GCSE Higher Mathematics
Practice Test 9: Probability

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Conditional Probability Fundamentals

1. A survey of 320 students shows:
   - 195 study Biology
   - 168 study Chemistry
   - 97 study both Biology and Chemistry

   (a) Draw a Venn diagram
   (b) Find P(studies Biology — studies Chemistry)
   (c) Find P(studies Chemistry — studies Biology)
   (d) Find P(studies exactly one subject)
   (e) Are studying Biology and Chemistry independent? Justify your answer

2. A jar contains 18 orange balls and 12 purple balls. Two balls are drawn without replacement.

   (a) Find P(second ball is orange — first ball is orange)
   (b) Find P(second ball is purple — first ball is orange)
   (c) Find P(both balls are the same color)
   (d) Find P(balls are different colors)
   (e) Verify that your probabilities sum to 1

3. Events A and B are such that:
   - P(A) = 0.67
   - P(B) = 0.38
   - P(A B) = 0.29

   Calculate:

   (a) P(A B)
   (b) P(A’)
   (c) P(A — B)
   (d) P(B — A)

(e) P(A' — B')
(f) P(A — B')

4. A card is drawn from a standard 52-card deck. Let A = "card is even numbered" (2, 4, 6, 8, 10) and B = "card is a King".
   (a) Find P(A), P(B), and P(A B)
   (b) Calculate P(A — B)
   (c) Calculate P(B — A)
   (d) Are events A and B independent? Show your working
   (e) Find P(A' B')

Section B: Tree Diagrams and Sequential Events

5. A bag contains 17 blue counters and 13 yellow counters. A counter is drawn, its color noted, and then replaced. This process is repeated twice more.
   (a) Draw a tree diagram for all three draws
   (b) Find P(all three counters are blue)
   (c) Find P(exactly two counters are yellow)
   (d) Find P(at least one counter is blue)
   (e) Find P(first counter is yellow — exactly two are yellow)
6. Box A contains 7 green balls and 3 orange balls. Box B contains 4 green balls and 8 orange balls. A fair die is rolled; if it shows 1 or 2, Box A is chosen, otherwise Box B is chosen. Then a ball is drawn.
   (a) Draw a tree diagram
   (b) Find P(green ball)
   (c) Find P(orange ball)
   (d) Given a green ball was drawn, find P(it came from Box A)
   (e) Given an orange ball was drawn, find P(it came from Box B)
7. Three factories produce widgets with different defect rates:
   - Factory A: produces 45% of widgets, 4% defective
   - Factory B: produces 33% of widgets, 9% defective
   - Factory C: produces 22% of widgets, 14% defective

   (a) Find the overall probability of a defective widget
   (b) If a widget is defective, find the probability it came from Factory A
   (c) If a widget is defective, find the probability it came from Factory C
   (d) If a widget is not defective, which factory most likely produced it?
8. A student takes three exams. The probability of passing each exam is 0.78, and the exams are independent.
   (a) Find P(passes all three exams)
   (b) Find P(fails all three exams)
   (c) Find P(passes exactly two exams)
   (d) Find P(passes at least one exam)
   (e) Given the student passed at least two exams, find P(passed all three)

Section C: Bayes' Theorem Applications

9. A diagnostic test for a disease has the following characteristics:

   - If a person has the disease, the test is positive 85% of the time
   - If a person doesn't have the disease, the test is negative 91% of the time
   - 1.7% of the population has the disease

   (a) Find P(positive test)

   (b) If someone tests positive, find P(they have the disease)

   (c) If someone tests negative, find P(they don't have the disease)

   (d) Comment on the reliability of a positive test result

   (e) How would the results change if 17% of the population had the disease?

10. An alarm system has three detectors. The probability each detector activates during a break-in is:

    - Detector A: 0.91
    - Detector B: 0.86
    - Detector C: 0.95

    The detectors operate independently.

    (a) Find P(all three detectors activate during a break-in)

    (b) Find P(at least one detector activates during a break-in)

    (c) Find P(exactly two detectors activate during a break-in)

    (d) If exactly two detectors activate, find P(Detector B failed)

    (e) Which single detector is most reliable?

11. A company manufactures products using two methods. Method A is used 68% of the time and produces 6% defective products. Method B is used 32% of the time and produces 17% defective products.

    (a) A random product is selected and found to be defective. Use Bayes' theorem to find P(produced by Method A)

    (b) If 800 products are manufactured, how many would you expect to be defective?

    (c) How many of the defective products would come from each method?

    (d) To reduce overall defect rate to 5%, what should Method B's defect rate be?

12. Three spam filters work independently to classify emails:

    - Filter A: 76% accurate for spam, 94% accurate for legitimate emails
    - Filter B: 87% accurate for spam, 89% accurate for legitimate emails
    - Filter C: 82% accurate for spam, 92% accurate for legitimate emails

    Historically, 38% of emails are spam.

    (a) If All three filters classify an email as spam, find P(it actually is spam)

    (b) If Filter A classifies as spam but Filters B and C classify as legitimate, find P(it's spam)

    (c) Which filter would you trust most for identifying spam?

    (d) Which filter would you trust most for identifying legitimate emails?

Section D: Introduction to Binomial Distribution

13. A biased coin shows heads with probability 0.4. The coin is flipped 18 times.
    (a) Find P(exactly 8 heads)
    (b) Find P(at most 6 heads)
    (c) Find P(at least 10 heads)
    (d) Find the expected number of heads
    (e) Find the most likely number of heads
    (f) Calculate the variance of the number of heads

14. A quiz has 24 questions, each with 4 possible answers. A student guesses randomly on all questions.
    (a) State the distribution of the number of correct answers
    (b) Find P(exactly 8 correct answers)
    (c) Find P(more than 10 correct answers)
    (d) Find the expected number of correct answers
    (e) Find P(passes the quiz) if the pass mark is 40%
    (f) Calculate the standard deviation of correct answers

15. The probability that a light bulb lasts more than 1000 hours is 0.73. A box contains 20 light bulbs.
    (a) Find P(all bulbs last more than 1000 hours)
    (b) Find P(exactly 16 bulbs last more than 1000 hours)
    (c) Find P(fewer than 12 bulbs last more than 1000 hours)
    (d) How many bulbs would you expect to last more than 1000 hours?
    (e) Find P(at least 80% of bulbs last more than 1000 hours)
    (f) What's the most likely number of bulbs to last more than 1000 hours?

16. A production line produces 12% defective components. Quality control inspects 40 components.
    (a) Find P(no defective components in the sample)
    (b) Find P(exactly 5 defective components)
    (c) Find P(more than 8 defective components)
    (d) Calculate the expected number of defective components
    (e) Find P(defect rate in sample exceeds 18%)
    (f) Calculate the probability that the sample defect rate is between 8% and 16%

Section E: Advanced Binomial Applications

17. A tennis player wins 76% of their serves. In a match, they serve 42 times.
    (a) Model this situation and state any assumptions
    (b) Find P(wins at least 35 serves)
    (c) Find P(wins between 30 and 35 serves inclusive)
    (d) Calculate the expected number of successful serves
    (e) Find the probability their success rate in this match is above 85%
    (f) What's the minimum number of serves needed for P(at least 1 success) 0.999999999?

18. A quality inspector examines 38 products per shift. The probability any product is faulty is 0.14.
    (a) Find P(finds exactly 6 faulty products in one shift)
    (b) Find P(finds no faulty products in one shift)
    (c) Over a 6-shift week, find the expected number of faulty products found
    (d) In what percentage of shifts would you expect to find more than 8 faulty products?
    (e) If the inspector finds 11 faulty products in one shift, comment on whether this is unusual

19. A tech company claims their app crashes for only 8% of users. A test involves 29 users.
    (a) If the claim is true, find P(app crashes for exactly 3 users)
    (b) Find P(app crashes for at most 4 users)
    (c) Calculate the expected number of users experiencing crashes
    (d) If the app crashes for 7 users, test whether this supports the company's claim
    (e) What's the maximum number of crashes that would support the 8% claim at 5% significance?

20. A poll shows 63% of voters favor a candidate. A random sample of 36 voters is surveyed.
    (a) Find P(exactly 22 voters favor the candidate)
    (b) Find P(fewer than 18 voters favor the candidate)
    (c) Calculate the expected number of supporters
    (d) Find P(between 55% and 75% of the sample favor the candidate)
    (e) If 27 voters in the sample favor the candidate, is this significantly different from expected?

Section F: Combined Probability Scenarios

21. An electronics store has two suppliers. Supplier A provides 65% of devices with 7% defect rate. Supplier B provides 35% of devices with 18% defect rate.
    (a) A customer buys 28 devices. Find P(exactly 3 are defective)
    (b) If a customer returns a defective device, find P(it came from Supplier B)
    (c) A shipment of 420 devices arrives. Find the expected number from each supplier
    (d) Calculate the overall defect rate
    (e) If the store wants to reduce defects to 6%, what should Supplier B's rate be?

22. A lottery game involves selecting 8 cards from a deck of 36 cards without replacement. The player wins if all 8 cards are hearts (9 hearts in the deck).
    (a) Calculate P(all 8 cards are hearts)
    (b) Calculate P(all 8 cards are the same suit) if there are 9 cards of each suit
    (c) If 8000 people play this game, how many would you expect to win?
    (d) What should be the payout ratio for this to be a fair game?
    (e) How does the probability change if cards are replaced after each draw?

23. A satellite communication system uses 12 independent transponders. Each transponder has probability 0.89 of successful operation.
    (a) Find P(all transponders operate successfully)
    (b) Find P(exactly one transponder fails)

(c) The system functions if at least 10 transponders work. Find P(system functions)

(d) If the system handles 150 transmissions, find P(fewer than 135 are successful)

(e) What should be the individual transponder reliability for 99.99% system reliability?

24. A medical clinic treats patients with a condition that occurs in 26% of cases. During one day, 45 patients are treated.

(a) Model the number of patients with the condition and state assumptions

(b) Find P(exactly 11 patients have the condition)

(c) Find P(no patients have the condition)

(d) Find P(more than 15 patients have the condition)

(e) Calculate the expected number of patients with the condition

(f) If 19 patients have the condition in one day, is this unusually high?

Section G: Advanced Problem Solving

25. A rare genetic condition affects 1 in 650 births. A prenatal test is 87% accurate for positive cases and 99.4% accurate for negative cases.

(a) Calculate the probability of testing positive

(b) If a test is positive, what's the probability the baby has the condition?

(c) How many false positives occur per 65,000 births?

(d) Design a two-stage testing procedure to reduce false positives

(e) Comment on the ethical implications of these probabilities

26. A cybersecurity firm develops malware detection software. The software correctly identifies 74% of malware but also flags 11% of safe files as potentially malicious. 42% of scanned files contain malware.

(a) If a file is flagged, find P(it actually contains malware)

(b) If a file passes scanning, find P(it's actually safe)

(c) In scanning 800 files, how many false alarms would you expect?

(d) Suggest improvements to the detection algorithm

(e) Calculate the overall accuracy of the detection system

27. A raffle has the following structure: pick 5 numbers from 1-35. You win if all 5 match.

(a) Calculate P(winning the raffle)

(b) Find P(matching exactly 4 numbers)

(c) Find P(matching exactly 3 numbers)

(d) If 18 million tickets are sold, find P(no one wins)

(e) Model the number of winners as a binomial distribution

28. An airport security system screens passengers. It correctly identifies 79% of security threats and incorrectly flags 6% of innocent passengers. On average, 0.3% of passengers pose security threats.

(a) Find the probability of triggering an alarm

(b) If an alarm triggers, find P(it's a real threat)

(c) In screening 2,200,000 passengers, how many false alarms occur?

(d) Design a cost-benefit analysis for this system

(e) How would increasing the detection rate to 84% affect false alarms?

29. Design and analyze a probability model for evaluating online review authenticity:

    (a) Define the scenario and identify variables for detecting fake reviews

    (b) State all assumptions about review patterns and detection methods

    (c) Choose appropriate probability distributions for modeling

    (d) Calculate probabilities for different detection scenarios

    (e) Discuss limitations and potential improvements to your model

    (f) Consider practical applications for e-commerce platforms

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

For more resources and practice materials, visit:
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