# GCSE Higher Mathematics Practice Test 2: Statistics

#### **Instructions:**

Answer all questions. Show your working clearly. Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

#### Section A: Advanced Averages and Spread

1. The table shows the distribution of daily temperatures:

Temperature (°C)	Frequency				
5-9	4				
10-14	7				
15-19	12				
20-24	19				
25-29	16				
30-34	11				
35-39	6				
40-44	2				

#### Calculate:

- (a) The total number of days recorded
- (b) An estimate of the mean temperature
- (c) The modal class
- (d) An estimate of the median temperature
- (e) An estimate of the range
- 2. For the data set: 8, 11, 14, 17, 19, 23, 26, 29, 32, 38
  - (a) Calculate the mean
  - (b) Find the median and quartiles (Q1 and Q3)
  - (c) Calculate the interquartile range
  - (d) Calculate the standard deviation
  - (e) Identify any outliers using the  $1.5 \times IQR$  rule
- 3. Two data sets have the following properties:
  - Set X: mean = 38, standard deviation = 6, n = 25
  - Set Y: mean = 44, standard deviation = 9, n = 35
  - (a) Calculate the combined mean

- (b) Calculate the combined standard deviation
- (c) Which set is more consistent? Explain.
- (d) Calculate the coefficient of variation for each set
- 4. The heights (in cm) of 40 students are summarized:

$$\sum x = 6800, \sum x^2 = 1162400$$

- (a) Calculate the mean height
- (b) Calculate the variance
- (c) Calculate the standard deviation
- (d) If each student grows 3 cm, find the new mean and standard deviation

#### Section B: Histograms and Frequency Density

5. The histogram shows the distribution of waiting times:

[Imagine a histogram with: 0-5 min (density 3.2), 5-10 min (density 4.8), 10-15 min (density 2.4), 15-25 min (density 1.6), 25-40 min (density 0.6)]

- (a) Complete the frequency table
- (b) Calculate the total number of customers
- (c) Estimate the mean waiting time
- (d) Find the modal class
- (e) What percentage of customers wait more than 15 minutes?
- 6. Draw a histogram for this data about car ages:

Age (years)	Frequency					
0-2	24					
2-4	32					
4-5	15					
5-8	27					
8-12	16					
12-20	12					

- (a) Calculate the frequency density for each class
- (b) Draw the histogram
- (c) Estimate the median car age
- (d) What fraction of cars are more than 5 years old?
- 7. A histogram shows data with unequal class widths. The class 15-18 has frequency density 8 and the class 18-24 has frequency 36.
  - (a) Find the frequency for the 15-18 class
  - (b) Find the frequency density for the 18-24 class
  - (c) If the total frequency is 150, suggest frequencies for other classes

#### Section C: Cumulative Frequency and Box Plots

8. The table shows the cumulative frequency of quiz scores:

Score	Cumulative Frequency
$\leq 15$	6
$\leq 25$	16
$\leq 35$	32
$\leq 45$	54
$\leq 55$	71
$\leq 65$	83
$\leq 75$	92
$\leq 85$	96

- (a) Draw the cumulative frequency curve
- (b) Find the median
- (c) Find the quartiles Q1 and Q3
- (d) Calculate the interquartile range
- (e) Draw a box plot
- (f) Estimate the 90th percentile
- 9. Two box plots show the distribution of ages for employees at two companies:

[Imagine box plots: Company A (min 22, Q1 28, median 35, Q3 42, max 58), Company B (min 24, Q1 31, median 38, Q3 46, max 62)]

Compare the distributions by commenting on:

- (a) Central tendency (medians)
- (b) Spread (ranges and IQRs)
- (c) Shape and outliers
- (d) Which company has more variable ages?
- 10. The cumulative frequency curve for delivery times (in minutes) passes through these points: (15, 0), (20, 8), (25, 22), (30, 38), (35, 51), (40, 61), (45, 65)
  - (a) Find the median delivery time
  - (b) Find the quartiles
  - (c) What percentage have delivery times between 22 minutes and 37 minutes?
  - (d) Draw the corresponding box plot

# Section D: Scatter Graphs and Correlation

11. The table shows data for 10 cars:

Engine size (L)	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8
Fuel consumption (L/100km)	5.2	5.8	6.4	6.9	7.5	8.0	8.6	9.1	9.7	10.2

- (a) Plot a scatter graph
- (b) Describe the correlation
- (c) Calculate the equation of the line of best fit
- (d) Use your line to predict fuel consumption for a 2.1L engine

- (e) Estimate the engine size for 7.8L/100km consumption
- (f) Calculate the correlation coefficient
- 12. The equation of a regression line is y = 2.3x + 8.
  - (a) Interpret the gradient
  - (b) Interpret the y-intercept
  - (c) If x = 18, predict y
  - (d) If y = 62, estimate x
  - (e) State assumptions made when using this model
- 13. Classify these correlation coefficients and describe the relationships:
  - (a) r = 0.89
  - (b) r = -0.76
  - (c) r = 0.23
  - (d) r = -0.94
  - (e) r = 0.58

## Section E: Advanced Probability

- 14. A box contains 6 yellow balls, 4 purple balls, and 3 orange balls. Two balls are drawn without replacement.
  - (a) Draw a tree diagram
  - (b) Find P(both yellow)
  - (c) Find P(both same color)
  - (d) Find P(at least one purple)
  - (e) Find P(different colors)
- 15. The probability that a machine breaks down on any day is 0.15, independently of other days.
  - (a) Find the probability it breaks down on exactly 1 out of 4 days
  - (b) Find the probability it breaks down on at least 2 out of 4 days
  - (c) Find the expected number of breakdowns in a week
  - (d) In a 20-day period, find P(more than 5 breakdowns)
- 16. A quiz has 12 true/false questions. A student guesses randomly.
  - (a) Find P(correct answer on one question)
  - (b) Find P(exactly 8 correct answers)
  - (c) Find P(at least 9 correct answers)
  - (d) Find the expected number of correct answers
  - (e) Find the most likely number of correct answers
- 17. Events C and D are such that P(C) = 0.45, P(D) = 0.35, and P(C D) = 0.18.
  - (a) Find P(C D)
  - (b) Find P(C')
  - (c) Find P(C D)
  - (d) Find P(D C)
  - (e) Are C and D independent? Justify your answer

#### Section F: Conditional Probability and Independence

- 18. A survey of 180 people about streaming services gives:
  - 105 subscribe to Netflix
  - 75 subscribe to Amazon Prime
  - 42 subscribe to both services
  - (a) Draw a Venn diagram
  - (b) Find P(subscribes to Netflix subscribes to Amazon Prime)
  - (c) Find P(subscribes to Amazon Prime subscribes to Netflix)
  - (d) Find P(subscribes to exactly one service)
  - (e) Are the subscriptions independent? Explain
- 19. In a workshop, 60% of products are made by Worker X and 40% by Worker Y. Worker X produces 3% defective items, Worker Y produces 7% defective items.
  - (a) Draw a tree diagram
  - (b) Find the probability a product is defective
  - (c) If a product is defective, find the probability it was made by Worker X
  - (d) If a product is not defective, find the probability it was made by Worker Y
- 20. A deck contains 10 black cards numbered 1-10 and 8 white cards numbered 1-8. A card is drawn at random.
  - (a) Find P(black and odd number)
  - (b) Find P(white even number)
  - (c) Find P(number ; 6)
  - (d) Are color and number parity independent?
- 21. A screening test is 92% accurate for positive cases and 96% accurate for negative cases. 3% of the population has the condition.
  - (a) Find the probability of testing positive
  - (b) If someone tests positive, find the probability they have the condition
  - (c) If someone tests negative, find the probability they don't have the condition
  - (d) Comment on the effectiveness of the test

# Section G: Hypothesis Testing and Sampling

- 22. A die is suspected of being biased towards sixes. It's rolled 30 times and shows six 8 times.
  - (a) State the null and alternative hypotheses
  - (b) Calculate the probability of getting 8 or more sixes if the die is fair
  - (c) At the 5% significance level, is there evidence the die is biased?
  - (d) What would be a Type I error in this context?
- 23. A sample of 60 batteries has mean lifetime 720 hours and standard deviation 85 hours.
  - (a) Calculate a 95% confidence interval for the population mean
  - (b) Interpret your confidence interval
  - (c) What assumptions are made?

- (d) How would the interval change with a smaller sample size?
- 24. A bakery claims 98% of cakes meet quality standards. In a sample of 80 cakes, 76 meet standards.
  - (a) Test at 5% level whether the claim is justified
  - (b) Calculate the critical value
  - (c) State your conclusion
  - (d) What is the p-value for this test?

#### Section H: Problem Solving and Integration

- 25. A restaurant records customer waiting times over 120 visits. The data shows:
  - Mean = 18 minutes
  - Standard deviation = 4.5 minutes
  - Distribution is approximately normal
  - (a) Find P(waiting time ; 25 minutes)
  - (b) Find the waiting time exceeded by only 5% of customers
  - (c) What percentage of customers wait within two standard deviations of the mean?
  - (d) If the restaurant serves 300 customers per week, estimate weekly complaints (assuming complaints when wait ¿ 20 minutes)
- 26. A production line samples 15 items every 2 hours. Over 6 sampling periods, the number of defective items found was: 1, 3, 0, 2, 1, 4.
  - (a) Calculate the mean and standard deviation
  - (b) Test whether the defect rate exceeds 8%
  - (c) Create a control chart with warning limits
  - (d) Comment on process stability
- 27. Compare these three portfolio options over 3 years:
  - Portfolio P: Mean return 12%, standard deviation 15%
  - Portfolio Q: Mean return 9%, standard deviation 10%
  - Portfolio R: Mean return 15%, standard deviation 22%
  - (a) Calculate the coefficient of variation for each
  - (b) Which offers the best risk-adjusted return?
  - (c) Using normal distribution, find P(return; 0%) for each
  - (d) Recommend an option for a conservative investor
- 28. A college tracks the relationship between study hours and exam grades. The correlation is 0.68.
  - (a) What does this correlation suggest?
  - (b) If study hours have mean 25 and standard deviation 8, and grades have mean 72 and standard deviation 14, find the regression equation
  - (c) Predict the grade for a student studying 30 hours
  - (d) Calculate the coefficient of determination and interpret it
- 29. Design a statistical investigation to compare the effectiveness of two training programs:

- (a) State hypotheses
- (b) Describe the sampling method
- (c) Identify variables and potential confounding factors
- (d) Outline the analysis plan
- (e) Discuss limitations and assumptions

### **Answer Space**

Use this space for your working and answers.

#### END OF TEST

Total marks: 100

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