A Level Mechanics
Practice Test 6: Statics and Dynamics of Particles

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw clear diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 50 minutes

Section A: Static Equilibrium and Force Systems [40 marks]

Question 1 [18 marks] A particle is in equilibrium under the action of four forces: 32 N at 20° north of east, 28 N at 70° west of north, 24 N due south, and a fourth force F.

(a) Choose a suitable coordinate system and resolve each of the three known forces into x and y components. **[6 marks]**

(b) Calculate the resultant of the three known forces in component form. **[3 marks]**

(c) Find the components of force F required for equilibrium. **[3 marks]**

(d) Calculate the magnitude and direction of force F. **[4 marks]**

(e) Verify your solution by checking that the vector sum of all four forces is zero. **[2 marks]**

Question 2 [22 marks] A stage light of mass 60 kg is suspended above a theater by three cables. Cable A makes an angle of 15° with the vertical, cable B makes an angle of 35° with the vertical, and cable C makes an angle of 55° with the vertical. The three cables meet at a single point directly above the light.

(a) Draw a force diagram showing all forces at the cable junction point. **[3 marks]**

(b) Write the equilibrium equations for the horizontal and vertical directions. **[3 marks]**

(c) If the tension in cable A is 350 N, calculate:

  (i) The horizontal component of this tension. **[2 marks]**

  (ii) The vertical component of this tension. **[2 marks]**

(d) Calculate the tension in cable B. **[6 marks]**

(e) Find the tension in cable C. **[4 marks]**

(f) Verify that the total vertical force equals the weight of the stage light. **[2 marks]**

Section B: Dynamics on Inclined Planes [45 marks]

Question 3 [25 marks] A wooden block of mass 18 kg is placed on a loading dock ramp inclined at 24° to the horizontal. The coefficient of static friction is 0.55 and the coefficient of kinetic friction is 0.40.

(a) Calculate the component of the block's weight acting parallel to the ramp surface. **[3 marks]**

(b) Find the normal reaction force between the block and ramp. **[3 marks]**

(c) Determine the maximum static friction force available. **[3 marks]**

(d) Will the block remain stationary on the ramp without additional forces? Show your reasoning. **[4 marks]**

(e) A rope is attached to pull the block up the ramp at constant velocity. Calculate the tension required in the rope. **[5 marks]**

(f) If the rope tension is increased to 120 N, calculate:

(i) The acceleration of the block up the ramp. **[4 marks]**

(ii) The velocity after the block has traveled 6 m up the ramp from rest. **[3 marks]**

Question 4 [20 marks] A sledge of mass 45 kg slides down a snow-covered hill inclined at 20° to the horizontal. The coefficient of kinetic friction between the sledge and snow is 0.15.

(a) Draw a force diagram for the sledge. **[3 marks]**

(b) Calculate the component of weight acting down the slope. **[3 marks]**

(c) Find the normal reaction force from the slope. **[3 marks]**

(d) Calculate the friction force opposing the motion. **[3 marks]**

(e) Determine the net force acting on the sledge down the slope. **[3 marks]**

(f) Find the acceleration of the sledge. **[2 marks]**

(g) If the sledge starts from rest and slides 40 m down the slope, calculate its final velocity. **[3 marks]**

Section C: Connected Particle Systems [50 marks]

Question 5 [25 marks] Three particles A, B, and C have masses 7 kg, 12 kg, and 5 kg respectively. Particle A rests on a rough horizontal table with coefficient of kinetic friction 0.3. It is connected by a light inextensible string over a smooth pulley to particle B, which hangs freely. Particle B is also connected by a second string over another smooth pulley to particle C, which rests on a smooth inclined plane at 40° to the horizontal.

(a) Draw separate force diagrams for all three particles. **[6 marks]**

(b) Calculate the component of C's weight acting down the inclined plane. **[3 marks]**

(c) Write the equation of motion for particle A in the horizontal direction. **[3 marks]**

(d) Write the equations of motion for particles B and C. **[4 marks]**

(e) Determine the direction of motion by analyzing the forces. **[3 marks]**

(f) Solve the system of equations to find the acceleration. **[4 marks]**

(g) Calculate the tensions in both connecting strings. **[2 marks]**

Question 6 [25 marks] Two particles P and Q are connected by a light inextensible string passing over smooth pulleys. Particle P (mass 9 kg) can slide on a rough inclined plane at 25° to the horizontal with coefficient of kinetic friction 0.25. Particle Q (mass 6 kg) can slide on a smooth inclined plane at 35° to the horizontal on the opposite side.

(a) Draw force diagrams for both particles. **[4 marks]**

(b) For particle P, calculate:

(i) The component of weight down the inclined plane. **[2 marks]**

(ii) The normal reaction force. **[2 marks]**

(iii) The friction force opposing motion. **[3 marks]**

(c) For particle Q, calculate the component of weight down the inclined plane. **[3 marks]**

(d) Compare the driving forces to determine which direction the system moves. **[3 marks]**

(e) Write equations of motion for both particles and solve for the acceleration. **[6 marks]**

(f) Find the tension in the connecting string. **[2 marks]**

Section D: Motion with Variable Forces [30 marks]

Question 7 [18 marks] A particle of mass 3.2 kg moves along a straight line under a driving force F = $(18 + 12t - 2t^2)$ N, where t is time in seconds. The particle also experiences a constant resistance of 8 N.

(a) Write an expression for the net force acting on the particle at time t. **[2 marks]**

(b) Calculate the acceleration at t = 1 s, t = 3 s, and t = 5 s. **[6 marks]**

(c) If the particle starts from rest, find expressions for:

(i) Velocity as a function of time. **[5 marks]**

(ii) Displacement as a function of time. **[3 marks]**

(d) Calculate the velocity and displacement when t = 4 s. **[2 marks]**

Question 8 [12 marks] A ball of mass 1.5 kg is dropped from rest and falls through air. The air resistance is proportional to the square of velocity: R = $0.3v^2$ N, where v is velocity in m/s.

(a) Write the equation of motion for the falling ball. **[3 marks]**

(b) Calculate the terminal velocity of the ball. **[4 marks]**

(c) When the ball has reached 60

(i) The air resistance force at this speed. **[2 marks]**

(ii) The acceleration at this instant. **[3 marks]**

Section E: Projectile Motion [40 marks]

Question 9 [**25 marks**] A projectile is fired from ground level at an angle of 48° above the horizontal with an initial speed of 50 m/s. A target is located 220 m away horizontally at the same level as the launch point.

(a) Calculate the initial horizontal and vertical components of velocity. [**3 marks**]

(b) Find the maximum height reached by the projectile. [**4 marks**]

(c) Calculate the time of flight of the projectile. [**4 marks**]

(d) Determine the horizontal range of the projectile. [**3 marks**]

(e) Find the time when the projectile is 220 m horizontally from the launch point. [**3 marks**]

(f) Calculate the height of the projectile at this horizontal distance. [**4 marks**]

(g) Determine whether the projectile hits the target or passes above it. [**2 marks**]

(h) Find the velocity components when the projectile returns to ground level. [**2 marks**]

Question 10 [**15 marks**] A stone is thrown horizontally from the top of a vertical cliff 85 m high with an initial speed of 18 m/s.

(a) Write expressions for the horizontal and vertical motion of the stone. [**3 marks**]

(b) Calculate the time taken for the stone to reach the base of the cliff. [**3 marks**]

(c) Find the horizontal distance from the base of the cliff where the stone lands. [**3 marks**]

(d) Calculate the velocity components just before the stone hits the ground. [**3 marks**]

(e) Find the magnitude and direction of the velocity at impact. [**3 marks**]

Section F: Equilibrium Optimization and Analysis [25 marks]

Question 11 [**15 marks**] A particle of mass 10 kg rests on a rough inclined plane at angle 30° to the horizontal. The coefficient of static friction is 0.5. A horizontal force P is applied to the particle.

(a) Draw a force diagram showing all forces acting on the particle. [**3 marks**]

(b) Without the horizontal force P, determine whether the particle will slide down the plane. [**3 marks**]

(c) Calculate the horizontal force P required to prevent the particle from sliding down the plane. [**4 marks**]

(d) Find the maximum horizontal force P that can be applied before the particle starts to slide up the plane. [**5 marks**]

Question 12 [**10 marks**] A particle is suspended in equilibrium by three strings. String A makes an angle of 40° with the vertical, string B makes an angle of 50° with the vertical, and string C makes an angle of 65° with the vertical. The particle has mass 4.5 kg.

(a) Draw a force diagram for the equilibrium situation. [**2 marks**]

(b) Calculate the tension in each of the three strings. [**6 marks**]

(c) Verify that the sum of vertical components equals the weight of the particle. [**2 marks**]

Physics Data and Formulae

Equilibrium and Forces:

Equilibrium conditions: $\sum F_x = 0$, $\sum F_y = 0$

Force components: $F_x = F\cos\theta$, $F_y = F\sin\theta$

Resultant force: $|\vec{R}| = \sqrt{F_x^2 + F_y^2}$

Direction of resultant: $\tan\alpha = \frac{F_y}{F_x}$

Newton's Laws:

First Law: $\sum \vec{F} = 0$ (equilibrium or constant velocity)

Second Law: $\sum \vec{F} = m\vec{a}$

Third Law: Forces between interacting objects are equal and opposite

Friction:

Static friction: $f_s \leq \mu_s N$, maximum value $f_{s,max} = \mu_s N$

Kinetic friction: $f_k = \mu_k N$ (opposes motion)

Limiting equilibrium: $f = \mu_s N$

Inclined Plane Forces:

Weight component parallel to plane: $mg\sin\theta$ (down the plane)

Weight component perpendicular to plane: $mg\cos\theta$ (into the plane)

Normal reaction: $N = mg\cos\theta$ (for simple case)

Kinematics:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{u+v}{2}t$

Projectile Motion:

Horizontal motion: $x = u_x t$ (constant velocity)

Vertical motion: $y = u_y t - \frac{1}{2}gt^2$

Velocity components: $v_x = u_x$, $v_y = u_y - gt$

Maximum height: $h_{max} = \frac{u_y^2}{2g}$

Time to maximum height: $t_{max} = \frac{u_y}{g}$

Range (level projection): $R = \frac{u^2 \sin 2\theta}{g}$

Time of flight (level): $T = \frac{2u\sin\theta}{g}$

Calculus for Variable Forces:

Acceleration: $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Velocity: $v = \int a\,dt + C_1$

Displacement: $s = \int v\,dt + C_2$

Constants:

Acceleration due to gravity: $g = 9.81$ m/s^2

Trigonometric Values:

$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$

$\sin 20 = 0.342$, $\cos 20 = 0.940$, $\tan 20 = 0.364$

$\sin 24 = 0.407$, $\cos 24 = 0.914$, $\tan 24 = 0.445$

$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$

$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$

$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$

$\sin 40 = 0.643$, $\cos 40 = 0.766$, $\tan 40 = 0.839$

$\sin 48 = 0.743$, $\cos 48 = 0.669$, $\tan 48 = 1.111$

$\sin 50 = 0.766$, $\cos 50 = 0.643$, $\tan 50 = 1.192$
$\sin 55 = 0.819$, $\cos 55 = 0.574$, $\tan 55 = 1.428$
$\sin 65 = 0.906$, $\cos 65 = 0.423$, $\tan 65 = 2.145$
$\sin 70 = 0.940$, $\cos 70 = 0.342$, $\tan 70 = 2.747$

END OF TEST

Total marks: 230

Grade boundaries: A* 207, A 184, B 161, C 138, D 115, E 92

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