

GCSE Higher Mathematics

Practice Test 9: Ratio, Proportion, and Rates of Change

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Advanced Ratios

- Express these ratios in their simplest form:
 - $9.6 : 6.4 : 3.2$
 - $\frac{4}{15} : \frac{8}{45} : \frac{2}{9}$
 - $0.72 : 0.48 : 0.24$
 - $5\frac{1}{4} : 2\frac{5}{8} : 3\frac{1}{2}$
- A renewable energy project distributes funding among solar, wind, hydro, and storage systems in the ratio $6 : 9 : 4 : 5$. If the total funding is £144,000:
 - How much funding does each system receive?
 - If additional grants of £36,000 are distributed in the same ratio, how much extra does each system get?
 - What percentage of the total funding goes to wind systems?
- The angles of a regular star polygon alternate between two values in the ratio $5 : 7$. If there are 10 angles total and they sum to 1440° , find each angle.
- A perfume blend combines lavender, rose, and jasmine oils in the ratio $5 : 8 : 2$ by volume.
 - How much of each oil is needed for 225ml of perfume?
 - If 96ml of rose oil is used, find the volume of lavender and jasmine oils needed
 - What fraction of the perfume is jasmine oil?
- Split £3150 among three departments so that Marketing gets £350 more than Sales, and Sales gets £200 more than Operations.

Section B: Direct Proportion

- y is directly proportional to x . When $x = 49$, $y = 42$.
 - Find the equation connecting y and x
 - Calculate y when $x = 63$
 - Calculate x when $y = 54$

- (d) Sketch the graph of y against x
7. R varies directly as the square of S . When $S = 11$, $R = 363$.
- (a) Express R in terms of S
 - (b) Find R when $S = 15$
 - (c) Find S when $R = 675$
 - (d) What happens to R when S is decreased by 40%?
8. The cost of event photography varies directly as the number of hours. 7 hours of photography costs £525.
- (a) Find the cost per hour
 - (b) How much does 11 hours of photography cost?
 - (c) How many hours can be booked for £900?
 - (d) Express the cost C in terms of hours H
9. The luminosity of a star varies directly as the fourth power of its surface temperature. A star at 6000K has luminosity 16 units.
- (a) Find the luminosity of a star at 7500K
 - (b) What temperature gives luminosity 81 units?
 - (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x . When $x = 21$, $y = 28$.
- (a) Find the equation connecting y and x
 - (b) Calculate y when $x = 35$
 - (c) Calculate x when $y = 42$
 - (d) Sketch the graph of y against x
11. The time to encrypt data is inversely proportional to the processor clock speed. At 3.2 GHz, encryption takes 25 minutes.
- (a) How long at 4.0 GHz?
 - (b) What clock speed is needed to encrypt in 16 minutes?
 - (c) Express time T in terms of clock speed C
 - (d) If the processor throttles from 3.6 GHz to 2.4 GHz, how much longer does encryption take?
12. X is inversely proportional to the square of Y . When $Y = 12$, $X = 25$.
- (a) Express X in terms of Y
 - (b) Find X when $Y = 20$
 - (c) Find Y when $X = 16$
 - (d) What happens to X when Y is increased by 20%?
13. The radiation intensity from a source is inversely proportional to the square of the distance from it. At 1.5 meters, the intensity is 400 units.
- (a) Find the intensity at 2.5 meters
 - (b) At what distance is the intensity 225 units?
 - (c) Write the relationship as an equation

Section D: Combined Proportion

14. t varies directly as u and inversely as v . When $u = 45$ and $v = 30$, $t = 54$.
- (a) Express t in terms of u and v
 - (b) Find t when $u = 60$ and $v = 24$
 - (c) Find v when $u = 75$ and $t = 45$
 - (d) What happens to t if u is decreased by 40% and v is increased by 25%?
15. The resonant frequency f of a pendulum varies directly as the square root of gravitational acceleration g and inversely as the square root of its length L . When $g = 9.8 \text{ m/s}^2$ and $L = 0.98 \text{ m}$, $f = 1 \text{ Hz}$.
- (a) Express f in terms of g and L
 - (b) Find f when $g = 3.2 \text{ m/s}^2$ and $L = 0.8 \text{ m}$
 - (c) Find L when $f = 1.5 \text{ Hz}$ and $g = 9.8 \text{ m/s}^2$
 - (d) What happens to f if g is halved and L is quadrupled?
16. The drag force D on a vehicle varies directly as the air density ρ , the square of velocity v , and the frontal area A . When $\rho = 1.2 \text{ kg/m}^3$, $v = 20 \text{ m/s}$, and $A = 2.5 \text{ m}^2$, $D = 600 \text{ newtons}$.
- (a) Express D in terms of ρ , v , and A
 - (b) Find D when $\rho = 1.0 \text{ kg/m}^3$, $v = 30 \text{ m/s}$, and $A = 3.0 \text{ m}^2$
 - (c) Find A when $D = 800 \text{ newtons}$, $\rho = 1.1 \text{ kg/m}^3$, and $v = 25 \text{ m/s}$
17. m varies directly as n^2 and inversely as p^3 . When $n = 6$ and $p = 2$, $m = 54$.
- (a) Find the constant of proportionality
 - (b) Express m in terms of n and p
 - (c) Calculate m when $n = 8$ and $p = 4$

Section E: Speed, Distance, and Time

18. A space probe travels 456 km in 2 hours 16 minutes. Calculate:
- (a) The average speed in km/h
 - (b) The average speed in m/s
 - (c) How far it travels in 1 hour 45 minutes at this speed
 - (d) How long it takes to travel 342 km at this speed
19. Convert these speeds:
- (a) 39 m/s to km/h
 - (b) 153 km/h to m/s
 - (c) 47 mph to km/h (use 1 mile = 1.6 km)
 - (d) 128 km/h to mph
20. A racing car drives from pit to checkpoint at 180 km/h and returns at 120 km/h. The total drive time is 1.25 hours.
- (a) Find the distance from pit to checkpoint
 - (b) Calculate the average speed for the whole drive
 - (c) How much time was lost due to the slower return speed?

21. Two rockets launch simultaneously from bases 900 km apart and fly towards each other. One travels at 450 km/h and the other at 550 km/h.
- (a) When do they meet?
 - (b) How far from each starting base do they meet?
 - (c) What is their relative speed of approach?
22. A mountain biker covers a 24 km course. For the first 15 km uphill, they cycle at 10 km/h. For the remaining 9 km downhill, they increase to 18 km/h.
- (a) Calculate the total time taken
 - (b) Find the average speed for the whole course
 - (c) How much slower would they need to cycle downhill to achieve an overall average of 11 km/h?

Section F: Density and Flow Rates

23. A crystalline sample has volume 480 cm^3 and mass 1.44 kg.
- (a) Calculate its density in g/cm^3
 - (b) Calculate its density in kg/m^3
 - (c) What mass of this crystal would have volume 640 cm^3 ?
 - (d) What volume would 2.16 kg of this crystal occupy?
24. Different woods have these densities:
- Maple: 0.63 g/cm^3
 - Cherry: 0.58 g/cm^3
 - Walnut: 0.65 g/cm^3
- (a) Compare the masses of 300 cm^3 of each wood
 - (b) What volume of walnut has the same mass as 260 cm^3 of cherry?
 - (c) A sculpture weighs 945 g and has volume 1500 cm^3 . Which wood is it most likely made from?
25. Molten metal flows through a casting system at a rate of 9.6 liters per minute.
- (a) How much metal flows in 22 minutes?
 - (b) How long to cast 432 liters of metal?
 - (c) Express the flow rate in m^3 per hour
 - (d) If the mold design reduces flow area by 20%, what happens to the flow velocity?
26. A pharmaceutical packaging line operates at different rates during the day:
- Setup phase (1.5 hours): 240 bottles/hour
 - Main production (7 hours): 600 bottles/hour
 - Quality control (1.5 hours): 360 bottles/hour
- (a) Calculate total daily production
 - (b) Find the average production rate
 - (c) How long to package 4500 bottles at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar stars (5-pointed) have corresponding diagonal lengths in the ratio 7:16.
- (a) If the smaller star has area 98 cm^2 , find the area of the larger star
 - (b) If the larger star has perimeter 320 cm, find the perimeter of the smaller star
 - (c) Find the ratio of their areas
28. A scale model of a wind turbine is built to scale 1:180. The real turbine is 126 m tall with 54 m blades.
- (a) Find the dimensions of the model
 - (b) If the model uses 0.35 m^2 of blade material, how much does the real turbine need?
 - (c) The model weighs 2.8 kg. Estimate the mass of the real turbine if made from the same material
29. A topographic map has scale 1:30000. Two mountain peaks are 21.6 cm apart on the map.
- (a) Calculate the actual distance in km
 - (b) What map distance represents 4.8 km?
 - (c) A valley covers 14.4 cm^2 on the map. Find its actual area in hectares
30. Two similar icosahedra have volume ratio 8:125.
- (a) Find the ratio of their edge lengths
 - (b) Find the ratio of their surface areas
 - (c) If the smaller icosahedron has edge length 4 cm, what is the edge length of the larger one?
 - (d) If the larger icosahedron has surface area 1250 cm^2 , what is the surface area of the smaller one?

Section H: Advanced Rate Problems

31. A cryptocurrency's value increases by 60% every 4 hours during a bull run. Starting at £500:
- (a) Write an expression for value after t hours
 - (b) What is the value after 16 hours?
 - (c) When will the value exceed £8000?
 - (d) What is the rate of increase after 8 hours (£per hour)?
32. A water recycling plant has six input streams and four output channels. The streams provide 18, 24, 30, 15, 27, and 21 liters/minute respectively. The channels remove 22, 28, 19, and 25 liters/minute respectively.
- (a) What is the net rate of filling when all systems operate?
 - (b) How long to fill a 2520-liter treatment tank from empty?
 - (c) If the three largest input streams are blocked, what is the new net rate?
 - (d) What capacity would a fifth output channel need to exactly balance all inputs?
33. Currency exchange rates:
- £1 = \$1.37
 - £1 = €1.23
 - \$1 = ¥121

- (a) Convert £640 to dollars
 - (b) Convert €369 to pounds
 - (c) Convert \$275 to yen
 - (d) Find the exchange rate from euros to dollars
34. A space manufacturing facility operates in cycles:
- Preparation: 3 hours at 30% capacity (full capacity is 80 components/hour)
 - Production: 9 hours at full capacity
 - Inspection: 2 hours at 50% capacity
- (a) Calculate total components produced
 - (b) Find the average production rate over the whole cycle
 - (c) How long would it take to produce the same quantity at 70% capacity?

Section I: Problem Solving and Applications

35. A cookie recipe for 24 cookies uses:
- 360g flour
 - 180g butter
 - 120g sugar
 - 2 eggs
- (a) Adapt the recipe for 36 cookies
 - (b) How much of each ingredient for 18 cookies?
 - (c) If you have 900g flour, what's the maximum number of cookies you can make?
36. A shopping mall has 8100 parking spaces. The ratio of ground level to first floor to roof spaces is 11:7:3. Due to construction, 405 ground level spaces are temporarily removed.
- (a) How many spaces of each type were there originally?
 - (b) What is the new ratio of ground level to first floor to roof spaces?
 - (c) How many first floor spaces would need to be added to restore the original ratio?
37. The signal strength of a radio transmitter varies inversely as the cube of the distance from it. At 2 km, the signal strength is 54 units.
- (a) Find the signal strength at 3 km
 - (b) At what distance is the signal strength 16 units?
 - (c) Compare signal strengths at 1.5 km and 4.5 km
38. A mining company's extraction cost C (in thousands) varies as the square of depth D (in hundreds of meters) up to 500m, then becomes linear. When $D = 2$, $C = 96$. When $D = 8$, $C = 1740$.
- (a) Find the cost function for $D \leq 5$
 - (b) Find the cost function for $D > 5$
 - (c) What depth minimizes cost per meter for $D \leq 5$?
 - (d) What is the linear rate of increase for depths beyond 500m?

39. A robotic assembly system has ten interconnected gears. Gear 1 has 45 teeth and rotates at 150 rpm. The subsequent gears have 30, 60, 20, 75, 15, 50, 25, 80, and 40 teeth respectively, connected in sequence.
- (a) Find the rotation speed of each gear
 - (b) What is the overall gear ratio from gear 1 to gear 10?
 - (c) If gear 1's speed changes to 225 rpm, find the new speed of gear 10
 - (d) How would the final speed change if gear 8 had 30 teeth instead of 25?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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