A Level Pure Mathematics
Practice Test 5: Vectors

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Vector Basics and Notation

1. Given vectors $\mathbf{r} = \begin{pmatrix} 7 \\ -3 \\ 4 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$, calculate:

   (a) $\mathbf{r} + \mathbf{s}$

   (b) $\mathbf{r} - \mathbf{s}$

   (c) $3\mathbf{r} + 4\mathbf{s}$

   (d) $2\mathbf{r} - 5\mathbf{s}$

   (e) $|\mathbf{r}|$ and $|\mathbf{s}|$

   (f) A unit vector in the direction of $\mathbf{r}$

2. Express these vectors in component form:

   (a) $\overrightarrow{PQ}$ where $P(5, 1, -2)$ and $Q(2, 4, 3)$

   (b) $\overrightarrow{UV}$ where $U(-1, 3, 2)$ and $V(3, -2, 4)$

   (c) The position vector of point W if $\overrightarrow{OW} = 5\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$

   (d) $\overrightarrow{QP}$ where $P(3, -2, 5)$ and $Q(1, 4, -1)$

3. Given $\mathbf{e} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{f} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$:

   (a) Find $|\mathbf{e}|$ and $|\mathbf{f}|$

   (b) Calculate $\mathbf{e} + \mathbf{f}$ and $\mathbf{e} - \mathbf{f}$

   (c) Find scalars x and y such that $x\mathbf{e} + y\mathbf{f} = \begin{pmatrix} 4 \\ -8 \\ 11 \end{pmatrix}$

   (d) Determine if $\mathbf{e}$ and $\mathbf{f}$ are parallel

4. Points L, M, and N have position vectors $\mathbf{l} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$, $\mathbf{m} = \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix}$, and $\mathbf{n} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$.

   (a) Find vectors $\overrightarrow{LM}$ and $\overrightarrow{LN}$

   (b) Calculate the lengths $|LM|$ and $|LN|$

(c) Find the position vector of the midpoint of MN

(d) Determine if triangle LMN is isosceles

5. Find the values of t for which these vectors are perpendicular:

(a) $\mathbf{g} = \begin{pmatrix} 3 \\ t \\ 5 \end{pmatrix}$ and $\mathbf{h} = \begin{pmatrix} t \\ 4 \\ -2 \end{pmatrix}$

(b) $\mathbf{j} = \begin{pmatrix} 4 \\ 2t \\ 3 \end{pmatrix}$ and $\mathbf{k} = \begin{pmatrix} 1 \\ -2 \\ t \end{pmatrix}$

(c) $\mathbf{l} = t\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{m} = 4\mathbf{i} + t\mathbf{j} + 3\mathbf{k}$

Section B: Dot Product (Scalar Product)

6. Calculate the dot product of these vectors:

(a) $\mathbf{n} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{o} = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$

(b) $\mathbf{p} = 5\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{q} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$

(c) $\mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$

(d) $\mathbf{t} = 6\mathbf{i} + 3\mathbf{j}$ and $\mathbf{u} = 4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$

7. Find the angle between these pairs of vectors:

(a) $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(b) $\mathbf{x} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

(c) $\mathbf{z} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

(d) $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$

8. Use the dot product to verify these properties:

(a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ (commutative)

(b) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$ (distributive)

(c) $(k\mathbf{u}) \cdot \mathbf{v} = k(\mathbf{u} \cdot \mathbf{v})$ for scalar k

(d) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$

9. Given vectors $\mathbf{d} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$, $\mathbf{e} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$, and $\mathbf{f} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$:

(a) Show that $\mathbf{d}$ and $\mathbf{e}$ are perpendicular

(b) Find the component of $\mathbf{f}$ in the direction of $\mathbf{d}$

(c) Calculate $|\mathbf{d} + \mathbf{e} + \mathbf{f}|$

(d) Find the angle between $\mathbf{d} + \mathbf{e}$ and $\mathbf{f}$

10. A triangle has vertices at $G(5, 2, 1)$, $H(3, 6, 2)$, and $I(4, 3, 5)$.
    (a) Find the vectors $\overrightarrow{GH}$ and $\overrightarrow{GI}$
    (b) Calculate the angle $\angle HGI$
    (c) Find the area of triangle GHI
    (d) Determine if the triangle is right-angled

Section C: Cross Product (Vector Product)

11. Calculate the cross product of these vectors:
    (a) $\mathbf{g} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{h} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$
    (b) $\mathbf{i} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ and $\mathbf{j} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
    (c) $\mathbf{k} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{l} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$
    (d) $\mathbf{m} = 5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{n} = 3\mathbf{i} + 4\mathbf{k}$

12. Verify these properties of the cross product:
    (a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$ (anti-commutative)
    (b) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$ (distributive)
    (c) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$
    (d) $|\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2|\mathbf{v}|^2 - (\mathbf{u} \cdot \mathbf{v})^2$

13. Find the area of the parallelogram spanned by:
    (a) $\mathbf{o} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$
    (b) $\mathbf{q} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
    (c) Vectors from origin to points $(2, 5, 3)$ and $(4, 1, 2)$
    (d) $\overrightarrow{JK}$ and $\overrightarrow{JL}$ where $J(5, 1, 3)$, $K(2, 4, 1)$, $L(4, 2, 6)$

14. Given $\mathbf{x} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$:
    (a) Calculate $\mathbf{x} \times \mathbf{y}$
    (b) Verify that $\mathbf{x} \times \mathbf{y}$ is perpendicular to both $\mathbf{x}$ and $\mathbf{y}$
    (c) Find a unit vector perpendicular to both $\mathbf{x}$ and $\mathbf{y}$
    (d) Calculate the area of triangle with sides $\mathbf{x}$ and $\mathbf{y}$

15. Use the scalar triple product $\mathbf{x} \cdot (\mathbf{y} \times \mathbf{z})$ to find:
    (a) The volume of parallelepiped with edges $\mathbf{x} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$, $\mathbf{z} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$
    (b) Whether points $A(5, 1, 3)$, $B(2, 6, 1)$, $C(4, 3, 2)$, $D(3, 5, 4)$ are coplanar
    (c) The volume of tetrahedron with vertices at $(0, 0, 0)$, $(5, 1, 3)$, $(2, 4, 1)$, $(3, 2, 5)$

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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