

GCSE Higher Mathematics

Practice Test 6: Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

(a) $8(x - 6) = 4x + 12$

(b) $\frac{x+11}{5} - \frac{x-9}{3} = 4$

(c) $\frac{7x-6}{12} = \frac{4x+5}{8} - 2$

(d) $0.8x + 1.6 = 0.6x - 2.4$

2. Solve these simultaneous equations by elimination:

(a)
$$\begin{cases} 7x + 6y = 37 \\ 5x - 4y = 3 \end{cases}$$

(b)
$$\begin{cases} 4x + 9y = 38 \\ 8x - 5y = 14 \end{cases}$$

(c)
$$\begin{cases} 7x + 4y = 29 \\ 5x - 8y = 17 \end{cases}$$

(d)
$$\begin{cases} 9x + 8y = 52 \\ 4x - 6y = 8 \end{cases}$$

3. Solve these simultaneous equations by substitution:

(a)
$$\begin{cases} y = 7x - 11 \\ 6x + 4y = 38 \end{cases}$$

(b)
$$\begin{cases} x = 6y - 8 \\ 7x - 5y = 34 \end{cases}$$

(c)
$$\begin{cases} y = 15 - 6x \\ 5x + 8y = 65 \end{cases}$$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

(a) $y = 7x - 5$ and $y = 16 - 4x$

(b) $y = x^2 - 11$ and $y = 5x + 1$

(c) $x^2 + y^2 = 29$ and $y = 5x - 3$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

(a) $x^2 + 17x + 42$

(b) $x^2 - 15x - 34$

(c) $x^2 - 19x + 84$

(d) $x^2 + 7x - 30$

(e) $x^2 - 196$

(f) $x^2 - 28x + 196$

6. Solve these quadratic equations by factorizing:

(a) $x^2 + 17x + 70 = 0$

(b) $x^2 - 16x - 17 = 0$

(c) $x^2 - 13x = 0$

(d) $x^2 - 225 = 0$

(e) $x^2 + 30x + 225 = 0$

(f) $7x^2 - 28x = 0$

7. Factorize these harder quadratics:

(a) $5x^2 + 12x + 7$

(b) $7x^2 - 19x + 6$

(c) $64x^2 - 81$

(d) $6x^2 + 17x - 28$

(e) $64x^2 - 112x + 49$

(f) $10x^2 - 27x - 9$

8. Solve by factorizing:

(a) $7x^2 + 15x - 8 = 0$

(b) $8x^2 - 17x + 2 = 0$

(c) $49x^2 - 36 = 0$

(d) $5x^2 + 14x - 3 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

(a) $x^2 + 16x + 15$

(b) $x^2 - 20x + 7$

(c) $x^2 + 12x - 9$

(d) $x^2 - 20x + 17$

(e) $7x^2 + 28x + 13$

(f) $3x^2 - 24x + 11$

10. Solve by completing the square:

(a) $x^2 + 16x + 13 = 0$

(b) $x^2 - 14x - 7 = 0$

(c) $x^2 + 10x - 5 = 0$

(d) $7x^2 + 14x - 3 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 13x - 6 = 0$

(b) $7x^2 - 15x + 6 = 0$

(c) $x^2 - 16x + 11 = 0$

(d) $6x^2 + 11x - 7 = 0$

(e) $5x^2 + 12x + 4 = 0$

(f) $9x^2 - 13x - 4 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 15x + 36 = 0$

(b) $x^2 - 14x + 49 = 0$

(c) $x^2 + 7x + 15 = 0$

(d) $6x^2 - 13x + 6 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 14x + 45$:

- (a) Find the y-intercept
- (b) Find the x-intercepts by factorizing
- (c) Complete the square to find the vertex
- (d) Sketch the graph
- (e) State the line of symmetry

14. For the quadratic $y = 6x^2 + 18x - 8$:

- (a) Complete the square
- (b) Find the coordinates of the vertex
- (c) Find the y-intercept
- (d) State the line of symmetry
- (e) Sketch the graph

15. A tennis ball is hit upward. Its height h (in meters) after t seconds is given by: $h = -5t^2 + 50t + 5$

- (a) What is the initial height?
- (b) At what times is the tennis ball at ground level?
- (c) What is the maximum height reached?
- (d) At what time does it reach maximum height?

16. The revenue R (in thousands of pounds) from selling x thousand units is: $R = -4x^2 + 20x - 16$

- (a) How many units should be sold to maximize revenue?
- (b) What is the maximum revenue?
- (c) At what production levels is the revenue £5,000?

Section E: Linear Inequalities

17. Solve these linear inequalities:

(a) $8x + 15 > 47$

(b) $7x - 17 \leq 32$

(c) $11 - 8x < 3$

(d) $\frac{x+9}{5} \geq 4$

(e) $9 - 8x > 7x + 24$

(f) $\frac{7x-6}{8} < \frac{x+9}{5}$

18. Show these inequalities on number lines:

(a) $x > -7$

(b) $x \leq 9$

(c) $-4 < x \leq 10$

(d) $x < 6$ or $x > 11$

19. Solve these compound inequalities:

(a) $-9 < 7x + 5 < 19$

(b) $13 \leq 6x - 5 \leq 31$

(c) $-4 \leq \frac{7x+5}{5} < 11$

20. Find the integer solutions to:

(a) $7x + 11 > 25$ and $x < 10$

(b) $-5 \leq x + 3 < 9$

(c) $x^2 < 81$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

(a) $x^2 - 15x + 36 > 0$

(b) $x^2 - 16x + 39 \leq 0$

(c) $x^2 - 121 < 0$

(d) $x^2 + 7x - 18 \geq 0$

(e) $x^2 - 16x + 64 > 0$

(f) $5x^2 - 6x - 8 < 0$

22. Solve and show on number lines:

(a) $x^2 - 144 \leq 0$

(b) $x^2 + 6x - 16 > 0$

(c) $4x^2 - 11x + 6 \geq 0$

23. Find the values of x for which:

(a) $x^2 < 8x + 9$

(b) $5x^2 + 11x \geq 12$

(c) $x^2 + 14x + 49 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

(a) $(x + 10)(7x - 5)$

(b) $(6x - 7)(x + 11)$

(c) $(5x + 10)^2$

(d) $(9x - 7)^2$

(e) $(x + 15)(x - 15)$

(f) $(8x + 5)(8x - 5)$

25. Expand these expressions:

(a) $(x + 7)(x^2 - 6x + 8)$

(b) $(5x - 6)(x^2 + 5x - 4)$

(c) $(x + 6)^3$

(d) $(6x - 4)^3$

26. Factorize completely:

(a) $16x^2 + 24x$

(b) $49x^2 - 169$

(c) $x^3 - 36x$

(d) $7x^3 + 28x^2 + 28x$

(e) $x^3 - 512$

(f) $343x^3 + 216$

27. Simplify these algebraic fractions:

(a) $\frac{x^2-49}{x+7}$

(b) $\frac{x^2+15x+56}{x+8}$

(c) $\frac{8x^2-32}{x^2-4}$

(d) $\frac{x^3-512}{x^2-64}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

(a) $\frac{8}{x} + \frac{5}{x}$

(b) $\frac{12}{x} - \frac{7}{x}$

(c) $\frac{5}{8x} + \frac{1}{16x}$

(d) $\frac{7}{x+5} + \frac{4}{x-6}$

(e) $\frac{x}{x+7} - \frac{6}{x-4}$

(f) $\frac{7x}{x^2-36} + \frac{6}{x+6}$

29. Multiply and divide these algebraic fractions:

(a) $\frac{x}{8} \times \frac{24}{x^2}$

(b) $\frac{x+7}{9} \times \frac{18}{x+7}$

(c) $\frac{x^2-49}{x+6} \div \frac{x-7}{x+6}$

(d) $\frac{7x+21}{x^2-64} \times \frac{x-8}{14}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{8} + \frac{x}{6} = 21$

(b) $\frac{7x+6}{9} = \frac{x-5}{6}$

(c) $\frac{8}{x} = \frac{7}{x-6}$

(d) $\frac{x+6}{x-7} = \frac{5x}{x+6}$

31. Make the subject of these formulae:

(a) $D = \frac{m}{V}$, make m the subject

(b) $C = \frac{5}{9}(F - 32)$, make F the subject

(c) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, make u the subject

(d) $I = \frac{PRT}{100}$, make T the subject

Section I: Sequences

32. Find the n th term for these arithmetic sequences:

(a) 17, 23, 29, 35, ...

(b) 10, 17, 24, 31, ...

(c) 50, 46, 42, 38, ...

(d) $\frac{3}{10}, \frac{7}{10}, \frac{11}{10}, \frac{15}{10}, \dots$

33. For these geometric sequences, find the n th term:

(a) 7, 21, 63, 189, ...

(b) 10, 40, 160, 640, ...

(c) 384, 192, 96, 48, ...

(d) 6, -24, 96, -384, ...

34. Find the sum of these series:

(a) First 45 terms of $11 + 15 + 19 + 23 + \dots$

(b) First 4 terms of $9 + 27 + 81 + 243 + \dots$

(c) $6 + 10 + 14 + \dots + 102$ (arithmetic series)

(d) $8 + 16 + 32 + \dots + 512$ (geometric series)

35. These are quadratic sequences. Find the n th term:

(a) 7, 28, 63, 112, 175, ...

(b) 6, 17, 34, 57, 86, ...

(c) 5, 20, 45, 80, 125, ...

(d) 9, 26, 49, 78, 113, ...

36. A sequence is defined by $u_1 = 7$ and $u_{n+1} = 5u_n - 6$.

(a) Find the first 5 terms

(b) Find a formula for u_n

(c) Calculate u_4

Section J: Problem Solving

37. The sum of two numbers is 34 and their product is 280. Find the two numbers.
38. A rectangular field has perimeter 80m. If the length is 16m more than the width, find the dimensions.
39. The difference between a positive number and its reciprocal is $\frac{63}{8}$. Find the number.
40. A soccer ball's height h (in meters) after t seconds is given by: $h = 160t - 5t^2$
- (a) When does it hit the ground?
 - (b) What is its maximum height?
 - (c) When is it 1200m high?
41. Prove that the sum of the first n terms of the sequence 4, 8, 12, 16, ... is $2n(n + 1)$.
42. The quadratic $px^2 + qx + r = 0$ has roots α and β .
- (a) Show that $\alpha + \beta = -\frac{q}{p}$
 - (b) Show that $\alpha\beta = \frac{r}{p}$
 - (c) If the roots are 8 and -5, find p , q , and r when $p = 5$
43. A function is defined as $m(x) = x^2 + fx + g$. If $m(4) = 21$ and $m(6) = 41$, find f and g .
44. The sum of the first n terms of a sequence is $S_n = 7n^2 - 4n$. Find the n th term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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