A Level Mechanics
Practice Test 4: Kinematics

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 45 minutes

Section A: Complex Mathematical Motion [35 marks]

Question 1 [**20 marks**] A particle moves along a straight line such that its velocity at time t seconds is given by $v = t^3 - 6t^2 + 9t$ m/s.

(a) Find the acceleration of the particle at time t. [**3 marks**]

(b) Calculate the times when the particle is at rest. [**4 marks**]

(c) Determine the times when the acceleration is zero. [**3 marks**]

(d) Given that the particle starts at position s = 4 m when t = 0, find the position equation. [**4 marks**]

(e) Calculate the particle's position when it is at rest. [**3 marks**]

(f) Find the total distance traveled between t = 0 and t = 5 seconds. [**3 marks**]

Question 2 [**15 marks**] A particle's displacement from origin O is given by $s = 4\sin(2t) + 3\cos(2t)$ metres, where t is in seconds.

(a) Find expressions for the velocity and acceleration at time t. [**4 marks**]

(b) Calculate the maximum displacement from the origin. [**4 marks**]

(c) Determine the maximum speed of the particle. [**3 marks**]

(d) Find the times when the particle passes through the origin in the first 2 seconds. [**4 marks**]

Section B: Advanced SUVAT and Motion Analysis [30 marks]

Question 3 [**18 marks**] A high-speed train operates between two cities 240 km apart. The journey profile is: - Accelerate from rest at 1.2 m/s² until reaching maximum speed of 90 m/s - Travel at maximum speed - Decelerate at 1.8 m/s² to rest at the destination

(a) Calculate the time taken to reach maximum speed. [**2 marks**]

(b) Find the distance covered during acceleration. [**3 marks**]

(c) Calculate the distance covered during deceleration. **[3 marks]**

(d) Determine the distance traveled at constant speed. **[2 marks]**

(e) Find the time spent at constant speed. **[2 marks]**

(f) Calculate the total journey time. **[2 marks]**

(g) Determine the average speed for the entire journey. **[2 marks]**

(h) Sketch a speed-time graph for the journey. **[2 marks]**

Question 4 [12 marks] Three runners A, B, and C start a race simultaneously. Runner A maintains constant speed 8 m/s. Runner B accelerates from rest at 0.5 m/s² for 20 seconds, then maintains constant speed. Runner C starts 10 m behind the start line and accelerates at 0.8 m/s² throughout.

(a) Write position equations for all three runners. **[6 marks]**

(b) Find when runner C overtakes runner A. **[3 marks]**

(c) Determine the order of runners at t = 25 seconds. **[3 marks]**

Section C: Vertical Motion and Timing Problems [35 marks]

Question 5 [20 marks] A ball is thrown vertically upward from the top of a building 45 m high with initial speed 12 m/s. At the same instant, another ball is dropped from rest from the top of a taller building 80 m high, located 30 m horizontally away.

(a) Write height equations for both balls. **[4 marks]**

(b) Calculate when the first ball reaches its maximum height above ground. **[3 marks]**

(c) Find the maximum height above ground reached by the first ball. **[3 marks]**

(d) Determine when each ball hits the ground. **[6 marks]**

(e) Calculate the velocities of both balls just before they hit the ground. **[4 marks]**

Question 6 [15 marks] From ground level, a ball is thrown vertically upward and returns to the thrower's hand after 4.6 seconds. During its flight, it passes a window 18 m above the ground.

(a) Calculate the initial velocity of the ball. **[3 marks]**

(b) Find the maximum height reached by the ball. **[3 marks]**

(c) Determine the two times when the ball is at the window level. **[6 marks]**

(d) Calculate the ball's velocities when passing the window on both occasions. **[3 marks]**

Section D: Inclined Plane and Friction Analysis [25 marks]

Question 7 [15 marks] A block is projected up a rough inclined plane with initial velocity 14 m/s. The plane makes an angle of 22° with the horizontal, and the coefficient of kinetic friction is 0.15.

(a) Calculate the deceleration of the block as it moves up the plane. **[5 marks]**

(b) Find the distance traveled up the plane before the block comes to rest. **[4 marks]**

(c) Calculate the time taken to reach the highest point. **[3 marks]**

(d) Determine the acceleration as the block slides back down. **[3 marks]**

Question 8 [10 marks] A particle slides down a smooth inclined plane of length 15 m, inclined at 35° to the horizontal. At the bottom, it moves onto a horizontal rough surface where it decelerates at 4.5 m/s².

(a) Calculate the speed at the bottom of the incline. **[4 marks]**

(b) Find how far the particle travels on the horizontal surface before stopping. **[3 marks]**

(c) Calculate the total time from release until the particle stops. **[3 marks]**

Section E: Complex Projectile Motion [40 marks]

Question 9 [25 marks] A catapult launches a projectile from ground level with initial velocity 32 m/s at 42° to the horizontal. The target is a castle wall 2.8 m high located 85 m away horizontally.

(a) Calculate the horizontal and vertical components of initial velocity. **[3 marks]**

(b) Write parametric equations for the projectile's motion. **[3 marks]**

(c) Find the time to reach maximum height and the maximum height achieved. **[4 marks]**

(d) Determine if the projectile clears the castle wall. **[6 marks]**

(e) Calculate the projectile's height when it reaches the wall's horizontal position. **[3 marks]**

(f) Find the range of the projectile if there were no wall. **[3 marks]**

(g) Determine the angle at which the projectile crosses the wall. **[3 marks]**

Question 10 [15 marks] A ball is kicked from the edge of a cliff 25 m high with initial velocity 28 m/s at 35° above the horizontal toward the sea below.

(a) Write equations for the ball's position relative to the cliff base. **[4 marks]**

(b) Calculate when the ball reaches the same height as the cliff base. **[4 marks]**

(c) Find the horizontal distance from the cliff base when this occurs. **[2 marks]**

(d) Determine the ball's velocity components at this moment. **[3 marks]**

(e) Calculate when the ball hits the sea surface. **[2 marks]**

Section F: Multi-Dimensional Relative Motion [30 marks]

Question 11 [18 marks] Two aircraft are flying at the same altitude. Aircraft A flies due north at 120 m/s starting from the origin. Aircraft B flies northeast at 100 m/s starting from position (-2000 m, -1500 m).

(a) Write position vector equations for both aircraft. **[4 marks]**

(b) Calculate the velocity of aircraft A relative to aircraft B. **[4 marks]**

(c) Find when the aircraft are closest together. **[6 marks]**

(d) Calculate the minimum separation distance. **[4 marks]**

Question 12 [12 marks] A helicopter pilot needs to reach a point 150 km due east while there is a wind of 35 km/h from the southwest (45° south of west). The helicopter's airspeed is 180 km/h.

(a) Resolve the wind velocity into components. **[3 marks]**

(b) Calculate the heading the pilot should take to reach the destination. **[5 marks]**

(c) Find the ground speed for this direct flight. **[2 marks]**

(d) Calculate the flight time. **[2 marks]**

Section G: Integrated Motion Problems [20 marks]

Question 13 [20 marks] A soccer ball is kicked from ground level with velocity 26 m/s at 48° to the horizontal. Simultaneously, a player 40 m away starts running toward the ball's landing point with acceleration 1.8 m/s² from rest.

(a) Calculate the range of the soccer ball. **[5 marks]**

(b) Find the time of flight of the ball. **[3 marks]**

(c) Determine how far the player must run to reach the ball's landing point. **[2 marks]**

(d) Calculate if the player reaches the landing point before the ball lands. **[6 marks]**

(e) If the player maintains constant speed after reaching the landing point, what speed is needed to arrive exactly when the ball lands? **[4 marks]**

Physics Data and Formulae

Calculus Relations:

$v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$v = \int a\,dt + C$, $s = \int v\,dt + C$

SUVAT Equations:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{u+v}{2}t$

$s = vt - \frac{1}{2}at^2$

Projectile Motion:

Horizontal: $x = x_0 + u_x t$ (where $u_x = u\cos\theta$)

Vertical: $y = y_0 + u_y t - \frac{1}{2}gt^2$ (where $u_y = u\sin\theta$)

Time of flight: $t = \frac{2u\sin\theta}{g}$ (level ground)

Range: $R = \frac{u^2\sin(2\theta)}{g}$ (level ground)

Maximum height: $h = \frac{u^2\sin^2\theta}{2g}$

Inclined Plane Motion:

Smooth plane down: $a = g\sin\theta$

With friction up: $a = g(\sin\theta + \mu\cos\theta)$

With friction down: $a = g(\sin\theta - \mu\cos\theta)$

Vector Analysis:

Magnitude: $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

Direction: $\tan\alpha = \frac{v_y}{v_x}$

Relative velocity: $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Trigonometric Functions:
For harmonic motion: $\frac{d}{dt}[\sin(at)] = a\cos(at)$
$\frac{d}{dt}[\cos(at)] = -a\sin(at)$

Constants:
Acceleration due to gravity: $g = 9.8$ m/s^2

Trigonometric Values:
$\sin 22 = 0.375$, $\cos 22 = 0.927$
$\sin 35 = 0.574$, $\cos 35 = 0.819$
$\sin 42 = 0.669$, $\cos 42 = 0.743$
$\sin 48 = 0.743$, $\cos 48 = 0.669$
$\sin 45 = 0.707$, $\cos 45 = 0.707$

END OF TEST

Total marks: 215

Grade boundaries: A* 194, A 172, B 151, C 129, D 108, E 86

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