A Level Mechanics
Practice Test 2: Circular Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Circular Motion Fundamentals [26 marks]

Question 1 [9 marks]

(a) Define angular displacement and angular velocity, stating their SI units. **[3 marks]**

(b) Derive the relationship between linear speed and angular velocity: $v = r\omega$. **[3 marks]**

(c) Define centripetal acceleration and explain why it is directed toward the center of the circle. **[3 marks]**

Question 2 [11 marks] A wheel of radius 1.8 m rotates with constant angular velocity 3.5 rad/s.

(a) Calculate the linear speed of a point on the rim. **[2 marks]**

(b) Find the centripetal acceleration of this point. **[2 marks]**

(c) Calculate the angular displacement in 45 seconds. **[2 marks]**

(d) How many complete revolutions occur in this time? **[2 marks]**

(e) If a particle of mass 1.5 kg is attached to the rim, calculate the centripetal force. **[3 marks]**

Question 3 [6 marks] A racing car moves around a circular track of radius 180 m at constant speed 30 m/s.

(a) Calculate the centripetal acceleration. **[2 marks]**

(b) Find the angular velocity of the car. **[2 marks]**

(c) Calculate the time for one complete lap. **[2 marks]**

Section B: Centripetal Force [28 marks]

Question 4 [10 marks]

(a) Explain what provides the centripetal force and why it is always directed radially inward. **[3 marks]**

(b) State the formula for centripetal force in terms of mass, velocity, and radius. **[2 marks]**

(c) Distinguish between centripetal force (real force) and centrifugal force (fictitious force). **[3 marks]**

(d) List four different types of forces that can provide centripetal force in various situations. **[2 marks]**

Question 5 [18 marks] A stone of mass 0.8 kg is tied to a string of length 1.2 m and whirled in a horizontal circle.

(a) If the stone moves with angular velocity 4 rad/s, calculate the tension in the string. **[3 marks]**

(b) Find the linear speed of the stone. **[2 marks]**

(c) If the maximum tension the string can withstand is 60 N, find the maximum angular velocity. **[4 marks]**

(d) Calculate the corresponding maximum linear speed. **[2 marks]**

(e) When whirled in a vertical circle with the same string, find the minimum speed at the top to keep the string taut. **[4 marks]**

(f) Calculate the tension at the bottom when moving at this critical speed. **[3 marks]**

Section C: Horizontal Circular Motion [32 marks]

Question 6 [16 marks] A motorcycle of mass 250 kg travels around a flat circular track of radius 120 m.

(a) If the coefficient of static friction is 0.7, find the maximum speed without slipping. **[4 marks]**

(b) Calculate the centripetal acceleration at this maximum speed. **[2 marks]**

(c) Find the maximum friction force available. **[2 marks]**

(d) If the motorcycle travels at 18 m/s, calculate the required centripetal force. **[2 marks]**

(e) Determine if the motorcycle will slip at this speed. **[3 marks]**

(f) Find the minimum coefficient of friction needed to travel at 22 m/s safely. **[3 marks]**

Question 7 [16 marks] A skater of mass 60 kg moves in a horizontal circle of radius 8 m on ice. The coefficient of friction between skates and ice is 0.15.

(a) Calculate the maximum centripetal force available from friction. **[2 marks]**

(b) Find the maximum speed the skater can maintain in the circle. **[3 marks]**

(c) If the skater moves at 6 m/s, calculate the actual friction force required. **[3 marks]**

(d) Find the centripetal acceleration at this speed. **[2 marks]**

(e) Calculate the angular velocity. **[2 marks]**

(f) If the skater leans inward at angle 20° from vertical, find the normal force from the ice. **[4 marks]**

Section D: Banked Curves [24 marks]

Question 8 [**8 marks**]

(a) Explain the purpose of banking roads on curves and how it reduces dependence on friction. [**3 marks**]

(b) Derive the banking formula for the case with no friction: $\tan\theta = \frac{v^2}{rg}$. [**5 marks**]

Question 9 [**16 marks**] A highway curve has radius 250 m and is banked at 12°.

(a) Calculate the design speed for which no friction is required. [**3 marks**]

(b) A car of mass 1500 kg travels at 28 m/s on this curve. Calculate the required centripetal force. [**2 marks**]

(c) Resolve forces to find the normal force from the road surface. [**4 marks**]

(d) Calculate the friction force needed and state its direction. [**4 marks**]

(e) If the coefficient of friction is 0.6, determine if the car will slip. [**3 marks**]

Section E: Vertical Circular Motion [29 marks]

Question 10 [**11 marks**]

(a) Explain why the tension in a string varies during vertical circular motion. [**3 marks**]

(b) Derive the minimum speed condition at the top of a vertical circle: $v_{top} \geq \sqrt{gr}$. [**4 marks**]

(c) Using energy conservation, find the relationship between speeds at top and bottom of a vertical circle. [**4 marks**]

Question 11 [**18 marks**] A ball of mass 0.4 kg attached to a string of length 0.9 m moves in a vertical circle.

(a) Calculate the minimum speed at the top to complete the circle. [**2 marks**]

(b) Using energy conservation, find the minimum speed at the bottom. [**4 marks**]

(c) Calculate the tension at the top when moving at this minimum speed. [**2 marks**]

(d) Find the tension at the bottom at minimum speed. [**3 marks**]

(e) If the speed at the bottom is 7 m/s, calculate the speed at the top. [**3 marks**]

(f) Find the tensions at top and bottom for this case. [**4 marks**]

Section F: Conical Pendulums [22 marks]

Question 12 [**8 marks**]

(a) Describe a conical pendulum and identify the forces acting on the bob. [**3 marks**]

(b) Derive the formula relating the half-angle and angular velocity: $\cos\theta = \frac{g}{\omega^2 l}$. [**5 marks**]

Question 13 [**14 marks**] A conical pendulum has a bob of mass 0.5 kg suspended by a string of length 2 m. The bob moves in a horizontal circle of radius 1.2 m.

(a) Find the angle the string makes with the vertical. [**3 marks**]

(b) Calculate the tension in the string. [**3 marks**]

(c) Find the angular velocity. **[3 marks]**

(d) Calculate the linear speed. **[2 marks]**

(e) Find the period of one revolution. **[2 marks]**

(f) Calculate the centripetal acceleration. **[1 mark]**

Section G: Motion in a Vertical Circle - Loops [27 marks]

Question 14 [16 marks] A roller coaster car of mass 600 kg approaches a vertical loop of radius 15 m.

(a) Find the minimum speed at the top to maintain contact with the track. **[3 marks]**

(b) Calculate the minimum speed at the bottom using energy conservation. **[4 marks]**

(c) If the car enters at 28 m/s, find its speed at the top. **[3 marks]**

(d) Calculate the normal force at the top for this speed. **[3 marks]**

(e) Find the normal force at the bottom. **[3 marks]**

Question 15 [11 marks] A particle slides on the inside of a smooth vertical circle of radius 1.5 m. It starts from rest at the top.

(a) Calculate the speed at the bottom using energy conservation. **[3 marks]**

(b) Find the normal force at the bottom if the particle has mass 2.5 kg. **[3 marks]**

(c) Calculate the speed when the particle is level with the center. **[3 marks]**

(d) Find the normal force at this position. **[2 marks]**

Section H: Applications and Problem Solving [28 marks]

Question 16 [14 marks] A communications satellite orbits Earth at altitude 300 km. Take Earth's radius as 6400 km and surface gravity as 9.8 m/s^2.

(a) Calculate the orbital radius from Earth's center. **[1 mark]**

(b) Find the gravitational acceleration at this altitude. **[3 marks]**

(c) Calculate the orbital speed. **[3 marks]**

(d) Find the orbital period. **[3 marks]**

(e) If the satellite has mass 800 kg, calculate the centripetal force required. **[2 marks]**

(f) Verify this equals the gravitational force. **[2 marks]**

Question 17 [14 marks] A fairground ride has seats on 4 m chains hanging from a rotating platform. When operating, the chains make 25° with the vertical.

(a) Calculate the radius of the circular path. **[2 marks]**

(b) Find the angular velocity of the platform. **[4 marks]**

(c) Calculate the linear speed of the seats. **[2 marks]**

(d) If a seat and rider have total mass 90 kg, find the tension in the chain. **[3 marks]**

(e) Calculate the centripetal acceleration. **[2 marks]**

(f) Find the frequency of rotation in revolutions per minute. **[1 mark]**

Section I: Advanced Circular Motion [26 marks]

Question 18 [**13 marks**] A car travels over a hill that can be modeled as part of a circle with radius 60 m.

(a) Find the maximum speed to maintain contact with the road at the crest. [**4 marks**]

(b) If the car has mass 1200 kg and travels at 15 m/s, calculate the normal force at the crest. [**4 marks**]

(c) Find the apparent weight of an 80 kg passenger at this speed. [**3 marks**]

(d) Calculate the centripetal acceleration. [**2 marks**]

Question 19 [**13 marks**] A bead slides on a smooth wire bent into a vertical circle of radius 1.2 m. It is released from rest at a height of 1.5 m above the bottom.

(a) Calculate the speed at the bottom using energy conservation. [**3 marks**]

(b) Find the normal force from the wire at the bottom if the bead has mass 0.3 kg. [**3 marks**]

(c) Calculate the speed at the top of the circle. [**3 marks**]

(d) Determine if the bead maintains contact with the wire at the top. [**4 marks**]

Section J: Comprehensive Applications [28 marks]

Question 20 [**16 marks**] A small object sits on a horizontal turntable at distance 0.4 m from the center. The coefficient of static friction is 0.25.

(a) Calculate the maximum angular velocity before the object starts to slip. [**4 marks**]

(b) Find the corresponding linear speed. [**2 marks**]

(c) If the turntable rotates at 3 rad/s, calculate the friction force on a 0.2 kg object. [**3 marks**]

(d) Determine if this object will slip. [**2 marks**]

(e) If the object is moved to 0.6 m from the center, will it slip at 3 rad/s? [**3 marks**]

(f) Find the minimum coefficient of friction needed at this new position. [**2 marks**]

Question 21 [**12 marks**] Two particles of masses 1.5 kg and 2.5 kg are connected by a rigid rod of negligible mass and length 1 m. The system rotates about an axis perpendicular to the rod.

(a) Find the center of mass of the system. [**3 marks**]

(b) If the axis passes through the center of mass, calculate the moment of inertia. [**3 marks**]

(c) When rotating at 5 rad/s, find the kinetic energy of the system. [**2 marks**]

(d) Calculate the centripetal force on each mass. [**4 marks**]

Physics Data and Formulae

Circular Motion:

Angular velocity: $\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Vertical Circular Motion:

At top: $T + mg = \frac{mv^2}{r}$ (string) or $N + mg = \frac{mv^2}{r}$ (track)

At bottom: $T - mg = \frac{mv^2}{r}$ (string) or $N - mg = \frac{mv^2}{r}$ (track)

Minimum speed at top: $v_{min} = \sqrt{gr}$

Banking:

No friction: $\tan\theta = \frac{v^2}{rg}$

With friction up slope: $\tan\theta = \frac{v^2/rg - \mu}{1 + \mu v^2/rg}$

With friction down slope: $\tan\theta = \frac{v^2/rg + \mu}{1 - \mu v^2/rg}$

Conical Pendulum:

$\cos\theta = \frac{g}{\omega^2 l}$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 r$

$r = l\sin\theta$

Energy Conservation:

$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

Orbital Motion:

$g_h = g\left(\frac{R}{R+h}\right)^2$ where h is altitude

Orbital speed: $v = \sqrt{g_h r}$ where $r = R + h$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s²

Earth's radius: $R = 6.4 \times 10^6$ m

Trigonometric Values:

$\sin 12 = 0.208$, $\cos 12 = 0.978$, $\tan 12 = 0.213$

$\sin 20 = 0.342$, $\cos 20 = 0.940$, $\tan 20 = 0.364$

$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$

$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$

END OF TEST

Total marks: 280

Grade boundaries: A* 252, A 224, B 196, C 168, D 140, E 112

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