GCSE Higher Mathematics
Practice Test 3: Number

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:
   (a) 4^3
   (b) 5^{-2}
   (c) 9^0
   (d) $(-4)^2$
   (e) $(-5)^3$
   (f) 6^{-2}

2. Write these as single powers:
   (a) $4^2 \times 4^7$
   (b) $6^8 \div 6^5$
   (c) $(2^4)^3$
   (d) $3^{-5} \times 3^9$
   (e) $\frac{7^6}{7^{-4}}$
   (f) $(5^3)^{-3}$

3. Evaluate these fractional indices:
   (a) $36^{\frac{1}{2}}$
   (b) $64^{\frac{1}{3}}$
   (c) $625^{\frac{3}{4}}$
   (d) $1024^{\frac{2}{5}}$
   (e) $216^{\frac{2}{3}}$
   (f) $32^{-\frac{3}{5}}$

4. Simplify these expressions:
   (a) $\sqrt{100}$
   (b) $\sqrt[3]{216}$

(c) $\sqrt[4]{625}$
(d) $\sqrt{0.49}$
(e) $\sqrt[3]{-125}$
(f) $\sqrt[5]{1024}$

5. Express in index form:

(a) $\sqrt{13}$
(b) $\sqrt[3]{9}$
(c) $\frac{1}{\sqrt{5}}$
(d) $\sqrt[4]{z^7}$
(e) $\frac{1}{\sqrt[3]{c^5}}$
(f) $\sqrt{z} \times \sqrt[3]{z}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

(a) $4^3 \times 4^{-5} \times 4^6$
(b) $\frac{6^8 \times 6^{-4}}{6^{-3}}$
(c) $(3^4)^{-3} \times 3^{11}$
(d) $\frac{5^{-4} \times 5^9}{5^3}$

7. Evaluate these expressions:

(a) $125^{\frac{2}{3}}$
(b) $16^{-\frac{5}{4}}$
(c) $243^{\frac{2}{5}}$
(d) $64^{-\frac{4}{3}}$
(e) $216^{\frac{1}{3}}$
(f) $625^{-\frac{3}{4}}$

8. Simplify these expressions:

(a) $x^{\frac{4}{5}} \times x^{\frac{1}{2}}$
(b) $\frac{w^{\frac{7}{3}}}{w^{\frac{1}{6}}}$
(c) $(c^{\frac{1}{4}})^8$
(d) $\sqrt{z} \times z^{\frac{1}{5}}$
(e) $\frac{\sqrt[3]{p^5}}{\sqrt{p}}$
(f) $(r^{-\frac{1}{4}})^{-8}$

9. Write these in the form a^n where a and n are rational:

(a) $\sqrt{5} \times 5^4$
(b) $\frac{9^2}{\sqrt[3]{9}}$
(c) $\sqrt[4]{6^7} \times 6^{-\frac{1}{2}}$
(d) $\frac{\sqrt{13}}{\sqrt[3]{13^5}}$

Section C: Surds

10. Simplify these surds:
    (a) $\sqrt{45}$
    (b) $\sqrt{63}$
    (c) $\sqrt{80}$
    (d) $\sqrt{112}$
    (e) $\sqrt{162}$
    (f) $\sqrt{500}$

11. Simplify these expressions:
    (a) $5\sqrt{3} + 8\sqrt{3}$
    (b) $11\sqrt{7} - 4\sqrt{7}$
    (c) $\sqrt{20} + \sqrt{45}$
    (d) $\sqrt{63} - \sqrt{28}$
    (e) $4\sqrt{8} + 5\sqrt{32}$
    (f) $\sqrt{125} - \sqrt{80} + \sqrt{20}$

12. Multiply and simplify:
    (a) $\sqrt{6} \times \sqrt{24}$
    (b) $\sqrt{15} \times \sqrt{60}$
    (c) $4\sqrt{3} \times 5\sqrt{12}$
    (d) $\sqrt{10} \times \sqrt{40}$
    (e) $\sqrt{5} \times \sqrt{20} \times \sqrt{80}$
    (f) $5\sqrt{8} \times 3\sqrt{18}$

13. Expand and simplify:
    (a) $(4 + \sqrt{5})(3 - \sqrt{5})$
    (b) $(3 + \sqrt{11})(2 + 4\sqrt{11})$
    (c) $(6 - \sqrt{7})^2$
    (d) $(\sqrt{13} + \sqrt{2})(\sqrt{13} - \sqrt{2})$
    (e) $(4\sqrt{5} + 1)(4\sqrt{5} - 1)$
    (f) $(\sqrt{11} + 4)^2$

14. Rationalize the denominators:
    (a) $\frac{1}{\sqrt{11}}$
    (b) $\frac{7}{\sqrt{13}}$
    (c) $\frac{\sqrt{5}}{\sqrt{20}}$
    (d) $\frac{8}{4\sqrt{3}}$
    (e) $\frac{1}{3+\sqrt{5}}$
    (f) $\frac{4}{1-\sqrt{11}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

    (a) $\frac{5}{4+\sqrt{7}}$

    (b) $\frac{9}{3-\sqrt{17}}$

    (c) $\frac{\sqrt{5}}{1+\sqrt{5}}$

    (d) $\frac{4\sqrt{11}}{3+\sqrt{11}}$

    (e) $\frac{1}{\sqrt{12}-\sqrt{5}}$

    (f) $\frac{\sqrt{11}+3}{\sqrt{11}-4}$

16. Simplify these expressions completely:

    (a) $\frac{\sqrt{20}+\sqrt{45}}{\sqrt{5}}$

    (b) $\frac{\sqrt{54}-\sqrt{24}}{\sqrt{6}}$

    (c) $\sqrt{(5+\sqrt{11})(5-\sqrt{11})}$

    (d) $\sqrt{80}-4\sqrt{5}+\sqrt{45}$

    (e) $(\sqrt{5}+\sqrt{20})^2$

    (f) $\frac{\sqrt{72}}{\sqrt{8}}+\frac{\sqrt{50}}{\sqrt{8}}$

17. Prove that:

    (a) $(\sqrt{u}+\sqrt{v})(\sqrt{u}-\sqrt{v})=u-v$

    (b) $\frac{1}{\sqrt{s}+\sqrt{t}}=\frac{\sqrt{s}-\sqrt{t}}{s-t}$

    (c) $(s+t\sqrt{w})^2=s^2+2st\sqrt{w}+t^2w$

Section E: Standard Form

18. Write these numbers in standard form:

    (a) 567000

    (b) 0.000083

    (c) 3450000000

    (d) 0.0000000156

    (e) 892.7

    (f) 0.00834

19. Write these in ordinary form:

    (a) 6.9×10^4

    (b) 4.58×10^{-6}

    (c) 7.432×10^{-9}

    (d) 3.25×10^{12}

    (e) 8.7×10^{-4}

    (f) 5.67×10^7

20. Calculate, giving answers in standard form:

(a) $(5 \times 10^6) \times (4 \times 10^8)$
(b) $(9 \times 10^{-4}) \times (7 \times 10^9)$
(c) $(12 \times 10^7) \div (3 \times 10^{-4})$
(d) $(8 \times 10^{-6}) \div (4 \times 10^{-9})$
(e) $(6 \times 10^5)^2$
(f) $\sqrt{25 \times 10^{12}}$

21. Calculate these more complex expressions:

(a) $(4.8 \times 10^5) \times (1.25 \times 10^{-7})$
(b) $\frac{9.6 \times 10^8}{3.2 \times 10^{-5}}$
(c) $(5.4 \times 10^{-4}) + (6.7 \times 10^{-5})$
(d) $(7.2 \times 10^7) - (4.8 \times 10^6)$
(e) $\frac{(4.5 \times 10^3) \times (3.6 \times 10^{-4})}{9 \times 10^{-2}}$
(f) $(3.24 \times 10^{10})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

(a) $\frac{5}{7} + \frac{3}{14}$
(b) $\frac{7}{12} - \frac{5}{18}$
(c) $\frac{8}{9} \times \frac{15}{16}$
(d) $\frac{9}{14} \div \frac{18}{35}$
(e) $\frac{7}{8} - \frac{3}{10} + \frac{5}{20}$
(f) $\left(\frac{4}{7}\right)^{-2}$

23. Convert these recurring decimals to fractions:

(a) $0.\overline{5}$
(b) $0.\overline{81}$
(c) $0.3\overline{4}$
(d) $0.\overline{571428}$
(e) $2.5\overline{7}$
(f) $0.62\overline{5}$

24. Work out these percentage calculations:

(a) Increase 480 by 35%
(b) Decrease 560 by 16%
(c) Find 22.5% of 720
(d) What percentage is 96 out of 120?
(e) If 55% of a number is 143, find the number
(f) A price increases from £75 to £87. Find the percentage increase

25. Solve these percentage problems:

(a) After a 40% increase, a price is £182. Find the original price
(b) After a 35% decrease, a quantity is 91. Find the original quantity
(c) The value of a car decreases by 30% each year. If it's worth £14700 now, what was it worth 2 years ago?
(d) An investment grows by 4% per year. After 2 years it's worth £2163.20. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

    (a) $4^{-2} + 5^0 - 6^{-1}$

    (b) $\sqrt{36} \times 125^{\frac{1}{3}} - 4^{-2}$

    (c) $\frac{216^{\frac{2}{3}} - 256^{\frac{3}{4}}}{25^{\frac{1}{2}}}$

    (d) $16^{-\frac{1}{2}} + 49^{\frac{1}{2}} \times 3^{-1}$

27. Calculate exactly (leave surds in your answer):

    (a) $\frac{5}{\sqrt{5}} + \frac{4}{\sqrt{20}}$

    (b) $\sqrt{24} \times \sqrt{54} - \sqrt{96}$

    (c) $\frac{\sqrt{98}+\sqrt{72}}{\sqrt{2}}$

    (d) $(4\sqrt{3} - 3)^2$

28. Work with standard form in context:

    (a) The mass of a neutron is 1.675×10^{-27} kg. Find the mass of 6.02×10^{23} neutrons

    (b) Radio waves travel at 3×10^8 m/s. How far do they travel in one day (use 1 day $= 8.64 \times 10^4$ seconds)?

    (c) The width of a virus is approximately 5×10^{-8} m. How many viruses would fit across a distance of 2 mm?

    (d) A quantum computer processes 4.8×10^{11} operations per second. How many operations in 12 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{5}$ is irrational. (Use proof by contradiction: assume $\sqrt{5} = \frac{r}{s}$ where r and s are integers with no common factors)

30. The number α satisfies $\alpha^2 = 2\alpha + 1$.

    (a) Show that $\alpha = 1 + \sqrt{2}$

    (b) Calculate α to 4 decimal places

    (c) Find $\frac{1}{\alpha}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{2}+\sqrt{6}+\sqrt{8}}$. (Hint: First rationalize using $(\sqrt{2} + \sqrt{6}) - \sqrt{8}$)

32. A rectangle has sides of length $(4 + \sqrt{3})$ cm and $(4 - \sqrt{3})$ cm.

    (a) Find the exact area

    (b) Find the exact perimeter

    (c) Show that the area is rational but the perimeter is irrational

33. The population of yeast quadruples every 5 hours. If there are initially 6×10^2 yeast cells:

    (a) How many yeast cells after 20 hours?

    (b) Express your answer in standard form

    (c) After how many hours will there be more than 3×10^7 yeast cells?

34. Show that $\frac{1}{\sqrt{r}+\sqrt{s}} + \frac{1}{\sqrt{r}-\sqrt{s}} = \frac{2\sqrt{r}}{r-s}$

35. A cylinder has volume $V = \pi r^2 h$. If the volume is 432π cm^3 and $h = 12$ cm:

    (a) Find the radius in surd form

    (b) Find the surface area (use $A = 2\pi r^2 + 2\pi rh$)

    (c) Express both answers exactly

36. The equation $x^2 - 8x + 1 = 0$ has solutions $x = 4 \pm \sqrt{15}$.

    (a) Verify this by substitution

    (b) Find $\frac{1}{4+\sqrt{15}} + \frac{1}{4-\sqrt{15}}$ without using a calculator

    (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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