A Level Pure Mathematics
Practice Test 2: Trigonometry

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Radian Measure and Basic Functions

1. Convert these angles from degrees to radians (leave answers in terms of π):
   (a) $36°$
   (b) $72°$
   (c) $120°$
   (d) $150°$
   (e) $210°$
   (f) $300°$

2. Convert these angles from radians to degrees:
   (a) $\frac{\pi}{10}$
   (b) $\frac{\pi}{5}$
   (c) $\frac{3\pi}{4}$
   (d) $\frac{4\pi}{3}$
   (e) $\frac{5\pi}{3}$
   (f) $\frac{11\pi}{6}$

3. Find the exact values of these trigonometric ratios (without calculator):
   (a) $\sin 0$, $\cos 0$, $\tan 0$
   (b) $\sin \frac{\pi}{2}$, $\cos \frac{\pi}{2}$, $\tan \frac{\pi}{2}$
   (c) $\sin \pi$, $\cos \pi$, $\tan \pi$
   (d) $\sin \frac{3\pi}{2}$, $\cos \frac{3\pi}{2}$, $\tan \frac{3\pi}{2}$

4. A circle has radius 12 cm. Find:
   (a) The arc length subtended by an angle of $\frac{3\pi}{4}$ radians
   (b) The area of the sector with angle $\frac{2\pi}{3}$ radians
   (c) The angle (in radians) that subtends an arc of length 18 cm
   (d) The radius of a circle where an angle of $\frac{5\pi}{6}$ radians subtends an arc of length 20 cm

5. Find the exact values:

(a) $\sin \frac{5\pi}{6}$
(b) $\cos \frac{2\pi}{3}$
(c) $\tan \frac{3\pi}{4}$
(d) $\sin \frac{11\pi}{6}$
(e) $\cos \frac{4\pi}{3}$
(f) $\tan \frac{7\pi}{6}$

Section B: Graphs of Trigonometric Functions

6. For the function $f(x) = 2\sin x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$
7. For the function $g(x) = \cos 3x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [0, 2\pi]$
8. For the function $h(x) = \tan 2x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the asymptotes in the interval $[0, \pi]$
   (d) Find the zeros in the interval $[0, \pi]$
   (e) Sketch the graph for $x \in [0, \pi]$
9. Sketch the graphs of these transformed functions for $x \in [0, 4\pi]$:
   (a) $y = 3\cos x$
   (b) $y = \sin 3x$
   (c) $y = \cos(x - \frac{\pi}{6})$
   (d) $y = \cos x - 2$
   (e) $y = -\sin x$
   (f) $y = \sin(x + \frac{\pi}{2})$
10. For the function $y = 2\cos(3x + \frac{\pi}{4}) - 1$:
   (a) Identify the amplitude
   (b) Find the period
   (c) Determine the phase shift
   (d) Find the vertical shift
   (e) State the range
   (f) Sketch the graph for $x \in [0, 2\pi]$

Section C: Fundamental Trigonometric Identities

11. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to find:

    (a) $\sin\theta$ if $\cos\theta = \frac{4}{5}$ and θ is acute
    (b) $\cos\theta$ if $\sin\theta = -\frac{5}{13}$ and θ is in the third quadrant
    (c) $\tan\theta$ if $\cos\theta = \frac{7}{25}$ and $\sin\theta > 0$
    (d) $\sin\theta$ if $\tan\theta = -\frac{15}{8}$ and $\cos\theta > 0$

12. Prove these identities:

    (a) $\sec^2\theta - \tan^2\theta = 1$
    (b) $\csc^2\theta - \cot^2\theta = 1$
    (c) $\frac{\cos\theta}{\sin\theta} = \cot\theta$
    (d) $\frac{1}{\cos\theta} = \sec\theta$

13. Simplify these expressions:

    (a) $\tan^2\theta - \sin^2\theta\tan^2\theta$
    (b) $\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$
    (c) $(\sec\theta + \tan\theta)^2$
    (d) $\frac{1-\cos^2\theta}{\sin\theta}$

14. Express in terms of $\cos\theta$ only:

    (a) $\sin^2\theta$
    (b) $\tan^2\theta$
    (c) $\csc^2\theta$
    (d) $\sin^2\theta + \cos^2\theta\cot^2\theta$

15. Prove that:

    (a) $\frac{1-\sin^2\theta}{\cos\theta} = \cos\theta$
    (b) $\sin^4\theta - \cos^4\theta = -\cos 2\theta$
    (c) $\frac{\sec\theta+\csc\theta}{\sec\theta-\csc\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$
    (d) $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1$

Section D: Addition and Double Angle Formulas

16. Use the addition formulas to find the exact values:

    (a) $\cos 75°$ (using $\cos(45° + 30°)$)
    (b) $\sin 15°$ (using $\sin(45° - 30°)$)
    (c) $\tan 15°$ (using $\tan(45° - 30°)$)
    (d) $\cos\frac{7\pi}{12}$ (using $\cos(\frac{\pi}{3} + \frac{\pi}{4})$)

17. Given $\cos A = \frac{4}{5}$ with A acute and $\sin B = \frac{12}{13}$ with B acute:

    (a) Find $\sin A$ and $\cos B$
    (b) Calculate $\cos(A + B)$
    (c) Calculate $\sin(A + B)$
    (d) Find $\tan(A - B)$

18. Use double angle formulas to find:
    (a) $\cos 2\theta$ if $\cos\theta = \frac{3}{5}$ and θ is acute
    (b) $\sin 2\theta$ if $\sin\theta = \frac{8}{17}$ and θ is acute
    (c) $\tan 2\theta$ if $\tan\theta = \frac{3}{4}$
    (d) $\sin 2\theta$ if $\cos\theta = -\frac{5}{13}$ and θ is in the second quadrant
19. Prove these half angle identities:
    (a) $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$
    (b) $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$
    (c) $\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$
    (d) $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$
20. Express in terms of half angles:
    (a) $1+\cos\theta$ in terms of $\cos\frac{\theta}{2}$
    (b) $1-\cos\theta$ in terms of $\sin\frac{\theta}{2}$
    (c) $\sin^2\theta$ in terms of $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$
    (d) $\cos^2\theta$ in terms of $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$

Section E: Solving Trigonometric Equations

21. Solve these equations for $0 \le x \le 2\pi$:
    (a) $\cos x = \frac{\sqrt{3}}{2}$
    (b) $\sin x = -\frac{1}{2}$
    (c) $\tan x = -1$
    (d) $\cos x = -\frac{\sqrt{2}}{2}$
22. Solve these equations for $0° \le x \le 360°$:
    (a) $2\cos x + 1 = 0$
    (b) $3\sin x - 2 = 0$
    (c) $\tan x - \sqrt{3} = 0$
    (d) $2\cos^2 x = 1$
23. Solve these equations for $0 \le x \le 2\pi$:
    (a) $\cos 2x = \frac{1}{2}$
    (b) $\sin 3x = -\frac{\sqrt{3}}{2}$
    (c) $\tan 2x = \sqrt{3}$
    (d) $\cos(x - \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$
24. Solve these quadratic trigonometric equations for $0 \le x \le 2\pi$:
    (a) $2\cos^2 x + \cos x - 1 = 0$
    (b) $\sin^2 x - 2\sin x + 1 = 0$
    (c) $3\tan^2 x - \tan x = 0$
    (d) $2\cos^2 x + 3\cos x + 1 = 0$

25. Solve these equations involving multiple angles for $0 \leq x \leq 2\pi$:
    (a) $\cos x = \sin x$
    (b) $\cos 2x = \sin x$
    (c) $\sin 2x = 2\sin x \cos x$
    (d) $\cos 3x = \cos x$

Section F: Advanced Trigonometric Identities

26. Prove these factor formulas:
    (a) $\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$
    (b) $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
    (c) $\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$
    (d) $\cos B - \cos A = 2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$
27. Use factor formulas to simplify:
    (a) $\sin 6x + \sin 2x$
    (b) $\cos 8x - \cos 4x$
    (c) $\sin 80° + \sin 20°$
    (d) $\cos 100° + \cos 40°$
28. Prove these product-to-sum identities:
    (a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
    (b) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
    (c) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$
    (d) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
29. Express as sums or differences:
    (a) $\sin 5x\cos 3x$
    (b) $\cos 7x\cos 2x$
    (c) $2\sin 4x\sin x$
    (d) $\cos 8x\sin 3x$
30. Derive the half-angle formulas:
    (a) $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
    (b) $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
    (c) $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

Section G: Complex Trigonometric Problems

31. Solve these equations for $0 \leq x < 2\pi$:
    (a) $\cos x - \sin x = 1$
    (b) $\cos x + \cos 2x = 0$
    (c) $\sin x + \sin 2x + \sin 3x = 0$

(d) $\cot x + \cot 2x = 0$

32. Prove these complex identities:

(a) $\frac{\cos 3\theta}{\cos\theta} - \frac{\sin 3\theta}{\sin\theta} = 2$
(b) $\cos^2\theta + \cos^2(\theta + \frac{2\pi}{3}) + \cos^2(\theta + \frac{4\pi}{3}) = \frac{3}{2}$
(c) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$
(d) $\cot A + \cot B + \cot C = \cot A \cot B \cot C$ when $A + B + C = \pi$

33. Find the general solution to these equations:

(a) $\cos x = \frac{2}{3}$
(b) $\sin 3x = 0.4$
(c) $\tan 2x = -3$
(d) $\cos(3x - \frac{\pi}{4}) = \frac{1}{2}$

34. Express these in the form $R\sin(x + \alpha)$ or $R\cos(x + \alpha)$:

(a) $5\sin x + 12\cos x$
(b) $3\sin x - 4\cos x$
(c) $\cos x + \sqrt{3}\sin x$
(d) $3\cos x - 3\sqrt{3}\sin x$

35. Find the range of these functions:

(a) $f(x) = 5\sin x + 12\cos x$
(b) $g(x) = 3\sin 2x - 4\cos 2x + 5$
(c) $h(x) = \cos^2 x + 3\sin x$
(d) $k(x) = 4\sin x\cos x - 2$

Section H: Applications of Trigonometry

36. A particle moves in simple harmonic motion with displacement $s = 6\cos(2t - \frac{\pi}{3})$ meters, where t is time in seconds.

(a) Find the amplitude of the motion
(b) Determine the period of oscillation
(c) Find the phase shift
(d) Calculate the displacement when $t = 0$
(e) Find when the particle first reaches maximum displacement

37. The temperature in a city is modeled by $T(t) = 4\sin(\frac{\pi t}{12}) + 18$ degrees Celsius, where t is hours after midnight.

(a) Find the maximum and minimum temperatures
(b) Determine the period of the temperature cycle
(c) Find the temperature at 9 AM
(d) Calculate when the maximum temperature occurs
(e) Find when the temperature is exactly 20 degrees

38. An alternating voltage is given by $V = 240\cos(50\pi t - \frac{\pi}{6})$ volts.

(a) Find the maximum voltage
(b) Determine the frequency (cycles per second)
(c) Calculate the voltage when $t = 0.01$ seconds
(d) Find when the voltage first equals 120 volts
(e) Determine the period of the voltage

39. A pendulum has angular displacement $\theta = 0.2\sin(4t + \frac{\pi}{4})$ radians, where t is time in seconds.
    (a) Find the maximum angular displacement
    (b) Determine the period of oscillation
    (c) Calculate the angular displacement at $t = 0$
    (d) Find when the pendulum first passes through equilibrium
    (e) Determine the frequency of oscillation

40. Two sound waves with equations $y_1 = 2\sin 3x$ and $y_2 = 3\cos 3x$ interfere.
    (a) Find the equation of the resultant wave
    (b) Express the result in the form $R\cos(3x - \alpha)$
    (c) Determine the amplitude of the resultant wave
    (d) Find the phase difference between the original waves
    (e) Calculate the points where the waves interfere destructively

Section I: Advanced Problem Solving

41. In triangle PQR, $p = 9$, $q = 12$, and $\angle R = 45°$.
    (a) Use the cosine rule to find side r
    (b) Use the sine rule to find $\angle P$
    (c) Calculate the area of the triangle
    (d) Find the radius of the incircle
    (e) Determine the length of the altitude from Q to side PR

42. Prove that in any triangle PQR:
    (a) $\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R} = 2R$ (sine rule)
    (b) $p^2 = q^2 + r^2 - 2qr\cos P$ (cosine rule)
    (c) $\cos P + \cos Q + \cos R = 1 + \frac{r}{2R}$ where r is the inradius
    (d) Area $= \frac{1}{2}qr\sin P = \sqrt{s(s-p)(s-q)(s-r)}$ where $s = \frac{p+q+r}{2}$

43. A regular hexagon is inscribed in a circle of radius r.
    (a) Find the central angle for each sector
    (b) Calculate the side length of the hexagon
    (c) Find the area of the hexagon
    (d) Determine the apothem (distance from center to side)
    (e) Calculate the ratio of the hexagon's area to the circle's area

44. The function $g(x) = p\cos x + q\sin x$ has maximum value 10 and minimum value -10.
    (a) Express $g(x)$ in the form $R\cos(x - \beta)$

(b) Find the relationship between p and q

(c) If $g(\frac{\pi}{4}) = 6$, find the values of p and q

(d) Solve $g(x) = 8$ for $0 \leq x \leq 2\pi$

(e) Find the values of x where $g(x)$ achieves its minimum

45. Consider the identity $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.

(a) Verify this identity for $\theta = \frac{\pi}{3}$

(b) Use this to solve $\cos 5\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$

(c) Find the exact values of $\cos\frac{\pi}{5}$ and $\cos\frac{2\pi}{5}$

(d) Express $\sin 5\theta$ in terms of powers of $\sin\theta$

(e) Use these results to find the area of a regular pentagon

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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