

GCSE Higher Mathematics

Practice Test 7: Number

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:

- (a) 9^2
- (b) 7^{-2}
- (c) 19^0
- (d) $(-9)^2$
- (e) $(-5)^3$
- (f) 12^{-2}

2. Write these as single powers:

- (a) $8^6 \times 8^{11}$
- (b) $15^{10} \div 15^4$
- (c) $(6^8)^5$
- (d) $11^{-1} \times 11^{13}$
- (e) $\frac{17^9}{17^{-7}}$
- (f) $(9^6)^{-3}$

3. Evaluate these fractional indices:

- (a) $121^{\frac{1}{2}}$
- (b) $512^{\frac{1}{3}}$
- (c) $32768^{\frac{3}{5}}$
- (d) $10000000^{\frac{2}{7}}$
- (e) $2197^{\frac{2}{3}}$
- (f) $2048^{-\frac{3}{11}}$

4. Simplify these expressions:

- (a) $\sqrt{225}$
- (b) $\sqrt[3]{1000}$

- (c) $\sqrt[4]{4096}$
- (d) $\sqrt{2.25}$
- (e) $\sqrt[3]{-729}$
- (f) $\sqrt[5]{100000}$

5. Express in index form:

- (a) $\sqrt{29}$
- (b) $\sqrt[3]{19}$
- (c) $\frac{1}{\sqrt{17}}$
- (d) $\sqrt[4]{s^{15}}$
- (e) $\frac{1}{\sqrt[3]{h^{11}}}$
- (f) $\sqrt{s} \times \sqrt[3]{s}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

- (a) $8^7 \times 8^{-10} \times 8^{11}$
- (b) $\frac{12^{12} \times 12^{-8}}{12^{-7}}$
- (c) $(9^5)^{-7} \times 9^{20}$
- (d) $\frac{13^{-8} \times 13^{17}}{13^7}$

7. Evaluate these expressions:

- (a) $2197^{\frac{2}{3}}$
- (b) $16807^{-\frac{3}{5}}$
- (c) $100000^{\frac{2}{5}}$
- (d) $32768^{-\frac{5}{15}}$
- (e) $8000^{\frac{1}{3}}$
- (f) $1000000^{-\frac{2}{6}}$

8. Simplify these expressions:

- (a) $x^{\frac{9}{10}} \times x^{\frac{1}{5}}$
- (b) $\frac{n^{\frac{11}{4}}}{n^{\frac{1}{6}}}$
- (c) $(h^{\frac{1}{8}})^{16}$
- (d) $\sqrt{s} \times s^{\frac{1}{9}}$
- (e) $\frac{\sqrt[3]{t^{11}}}{\sqrt{t}}$
- (f) $(w^{-\frac{1}{8}})^{-16}$

9. Write these in the form a^n where a and n are rational:

- (a) $\sqrt{19} \times 19^8$
- (b) $\frac{23^6}{\sqrt[3]{23}}$
- (c) $\sqrt[4]{14^{15}} \times 14^{-\frac{5}{6}}$
- (d) $\frac{\sqrt{29}}{\sqrt[3]{29^{11}}}$

Section C: Surds

10. Simplify these surds:

- (a) $\sqrt{92}$
- (b) $\sqrt{136}$
- (c) $\sqrt{152}$
- (d) $\sqrt{245}$
- (e) $\sqrt{272}$
- (f) $\sqrt{900}$

11. Simplify these expressions:

- (a) $9\sqrt{19} + 15\sqrt{19}$
- (b) $19\sqrt{8} - 8\sqrt{8}$
- (c) $\sqrt{68} + \sqrt{153}$
- (d) $\sqrt{245} - \sqrt{180}$
- (e) $8\sqrt{28} + 11\sqrt{63}$
- (f) $\sqrt{245} - \sqrt{147} + \sqrt{45}$

12. Multiply and simplify:

- (a) $\sqrt{13} \times \sqrt{52}$
- (b) $\sqrt{28} \times \sqrt{112}$
- (c) $9\sqrt{6} \times 7\sqrt{24}$
- (d) $\sqrt{20} \times \sqrt{80}$
- (e) $\sqrt{9} \times \sqrt{36} \times \sqrt{144}$
- (f) $11\sqrt{8} \times 7\sqrt{32}$

13. Expand and simplify:

- (a) $(8 + \sqrt{17})(5 - \sqrt{17})$
- (b) $(7 + \sqrt{23})(4 + 8\sqrt{23})$
- (c) $(10 - \sqrt{19})^2$
- (d) $(\sqrt{29} + \sqrt{11})(\sqrt{29} - \sqrt{11})$
- (e) $(8\sqrt{17} + 1)(8\sqrt{17} - 1)$
- (f) $(\sqrt{23} + 8)^2$

14. Rationalize the denominators:

- (a) $\frac{1}{\sqrt{23}}$
- (b) $\frac{17}{\sqrt{29}}$
- (c) $\frac{\sqrt{17}}{\sqrt{68}}$
- (d) $\frac{16}{8\sqrt{2}}$
- (e) $\frac{1}{7+\sqrt{17}}$
- (f) $\frac{11}{1-\sqrt{23}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

(a) $\frac{13}{8+\sqrt{19}}$

(b) $\frac{19}{7-\sqrt{31}}$

(c) $\frac{\sqrt{17}}{1+\sqrt{17}}$

(d) $\frac{8\sqrt{23}}{7+\sqrt{23}}$

(e) $\frac{1}{\sqrt{24}-\sqrt{17}}$

(f) $\frac{\sqrt{23+7}}{\sqrt{23-8}}$

16. Simplify these expressions completely:

(a) $\frac{\sqrt{36}+\sqrt{81}}{\sqrt{9}}$

(b) $\frac{\sqrt{98}-\sqrt{72}}{\sqrt{2}}$

(c) $\sqrt{(9+\sqrt{23})(9-\sqrt{23})}$

(d) $\sqrt{152}-8\sqrt{38}+\sqrt{98}$

(e) $(\sqrt{17}+\sqrt{68})^2$

(f) $\frac{\sqrt{136}}{\sqrt{17}}+\frac{\sqrt{102}}{\sqrt{17}}$

17. Prove that:

(a) $(\sqrt{b}+\sqrt{c})(\sqrt{b}-\sqrt{c})=b-c$

(b) $\frac{1}{\sqrt{d}+\sqrt{f}}=\frac{\sqrt{d}-\sqrt{f}}{d-f}$

(c) $(d+f\sqrt{y})^2=d^2+2df\sqrt{y}+f^2y$

Section E: Standard Form

18. Write these numbers in standard form:

(a) 923000

(b) 0.000128

(c) 7890000000

(d) 0.0000000517

(e) 1258.3

(f) 0.01287

19. Write these in ordinary form:

(a) 1.08×10^0

(b) 8.91×10^{-10}

(c) 1.2543×10^{-13}

(d) 7.69×10^{16}

(e) 1.28×10^{-8}

(f) 9.23×10^{11}

20. Calculate, giving answers in standard form:

- (a) $(11 \times 10^{10}) \times (13 \times 10^{12})$
- (b) $(8 \times 10^{-8}) \times (15 \times 10^{13})$
- (c) $(24 \times 10^{11}) \div (8 \times 10^{-6})$
- (d) $(21 \times 10^{-10}) \div (14 \times 10^{-13})$
- (e) $(11 \times 10^9)^2$
- (f) $\sqrt{81 \times 10^{20}}$

21. Calculate these more complex expressions:

- (a) $(8.4 \times 10^9) \times (3.25 \times 10^{-11})$
- (b) $\frac{16.8 \times 10^{12}}{4.2 \times 10^{-9}}$
- (c) $(9.2 \times 10^{-8}) + (1.07 \times 10^{-7})$
- (d) $(11.9 \times 10^{11}) - (8.6 \times 10^{10})$
- (e) $\frac{(8.1 \times 10^7) \times (7.2 \times 10^{-8})}{12.15 \times 10^{-6}}$
- (f) $(1.0404 \times 10^{18})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

- (a) $\frac{8}{15} + \frac{11}{30}$
- (b) $\frac{19}{24} - \frac{13}{36}$
- (c) $\frac{18}{19} \times \frac{38}{27}$
- (d) $\frac{16}{31} \div \frac{24}{42}$
- (e) $\frac{13}{14} - \frac{9}{21} + \frac{17}{42}$
- (f) $(\frac{8}{15})^{-2}$

23. Convert these recurring decimals to fractions:

- (a) $0.\overline{9}$
- (b) $0.\overline{82}$
- (c) $0.\overline{73}$
- (d) $0.\overline{571428}$
- (e) $6.\overline{24}$
- (f) $0.8\overline{29}$

24. Work out these percentage calculations:

- (a) Increase 880 by 75%
- (b) Decrease 1080 by 36%
- (c) Find 42.5% of 1240
- (d) What percentage is 168 out of 224?
- (e) If 95% of a number is 247, find the number
- (f) A price increases from £135 to £148.5. Find the percentage increase

25. Solve these percentage problems:

- (a) After a 80% increase, a price is £324. Find the original price
- (b) After a 75% decrease, a quantity is 144. Find the original quantity
- (c) The value of a car decreases by 50% each year. If it's worth £7500 now, what was it worth 2 years ago?
- (d) An investment grows by 11% per year. After 2 years it's worth £2468.1. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

(a) $8^{-2} + 13^0 - 14^{-1}$

(b) $\sqrt{100} \times 1000^{\frac{1}{3}} - 8^{-2}$

(c) $\frac{2197^{\frac{2}{3}} - 32768^{\frac{3}{5}}}{121^{\frac{1}{2}}}$

(d) $64^{-\frac{1}{2}} + 121^{\frac{1}{2}} \times 7^{-1}$

27. Calculate exactly (leave surds in your answer):

(a) $\frac{13}{\sqrt{17}} + \frac{9}{\sqrt{68}}$

(b) $\sqrt{63} \times \sqrt{112} - \sqrt{252}$

(c) $\frac{\sqrt{196} + \sqrt{147}}{\sqrt{49}}$

(d) $(8\sqrt{3} - 7)^2$

28. Work with standard form in context:

(a) The mass of a hydrogen atom is 1.674×10^{-27} kg. Find the mass of 6.02×10^{23} hydrogen atoms

(b) Microwaves travel at 3×10^8 m/s. How far do they travel in one year (use 1 year = 3.154×10^7 seconds)?

(c) The diameter of a bacterium is approximately 2×10^{-6} m. How many bacteria would fit across a distance of 1.2 cm?

(d) A neural network processes 2.88×10^{15} operations per second. How many operations in 30 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{17}$ is irrational. (Use proof by contradiction: assume $\sqrt{17} = \frac{x}{y}$ where x and y are integers with no common factors)

30. The number ϵ satisfies $\epsilon^2 = 6\epsilon - 4$.

(a) Show that $\epsilon = 3 + \sqrt{5}$

(b) Calculate ϵ to 4 decimal places

(c) Find $\frac{1}{\epsilon}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{13} + \sqrt{17} + \sqrt{19}}$. (Hint: First rationalize using $(\sqrt{13} + \sqrt{17}) - \sqrt{19}$)

32. A rectangle has sides of length $(8 + \sqrt{17})$ cm and $(8 - \sqrt{17})$ cm.

(a) Find the exact area

(b) Find the exact perimeter

(c) Show that the area is rational but the perimeter is irrational

33. The population of fungi octuples every 12 hours. If there are initially 11×10^2 fungi:

(a) How many fungi after 48 hours?

(b) Express your answer in standard form

(c) After how many hours will there be more than 2×10^{11} fungi?

34. Show that $\frac{1}{\sqrt{b}+\sqrt{c}} + \frac{1}{\sqrt{b}-\sqrt{c}} = \frac{2\sqrt{b}}{b-c}$
35. A sphere has volume $V = \frac{4}{3}\pi r^3$. If the volume is $704\pi \text{ cm}^3$:
- (a) Find the radius in surd form
 - (b) Find the surface area (use $A = 4\pi r^2$)
 - (c) Express both answers exactly
36. The equation $x^2 - 16x + 1 = 0$ has solutions $x = 8 \pm 3\sqrt{7}$.
- (a) Verify this by substitution
 - (b) Find $\frac{1}{8+3\sqrt{7}} + \frac{1}{8-3\sqrt{7}}$ without using a calculator
 - (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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