

GCSE Higher Mathematics

Practice Test 9: Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

(a) $11(x - 9) = 7x + 19$

(b) $\frac{x+17}{8} - \frac{x-15}{6} = 7$

(c) $\frac{10x-9}{20} = \frac{7x+8}{15} - 5$

(d) $1.2x + 2.4 = 1.0x - 5.6$

2. Solve these simultaneous equations by elimination:

(a)
$$\begin{cases} 10x + 9y = 61 \\ 8x - 7y = 18 \end{cases}$$

(b)
$$\begin{cases} 7x + 15y = 55 \\ 11x - 8y = 35 \end{cases}$$

(c)
$$\begin{cases} 10x + 7y = 44 \\ 8x - 11y = 29 \end{cases}$$

(d)
$$\begin{cases} 12x + 11y = 82 \\ 7x - 9y = 19 \end{cases}$$

3. Solve these simultaneous equations by substitution:

(a)
$$\begin{cases} y = 10x - 17 \\ 9x + 7y = 65 \end{cases}$$

(b)
$$\begin{cases} x = 9y - 14 \\ 10x - 8y = 78 \end{cases}$$

(c)
$$\begin{cases} y = 21 - 9x \\ 8x + 11y = 104 \end{cases}$$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

(a) $y = 10x - 8$ and $y = 22 - 7x$

(b) $y = x^2 - 17$ and $y = 8x + 7$

(c) $x^2 + y^2 = 50$ and $y = 8x - 6$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

- (a) $x^2 + 23x + 60$
- (b) $x^2 - 21x - 46$
- (c) $x^2 - 25x + 126$
- (d) $x^2 + 10x - 39$
- (e) $x^2 - 400$
- (f) $x^2 - 40x + 400$

6. Solve these quadratic equations by factorizing:

- (a) $x^2 + 23x + 112 = 0$
- (b) $x^2 - 22x - 23 = 0$
- (c) $x^2 - 19x = 0$
- (d) $x^2 - 441 = 0$
- (e) $x^2 + 42x + 441 = 0$
- (f) $10x^2 - 40x = 0$

7. Factorize these harder quadratics:

- (a) $8x^2 + 17x + 9$
- (b) $10x^2 - 27x + 9$
- (c) $121x^2 - 144$
- (d) $9x^2 + 20x - 44$
- (e) $121x^2 - 220x + 100$
- (f) $13x^2 - 33x - 18$

8. Solve by factorizing:

- (a) $10x^2 + 21x - 11 = 0$
- (b) $11x^2 - 23x + 10 = 0$
- (c) $100x^2 - 81 = 0$
- (d) $8x^2 + 19x - 15 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

- (a) $x^2 + 22x + 21$
- (b) $x^2 - 26x + 13$
- (c) $x^2 + 18x - 15$
- (d) $x^2 - 26x + 23$
- (e) $10x^2 + 40x + 19$
- (f) $6x^2 - 48x + 23$

10. Solve by completing the square:

- (a) $x^2 + 22x + 19 = 0$
- (b) $x^2 - 20x - 13 = 0$

(c) $x^2 + 16x - 11 = 0$

(d) $10x^2 + 20x - 13 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 19x - 9 = 0$

(b) $10x^2 - 21x + 9 = 0$

(c) $x^2 - 22x + 17 = 0$

(d) $9x^2 + 17x - 10 = 0$

(e) $8x^2 + 17x + 7 = 0$

(f) $12x^2 - 19x - 7 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 21x + 54 = 0$

(b) $x^2 - 20x + 100 = 0$

(c) $x^2 + 10x + 27 = 0$

(d) $9x^2 - 19x + 10 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 20x + 75$:

- (a) Find the y-intercept
- (b) Find the x-intercepts by factorizing
- (c) Complete the square to find the vertex
- (d) Sketch the graph
- (e) State the line of symmetry

14. For the quadratic $y = 9x^2 + 27x - 13$:

- (a) Complete the square
- (b) Find the coordinates of the vertex
- (c) Find the y-intercept
- (d) State the line of symmetry
- (e) Sketch the graph

15. A discus is thrown upward. Its height h (in meters) after t seconds is given by: $h = -5t^2 + 80t + 11$

- (a) What is the initial height?
- (b) At what times is the discus at ground level?
- (c) What is the maximum height reached?
- (d) At what time does it reach maximum height?

16. The revenue R (in thousands of pounds) from manufacturing x thousand gadgets is: $R = -7x^2 + 35x - 32$

- (a) How many gadgets should be manufactured to maximize revenue?
- (b) What is the maximum revenue?
- (c) At what production levels is the revenue £6,000?

Section E: Linear Inequalities

17. Solve these linear inequalities:

- (a) $11x + 21 > 65$
- (b) $10x - 23 \leq 47$
- (c) $14 - 11x < 3$
- (d) $\frac{x+15}{8} \geq 1$
- (e) $12 - 11x > 10x + 33$
- (f) $\frac{10x-9}{11} < \frac{x+12}{8}$

18. Show these inequalities on number lines:

- (a) $x > -10$
- (b) $x \leq 12$
- (c) $-7 < x \leq 13$
- (d) $x < 9$ or $x > 14$

19. Solve these compound inequalities:

- (a) $-12 < 10x + 8 < 28$
- (b) $19 \leq 9x - 8 \leq 46$
- (c) $-7 \leq \frac{10x+8}{8} < 14$

20. Find the integer solutions to:

- (a) $10x + 17 > 37$ and $x < 13$
- (b) $-8 \leq x + 6 < 12$
- (c) $x^2 < 144$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

- (a) $x^2 - 21x + 54 > 0$
- (b) $x^2 - 22x + 57 \leq 0$
- (c) $x^2 - 289 < 0$
- (d) $x^2 + 10x - 24 \geq 0$
- (e) $x^2 - 22x + 121 > 0$
- (f) $8x^2 - 9x - 14 < 0$

22. Solve and show on number lines:

- (a) $x^2 - 324 \leq 0$
- (b) $x^2 + 9x - 22 > 0$
- (c) $7x^2 - 15x + 8 \geq 0$

23. Find the values of x for which:

- (a) $x^2 < 11x + 12$
- (b) $8x^2 + 17x \geq 21$
- (c) $x^2 + 20x + 100 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

(a) $(x + 13)(10x - 8)$

(b) $(9x - 10)(x + 14)$

(c) $(8x + 13)^2$

(d) $(12x - 11)^2$

(e) $(x + 21)(x - 21)$

(f) $(11x + 10)(11x - 10)$

25. Expand these expressions:

(a) $(x + 10)(x^2 - 9x + 11)$

(b) $(8x - 9)(x^2 + 8x - 7)$

(c) $(x + 9)^3$

(d) $(9x - 7)^3$

26. Factorize completely:

(a) $22x^2 + 33x$

(b) $100x^2 - 361$

(c) $x^3 - 81x$

(d) $10x^3 + 40x^2 + 40x$

(e) $x^3 - 1331$

(f) $1000x^3 + 729$

27. Simplify these algebraic fractions:

(a) $\frac{x^2-100}{x+10}$

(b) $\frac{x^2+21x+110}{x+11}$

(c) $\frac{11x^2-44}{x^2-4}$

(d) $\frac{x^3-1331}{x^2-121}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

(a) $\frac{11}{x} + \frac{8}{x}$

(b) $\frac{15}{x} - \frac{10}{x}$

(c) $\frac{8}{11x} + \frac{1}{22x}$

(d) $\frac{10}{x+8} + \frac{7}{x-9}$

(e) $\frac{x}{x+10} - \frac{9}{x-7}$

(f) $\frac{10x}{x^2-81} + \frac{9}{x+9}$

29. Multiply and divide these algebraic fractions:

(a) $\frac{x}{11} \times \frac{33}{x^2}$

(b) $\frac{x+10}{12} \times \frac{24}{x+10}$

(c) $\frac{x^2-100}{x+9} \div \frac{x-10}{x+9}$

(d) $\frac{10x+30}{x^2-121} \times \frac{x-11}{20}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{11} + \frac{x}{9} = 40$

(b) $\frac{10x+9}{12} = \frac{x-8}{9}$

(c) $\frac{11}{x} = \frac{10}{x-9}$

(d) $\frac{x+9}{x-10} = \frac{8x}{x+9}$

31. Make the subject of these formulae:

(a) $R = \frac{V}{I}$, make V the subject

(b) $A = \frac{\pi d^2}{4}$, make d the subject

(c) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, make f the subject

(d) $E = \frac{1}{2}mv^2 + mgh$, make m the subject

Section I: Sequences

32. Find the n th term for these arithmetic sequences:

(a) 23, 29, 35, 41, ...

(b) 13, 20, 27, 34, ...

(c) 65, 61, 57, 53, ...

(d) $\frac{4}{9}, \frac{6}{9}, \frac{8}{9}, \frac{10}{9}, \dots$

33. For these geometric sequences, find the n th term:

(a) 10, 30, 90, 270, ...

(b) 13, 52, 208, 832, ...

(c) 1024, 256, 64, 16, ...

(d) 9, -36, 144, -576, ...

34. Find the sum of these series:

(a) First 60 terms of $14 + 18 + 22 + 26 + \dots$

(b) First 1 term of $12 + 36 + 108 + 324 + \dots$

(c) $9 + 13 + 17 + \dots + 105$ (arithmetic series)

(d) $11 + 22 + 44 + \dots + 704$ (geometric series)

35. These are quadratic sequences. Find the n th term:

(a) 10, 40, 90, 160, 250, ...

(b) 9, 23, 43, 69, 101, ...

(c) 8, 32, 72, 128, 200, ...

(d) 12, 35, 64, 99, 140, ...

36. A sequence is defined by $u_1 = 10$ and $u_{n+1} = 8u_n - 9$.

(a) Find the first 5 terms

(b) Find a formula for u_n

(c) Calculate u_1

Section J: Problem Solving

37. The sum of two numbers is 46 and their product is 525. Find the two numbers.
38. A rectangular arena has perimeter 104m. If the length is 22m more than the width, find the dimensions.
39. The difference between a positive number and its reciprocal is $\frac{120}{11}$. Find the number.
40. A javelin's height h (in meters) after t seconds is given by: $h = 220t - 5t^2$
- (a) When does it hit the ground?
 - (b) What is its maximum height?
 - (c) When is it 2275m high?
41. Prove that the sum of the first n terms of the sequence 7, 14, 21, 28, ... is $\frac{7n(n+1)}{2}$.
42. The quadratic $ax^2 + bx + c = 0$ has roots ϕ and ψ .
- (a) Show that $\phi + \psi = -\frac{b}{a}$
 - (b) Show that $\phi\psi = \frac{c}{a}$
 - (c) If the roots are 11 and -8, find a , b , and c when $a = 8$
43. A function is defined as $q(x) = x^2 + rx + s$. If $q(7) = 42$ and $q(9) = 68$, find r and s .
44. The sum of the first n terms of a sequence is $S_n = 10n^2 - 7n$. Find the n th term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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