

GCSE Higher Mathematics

Practice Test 3: Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

(a) $5(2x - 3) = 3x + 11$

(b) $\frac{x+4}{5} - \frac{x-3}{2} = 1$

(c) $\frac{4x-3}{6} = \frac{x+2}{4} - 2$

(d) $0.5x + 0.9 = 0.2x - 0.6$

2. Solve these simultaneous equations by elimination:

(a)
$$\begin{cases} 4x + 3y = 22 \\ 2x - 5y = -8 \end{cases}$$

(b)
$$\begin{cases} 3x + 7y = 29 \\ 5x - 2y = 8 \end{cases}$$

(c)
$$\begin{cases} 5x + 2y = 21 \\ 3x - 4y = 5 \end{cases}$$

(d)
$$\begin{cases} 6x + 5y = 32 \\ 4x - 3y = 6 \end{cases}$$

3. Solve these simultaneous equations by substitution:

(a)
$$\begin{cases} y = 4x - 5 \\ 3x + 2y = 19 \end{cases}$$

(b)
$$\begin{cases} x = 2y + 3 \\ 4x - 3y = 23 \end{cases}$$

(c)
$$\begin{cases} y = 9 - 2x \\ 3x + 5y = 37 \end{cases}$$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

(a) $y = 4x + 2$ and $y = 10 - x$

(b) $y = x^2 - 5$ and $y = 2x + 3$

(c) $x^2 + y^2 = 20$ and $y = x + 2$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

(a) $x^2 + 11x + 18$

(b) $x^2 - 9x - 22$

(c) $x^2 - 13x + 36$

(d) $x^2 + 4x - 21$

(e) $x^2 - 64$

(f) $x^2 - 16x + 64$

6. Solve these quadratic equations by factorizing:

(a) $x^2 + 11x + 28 = 0$

(b) $x^2 - 10x - 11 = 0$

(c) $x^2 - 7x = 0$

(d) $x^2 - 81 = 0$

(e) $x^2 + 18x + 81 = 0$

(f) $4x^2 - 16x = 0$

7. Factorize these harder quadratics:

(a) $2x^2 + 9x + 7$

(b) $4x^2 - 13x + 3$

(c) $25x^2 - 36$

(d) $3x^2 + 14x - 24$

(e) $25x^2 - 30x + 9$

(f) $7x^2 - 19x - 6$

8. Solve by factorizing:

(a) $4x^2 + 9x - 9 = 0$

(b) $5x^2 - 11x + 2 = 0$

(c) $16x^2 - 9 = 0$

(d) $3x^2 + 10x - 8 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

(a) $x^2 + 10x + 9$

(b) $x^2 - 4x + 1$

(c) $x^2 + 6x - 3$

(d) $x^2 - 14x + 11$

(e) $2x^2 + 16x + 7$

(f) $4x^2 - 12x + 1$

10. Solve by completing the square:

(a) $x^2 + 10x + 7 = 0$

(b) $x^2 - 8x - 1 = 0$

(c) $x^2 + 2x - 4 = 0$

(d) $2x^2 + 8x - 3 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 7x - 3 = 0$

(b) $4x^2 - 9x + 3 = 0$

(c) $x^2 - 10x + 5 = 0$

(d) $3x^2 + 5x - 3 = 0$

(e) $2x^2 + 9x + 7 = 0$

(f) $6x^2 - 7x - 2 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 9x + 14 = 0$

(b) $x^2 - 8x + 16 = 0$

(c) $x^2 + 4x + 9 = 0$

(d) $2x^2 - 7x + 3 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 8x + 12$:

- (a) Find the y-intercept
- (b) Find the x-intercepts by factorizing
- (c) Complete the square to find the vertex
- (d) Sketch the graph
- (e) State the line of symmetry

14. For the quadratic $y = 2x^2 + 8x - 3$:

- (a) Complete the square
- (b) Find the coordinates of the vertex
- (c) Find the y-intercept
- (d) State the line of symmetry
- (e) Sketch the graph

15. A football is kicked upward. Its height h (in meters) after t seconds is given by: $h = -5t^2 + 25t + 1$

- (a) What is the initial height?
- (b) At what times is the football at ground level?
- (c) What is the maximum height reached?
- (d) At what time does it reach maximum height?

16. The cost C (in thousands of pounds) for producing x thousand widgets is: $C = x^2 - 14x + 60$

- (a) How many widgets should be produced to minimize cost?
- (b) What is the minimum cost?
- (c) At what production levels is the cost £11,000?

Section E: Linear Inequalities

17. Solve these linear inequalities:

(a) $5x + 9 > 29$

(b) $4x - 11 \leq 17$

(c) $8 - 5x < 3$

(d) $\frac{x+1}{4} \geq 3$

(e) $6 - 5x > 4x + 15$

(f) $\frac{4x-3}{5} < \frac{x+6}{2}$

18. Show these inequalities on number lines:

(a) $x > -4$

(b) $x \leq 6$

(c) $-1 < x \leq 7$

(d) $x < 3$ or $x > 8$

19. Solve these compound inequalities:

(a) $-6 < 4x + 2 < 10$

(b) $7 \leq 3x - 2 \leq 16$

(c) $-1 \leq \frac{4x+3}{5} < 7$

20. Find the integer solutions to:

(a) $4x + 5 > 13$ and $x < 7$

(b) $-1 \leq x + 4 < 6$

(c) $x^2 < 36$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

(a) $x^2 - 9x + 14 > 0$

(b) $x^2 - 10x + 21 \leq 0$

(c) $x^2 - 25 < 0$

(d) $x^2 + 4x - 12 \geq 0$

(e) $x^2 - 10x + 25 > 0$

(f) $2x^2 - 3x - 2 < 0$

22. Solve and show on number lines:

(a) $x^2 - 36 \leq 0$

(b) $x^2 + 3x - 10 > 0$

(c) $3x^2 - 8x + 4 \geq 0$

23. Find the values of x for which:

(a) $x^2 < 5x + 6$

(b) $2x^2 + 5x \geq 3$

(c) $x^2 + 8x + 16 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

(a) $(x + 6)(4x - 3)$

(b) $(3x - 4)(x + 7)$

(c) $(2x + 7)^2$

(d) $(6x - 1)^2$

(e) $(x + 9)(x - 9)$

(f) $(5x + 2)(5x - 2)$

25. Expand these expressions:

(a) $(x + 4)(x^2 - 3x + 5)$

(b) $(2x - 3)(x^2 + 2x - 1)$

(c) $(x + 3)^3$

(d) $(2x - 5)^3$

26. Factorize completely:

(a) $10x^2 + 15x$

(b) $16x^2 - 49$

(c) $x^3 - 9x$

(d) $4x^3 + 16x^2 + 16x$

(e) $x^3 - 125$

(f) $64x^3 + 27$

27. Simplify these algebraic fractions:

(a) $\frac{x^2-16}{x+4}$

(b) $\frac{x^2+9x+20}{x+5}$

(c) $\frac{5x^2-20}{x^2-4}$

(d) $\frac{x^3-125}{x^2-25}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

(a) $\frac{5}{x} + \frac{2}{x}$

(b) $\frac{8}{x} - \frac{5}{x}$

(c) $\frac{3}{4x} + \frac{1}{8x}$

(d) $\frac{4}{x+1} + \frac{3}{x-2}$

(e) $\frac{x}{x+4} - \frac{3}{x-1}$

(f) $\frac{4x}{x^2-9} + \frac{3}{x+3}$

29. Multiply and divide these algebraic fractions:

(a) $\frac{x}{5} \times \frac{15}{x^2}$

(b) $\frac{x+4}{6} \times \frac{12}{x+4}$

(c) $\frac{x^2-16}{x+3} \div \frac{x-4}{x+3}$

(d) $\frac{4x+12}{x^2-25} \times \frac{x-5}{8}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{5} + \frac{x}{2} = 21$

(b) $\frac{4x+3}{6} = \frac{x-2}{4}$

(c) $\frac{5}{x} = \frac{4}{x-3}$

(d) $\frac{x+3}{x-4} = \frac{2x}{x+3}$

31. Make the subject of these formulae:

(a) $E = mc^2$, make c the subject

(b) $S = \frac{n}{2}(2a + (n-1)d)$, make d the subject

(c) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, make v the subject

(d) $T = 2\pi\sqrt{\frac{l}{g}}$, make l the subject

Section I: Sequences

32. Find the n th term for these arithmetic sequences:

(a) 11, 17, 23, 29, ...

(b) 6, 13, 20, 27, ...

(c) 35, 31, 27, 23, ...

(d) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \dots$

33. For these geometric sequences, find the n th term:

(a) 4, 12, 36, 108, ...

(b) 7, 28, 112, 448, ...

(c) 128, 64, 32, 16, ...

(d) 3, -12, 48, -192, ...

34. Find the sum of these series:

(a) First 30 terms of $8 + 12 + 16 + 20 + \dots$

(b) First 7 terms of $6 + 18 + 54 + 162 + \dots$

(c) $3 + 7 + 11 + \dots + 99$ (arithmetic series)

(d) $4 + 8 + 16 + \dots + 256$ (geometric series)

35. These are quadratic sequences. Find the n th term:

(a) 4, 16, 36, 64, 100, ...

(b) 3, 11, 25, 45, 71, ...

(c) 2, 8, 18, 32, 50, ...

(d) 6, 17, 34, 57, 86, ...

36. A sequence is defined by $u_1 = 5$ and $u_{n+1} = 2u_n + 3$.

(a) Find the first 5 terms

(b) Find a formula for u_n

(c) Calculate u_7

Section J: Problem Solving

37. The sum of two numbers is 22 and their product is 112. Find the two numbers.
38. A rectangular pool has perimeter 48m. If the length is 10m more than the width, find the dimensions.
39. The difference between a positive number and its reciprocal is $\frac{15}{4}$. Find the number.
40. A cannon ball's height h (in meters) after t seconds is given by: $h = 100t - 5t^2$
- (a) When does it hit the ground?
 - (b) What is its maximum height?
 - (c) When is it 480m high?
41. Prove that the sum of the first n terms of the sequence $1, 3, 5, 7, \dots$ is n^2 .
42. The quadratic $ax^2 + bx + c = 0$ has roots p and q .
- (a) Show that $p + q = -\frac{b}{a}$
 - (b) Show that $pq = \frac{c}{a}$
 - (c) If the roots are 5 and -3, find a , b , and c when $a = 2$
43. A function is defined as $h(x) = x^2 + kx + l$. If $h(1) = 6$ and $h(3) = 18$, find k and l .
44. The sum of the first n terms of a sequence is $S_n = 4n^2 - n$. Find the n th term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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