velocity of the geostationary satellite. [**4 marks**]

(e) Explain why geostationary satellites are useful for telecommunications but not for polar observations. [**3 marks**]

(f) Calculate the energy required to place a 2000 kg geostationary satellite in orbit from Earth's surface. [**2 marks**]

Question 8 [15 marks] Jupiter has a mass of 1.90×10^2 kg. Its moon Europa orbits at a distance of 6.71×10 m from Jupiter's center with a period of 3.55 days.

(a) Use the data for Europa to verify Kepler's third law. [**6 marks**]

(b) Calculate Europa's orbital velocity. [**4 marks**]

(c) Another moon, Io, orbits Jupiter at a distance of 4.22×10 m. Predict Io's orbital period using Kepler's third law. [**5 marks**]

Section E: Escape Velocity and Energy Transfer [30 marks]

Question 9 [18 marks]

(a) Define escape velocity and explain its physical significance. [**3 marks**]

(b) Using conservation of energy, derive the expression for escape velocity $v_e = (2GM/R)fromaplanet'ssurface.$

(c) Calculate the escape velocity from Mars' surface (mass 6.39×10^{23} kg, radius 3.39×10 m). [**3 marks**]

(d) Explain why it requires less energy to launch spacecraft to Mars than to launch them from Mars back to Earth. [**3 marks**]

Question 10 [12 marks] A rocket is launched vertically upward from Earth's surface with an initial velocity of 10.0 km/s.

(a) Calculate Earth's escape velocity and determine whether this rocket will escape Earth's gravitational field. [**4 marks**]

(b) Using conservation of energy, calculate the maximum height reached by the rocket. [**6 marks**]

(c) Find the rocket's velocity when it reaches an altitude of 5000 km. [**2 marks**]

Section F: Gravitational Field Superposition [25 marks]

Question 11 [15 marks] Two planets, each with mass M, are separated by a distance d. A small mass m is placed at various points along the line joining their centers.

(a) Calculate the gravitational field strength at the midpoint between the two planets. [**4 marks**]

(b) Find the position along the line where the gravitational field strength is zero. [**3 marks**]

(c) Calculate the gravitational potential at the midpoint between the planets. [**4 marks**]

(d) Determine the gravitational potential at the point where the field strength is zero. [**4 marks**]

Question 12 [10 marks] The Earth-Moon system can be treated as two point masses for calculations involving their gravitational effects. Earth's mass is 5.97×10^{2} kg, the Moon's mass is 7.35×10^{22} kg, and they are separated by 3.84×10 m.

(a) Calculate the position of the center of mass of the Earth-Moon system measured from Earth's center. [**4 marks**]

(b) Find the distance from Earth's center to the point where the gravitational fields of Earth and Moon cancel out. [**6 marks**]

Physics Data and Formulae

Gravitational Force and Field:
Newton's Law: $F = \frac{Gm_1m_2}{r^2}$
Field strength: $g = \frac{F}{m} = \frac{GM}{r^2}$
Field-potential relation: $g = -\frac{dV}{dr}$

Gravitational Potential and Energy:
Potential: $V = -\frac{GM}{r}$
Potential energy: $U = mV = -\frac{GMm}{r}$
Work done: $W = m\Delta V = \Delta U$

Orbital Motion:
Circular orbital velocity: $v = \sqrt{\frac{GM}{r}}$
Orbital period: $T = 2\pi\sqrt{\frac{r^3}{GM}}$
Kepler's Third Law: $T^2 \propto r^3$
Escape velocity: $v_e = \sqrt{\frac{2GM}{r}}$

Energy in Orbits:
Kinetic energy: $E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$
Potential energy: $E_p = -\frac{GMm}{r}$
Total energy: $E = -\frac{GMm}{2r}$

Circular Motion:
Centripetal force: $F_c = \frac{mv^2}{r} = m\omega^2 r$
Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$
Angular velocity: $\omega = \frac{2\pi}{T}$

Physical Constants:
Gravitational constant: $G = 6.67 \times 10^{-11}$ Nm²/kg²

Earth's mass: $M_E = 5.97 \times 10^{24}$ kg
Earth's radius: $R_E = 6.37 \times 10^6$ m
Moon's mass: $M_M = 7.35 \times 10^{22}$ kg
Moon's radius: $R_M = 1.74 \times 10^6$ m
Earth-Moon distance: 3.84×10^8 m
Standard gravity: $g = 9.81$ m/s²

END OF TEST

Total marks: 210

Grade boundaries: A* 189, A 168, B 147, C 126, D 105, E 84

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