A Level Mechanics
Practice Test 6: Rotational Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Rotational Motion Fundamentals [29 marks]

Question 1 [**11 marks**]

(a) Define angular displacement, angular velocity, and angular acceleration as vector quantities, explaining the right-hand rule for determining their directions. [**4 marks**]

(b) Derive the complete set of relationships between linear and angular kinematic quantities for a particle in circular motion. [**4 marks**]

(c) State the rotational kinematic equations and show how they correspond to their linear motion analogues. [**3 marks**]

Question 2 [**15 marks**] A gas turbine blade rotates with angular velocity described by $\omega(t) = 2t^6 - 12t^5 + 20t^4 - 8t^2 + 5$ rad/s.

(a) Find the angular acceleration function $\alpha(t)$. [**3 marks**]

(b) Calculate the angular velocity and acceleration at $t = 1.8$ seconds. [**3 marks**]

(c) Determine all times when the angular acceleration is zero and identify the nature of these critical points. [**5 marks**]

(d) Calculate the angular displacement from $t = 0.5$ to $t = 2.2$ seconds. [**3 marks**]

(e) Find the time interval(s) where the angular acceleration is maximum. [**1 mark**]

Question 3 [**3 marks**] A point on a rotating mechanism at radius 1.5 m has tangential speed 24 m/s and experiences total acceleration of magnitude 30 m/s^2 at an angle of 37° to the radial direction.

(a) Calculate the tangential and centripetal components of acceleration. [**2 marks**]

(b) Determine the angular velocity and angular acceleration of the mechanism. [**1 mark**]

Section B: Moment of Inertia - Point Masses [32 marks]

Question 4 [**8 marks**]

(a) Define moment of inertia as a tensor quantity and explain how it depends on the choice of coordinate system and rotation axis. [**4 marks**]

(b) State the parallel axis theorem and prove it rigorously for a general mass distribution. [**4 marks**]

Question 5 [**24 marks**] Calculate moments of inertia for the following systems:

(a) Ten point masses arranged as: 3.2 kg at (1,5), 2.8 kg at (-4,2), 3.6 kg at (3,-3), 2.4 kg at (-2,-4), 3.1 kg at (5,1), 2.7 kg at (-3,4), 2.9 kg at (4,-2), 3.4 kg at (-1,-5), 2.6 kg at (2,3), and 3.8 kg at (-5,-1) meters. Find the moment of inertia about the z-axis. [**8 marks**]

(b) Eight masses of 4.2 kg each positioned at the vertices of a regular octagon with circumradius 2.8 m. Calculate the moment of inertia about the central axis perpendicular to the plane. [**4 marks**]

(c) Six point masses distributed along a line: 7 kg at x = -4 m, 11 kg at x = -2 m, 15 kg at x = 0, 9 kg at x = 1.5 m, 13 kg at x = 3 m, and 8 kg at x = 5 m. Find the moment of inertia about: (i) x = 0.5 m, (ii) the center of mass. [**7 marks**]

(d) A pentagonal arrangement has masses 6 kg at (0,0), 8 kg at (4,0), 10 kg at (4,3), 7 kg at (2,5), and 9 kg at (-1,3) meters. Calculate the moment of inertia about an axis through (2,2.5) parallel to the z-axis. [**4 marks**]

(e) Verify your calculation in part (d) using the parallel axis theorem by first finding the center of mass. [**1 mark**]

Section C: Moment of Inertia - Extended Bodies [36 marks]

Question 6 [**13 marks**]

(a) State the complete set of standard moment of inertia formulas for: thin rod (center and end), solid and hollow discs, solid and hollow cylinders, solid and hollow spheres, and rectangular plate. [**8 marks**]

(b) Derive the moment of inertia of a rectangular plate of mass M, length a, and width b about an axis through its center parallel to the length. [**4 marks**]

(c) Explain the physical significance of the radius of gyration and calculate it for a solid sphere. [**1 mark**]

Question 7 [**23 marks**] Calculate moments of inertia for composite objects:

(a) A layered disc system has four concentric sections: solid inner core (mass 12 kg, radius 0.15 m), first hollow ring (mass 8 kg, inner radius 0.15 m, outer radius 0.28 m), second hollow ring (mass 6 kg, inner radius 0.28 m, outer radius 0.38 m), and outer ring (mass 4 kg, inner radius 0.38 m, outer radius 0.45 m). Find the total moment of inertia. [**6 marks**]

(b) A complex framework consists of four identical uniform rods (each mass 5 kg, length 3.2 m) arranged as the edges of a square pyramid with the apex 2.4 m above the square base (side length 2.4 m). Calculate the moment of inertia about the vertical axis through the apex. [**7 marks**]

(c) A mechanical system has six solid spheres (each mass 4 kg, radius 0.08 m) connected by five uniform rods (each mass 2.5 kg, length 1.8 m) forming a linear chain with spheres at all connection points. Find the moment of inertia about an axis through the second sphere from one end, perpendicular to the chain. [**6 marks**]

(d) A wind turbine rotor consists of a central hub (solid cylinder, mass 45 kg, radius 0.6 m) with twelve uniform blades (each mass 15 kg, length 4.5 m) extending radially from the hub edge. Each blade tapers and can be modeled as a rod. Calculate the total moment of inertia about the central axis. [**4 marks**]

Section D: Torque and Rotational Dynamics [33 marks]

Question 8 [**12 marks**]

(a) Define torque as the cross product of position and force vectors, explaining the geometric and physical interpretation of this definition. [**4 marks**]

(b) Derive the fundamental equation of rotational dynamics from Newton's second law applied to a system of particles. [**5 marks**]

(c) Establish the relationship between torque and angular momentum, and derive the rotational impulse-momentum theorem with a practical example. [**3 marks**]

Question 9 [**21 marks**] A complex rotating assembly consists of five coaxial components: base disc (mass 40 kg, radius 2 m), first platform (mass 25 kg, radius 1.4 m), second platform (mass 18 kg, radius 1 m), third platform (mass 12 kg, radius 0.7 m), and top disc (mass 8 kg, radius 0.4 m).

(a) Calculate the total moment of inertia of the assembly. [**4 marks**]

(b) Six forces are applied simultaneously: 60 N tangentially at the base disc edge, 42 N at 25° to the radius at 1.6 m from center, 35 N tangentially at the first platform edge in reverse direction, 28 N at 50° to the radius at 0.8 m from center, 20 N tangentially at the second platform edge, and 12 N radially outward at the top disc edge. Calculate the net torque. [**9 marks**]

(c) Find the angular acceleration of the assembly. [**2 marks**]

(d) Starting from angular velocity 2.2 rad/s, calculate the angular velocity after 7 seconds. [**2 marks**]

(e) Determine the total angular displacement during these 7 seconds. [**2 marks**]

(f) Calculate the work done by the net torque and verify using the change in rotational kinetic energy. [**2 marks**]

Section E: Rotational Energy [34 marks]

Question 10 [**12 marks**]

(a) Derive the expression for rotational kinetic energy starting from the kinetic energy of individual mass elements in a rotating rigid body. [**4 marks**]

(b) Establish the work-energy theorem for rotational motion by showing that the work done by all torques equals the change in rotational kinetic energy. [**5 marks**]

(c) Derive the relationship between instantaneous power and torque, and show how it relates to the time rate of change of rotational kinetic energy. [**3 marks**]

Question 11 [**22 marks**] A compound rolling object consists of a solid inner cylinder (mass 16 kg, radius 0.2 m) with a hollow outer cylinder (mass 12 kg, inner radius 0.2 m, outer radius 0.4 m) rigidly attached. This object rolls without slipping down a spiral track that descends 6 m vertically over a track length of 20 m.

(a) Calculate the total moment of inertia of the compound object about its central axis. [**4 marks**]

(b) State and explain the physical meaning of the rolling constraint for this motion. [**3 marks**]

(c) Apply conservation of energy to find the linear speed at the bottom of the track. [**6 marks**]

(d) Calculate the corresponding angular velocity. [**2 marks**]

(e) If the object then rolls onto a rough horizontal surface with coefficient of friction 0.25, determine how far it travels before coming to rest. [**4 marks**]

(f) Calculate the translational and rotational kinetic energies at the bottom of the spiral track. [**2 marks**]

(g) Compare the rotational energy fraction with those of a solid cylinder and hollow cylinder of the same total mass and outer radius. [**1 mark**]

Section F: Angular Momentum [28 marks]

Question 12 [**8 marks**]

(a) Define angular momentum for both point particles and extended bodies, explaining its vector nature and dependence on the reference point. [**4 marks**]

(b) Prove the conservation of angular momentum principle starting from the rotational equation of motion and discuss its limitations. [**4 marks**]

Question 13 [**20 marks**] A competitive figure skater performs a complex routine, initially spinning with arms extended (moment of inertia 11.5 kg·m^2) at angular velocity 2.4 rad/s about a vertical axis.

(a) Calculate the initial angular momentum of the skater. [**2 marks**]

(b) The skater pulls arms inward and extends one leg, changing the moment of inertia to 3.8 kg·m^2. Find the new angular velocity. [**3 marks**]

(c) Calculate the change in rotational kinetic energy and explain the energy source and conversion mechanism. [**5 marks**]

(d) If this position change occurs over 1.1 seconds, calculate the average power delivered by the skater's muscular system. [**3 marks**]

(e) During the extended position, a 2.2 kg object moving horizontally at 10 m/s collides and adheres to the skater's extended arm at 1.6 m from the rotation axis. Find the resulting angular velocity. [**4 marks**]

(f) Analyze and compare the energy dissipation mechanisms in parts (b) and (e). [**3 marks**]

Section G: Conservation in Rotational Systems [27 marks]

Question 14 [**18 marks**] A large industrial turntable (uniform disc, mass 600 kg, radius 5 m) rotates at 0.3 rad/s. Six workers of varying masses jump onto it: 92 kg at 4.2 m from center, 78 kg at 3.8 m, 85 kg at 4.5 m, 71 kg at 3.2 m, 88 kg at 4.0 m, and 76 kg at 3.5 m from center.

(a) Calculate the initial angular momentum of the turntable. [**3 marks**]

(b) Find the total moment of inertia after all six workers are positioned on the turntable. [**6 marks**]

(c) Determine the final angular velocity using conservation of angular momentum. [**3 marks**]

(d) Calculate the kinetic energy before and after, and explain the thermodynamic processes responsible for energy dissipation. [**4 marks**]

(e) If the 92 kg worker runs tangentially onto the turntable at 8.5 m/s while others step on stationary, find the final angular velocity. [**2 marks**]

Question 15 [**9 marks**] Two sophisticated rotating discs can be engaged through a coupling mechanism. Disc Alpha (moment of inertia 6.8 kg·m^2) rotates at 22 rad/s clockwise, while disc Beta (moment of inertia 4.2 kg·m^2) rotates at 16 rad/s counterclockwise.

(a) Calculate the net angular momentum before the discs are coupled. [**3 marks**]

(b) Find the common angular velocity after engagement. [**3 marks**]

(c) Determine the energy dissipated during coupling and explain the microscopic mechanisms of energy loss. [**3 marks**]

Section H: Combined Motion [26 marks]

Question 16 [**17 marks**] A uniform solid sphere (mass 18 kg, radius 0.25 m) is suspended by a string wound around its equatorial circumference and released from rest to fall vertically while unwinding.

(a) Calculate the moment of inertia about the central axis. [**2 marks**]

(b) Apply Newton's second law for both translational and rotational motion to establish the equations of motion. [**8 marks**]

(c) Solve for the linear acceleration of the sphere. [**3 marks**]

(d) Calculate the tension in the string and express it as a fraction of the sphere's weight. [**2 marks**]

(e) If the sphere falls through a vertical distance of 3.2 m, calculate the final linear and angular velocities. [**2 marks**]

Question 17 [**9 marks**] A hollow sphere of mass 10 kg and radius 0.35 m rolls down a 26° incline with initial linear velocity 8 m/s.

(a) Calculate the initial total kinetic energy of the sphere. [**3 marks**]

(b) If the sphere rolls 10 m down the incline, use energy conservation to find the final linear velocity. [**4 marks**]

(c) Calculate the final angular velocity and verify the rolling constraint. [**2 marks**]

Section I: Complex Systems [28 marks]

Question 18 [**18 marks**] An intricate pulley system features two rigidly connected coaxial wheels with different radii. The primary wheel (radius 1.1 m) supports a 50 kg mass, while the secondary wheel (radius 0.65 m) supports a 32 kg mass hanging on the opposite side. The combined wheel assembly has moment of inertia 15.5 kg·m^2.

(a) Draw comprehensive free body diagrams for all system components. [**5 marks**]

(b) Develop the complete system of dynamic equations for all moving parts. [**6 marks**]

(c) Solve analytically for the angular acceleration of the wheel assembly. [**4 marks**]

(d) Calculate the tensions in both cables and verify equilibrium of forces and torques. [**3 marks**]

Question 19 [**10 marks**] A uniform rod of mass 11 kg and length 4.2 m is pivoted at a point 1.8 m from one end. It is released from rest when the longer section makes an angle of 25° below the horizontal.

(a) Calculate the moment of inertia about the pivot point using the parallel axis theorem. [**4 marks**]

(b) Find the initial angular acceleration using rotational dynamics. [**3 marks**]

(c) Apply energy conservation to find the angular velocity when the rod reaches its lowest point. [**3 marks**]

Section J: Advanced Applications [27 marks]

Question 20 [**18 marks**] An advanced space station consists of a main cylindrical habitat (mass 1200 kg, radius 2.5 m, length 8 m), six docking modules (each mass 180 kg, positioned 6 m from the central axis), three laboratory pods (each mass 220 kg, located 4.5 m from the axis), and two communication arrays (each mass 150 kg, positioned 3.8 m from the axis).

(a) Calculate the moment of inertia of the main habitat about its central axis. [**2 marks**]

(b) Find the total moment of inertia contributions from all auxiliary components. [**6 marks**]

(c) Calculate the total moment of inertia of the complete space station. [**2 marks**]

(d) If the station rotates at 0.04 rad/s, determine its angular momentum. [**2 marks**]

(e) Attitude control systems apply a net torque of 200 N·m for 75 seconds. Calculate the resulting change in angular velocity. [**3 marks**]

(f) During a meteoroid impact, two docking modules and one laboratory pod are completely destroyed. Find the new angular velocity of the remaining system. [**3 marks**]

Question 21 [**9 marks**] A precision machining spindle (solid disc, mass 28 kg, radius 0.18 m) operates at 540 rpm. During a cutting operation, the workpiece exerts a friction torque of 35 N·m opposing the rotation.

(a) Convert the operating speed to rad/s and calculate the initial angular momentum. [**3 marks**]

(b) Find the angular deceleration caused by the friction torque. [**3 marks**]

(c) Calculate the time required for the spindle speed to decrease to 360 rpm. [**3 marks**]

Physics Data and Formulae

Rotational Kinematics:

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $v = r\omega$, $a_t = r\alpha$, $a_c = \frac{v^2}{r} = \omega^2 r$

Moment of Inertia:

Point mass: $I = mr^2$

Parallel axis theorem: $I = I_{cm} + md^2$

Uniform rod (center): $I = \frac{1}{12}ML^2$, (end): $I = \frac{1}{3}ML^2$

Solid disc/cylinder: $I = \frac{1}{2}MR^2$, Thin ring: $I = MR^2$

Hollow cylinder: $I = \frac{1}{2}M(R_1^2 + R_2^2)$

Solid sphere: $I = \frac{2}{5}MR^2$, Hollow sphere: $I = \frac{2}{3}MR^2$

Rectangular plate: $I = \frac{1}{12}M(a^2 + b^2)$ (about center, to plate)

Rotational Dynamics:

$\tau = I\alpha$ (equation of rotational motion)

$\tau = \vec{r} \times \vec{F}$ (torque from force)

Work: $W = \tau\theta$, Power: $P = \tau\omega$

Rotational Energy:
$KE_{rot} = \frac{1}{2}I\omega^2$
Rolling motion: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
Rolling constraint: $v = r\omega$ (no slipping)

Angular Momentum:
$L = I\omega$ (rigid body), $L = \vec{r} \times \vec{p}$ (point particle)
Conservation: $L_i = L_f$ (when $\sum \tau_{ext} = 0$)

Constants:
Acceleration due to gravity: $g = 9.8$ m/s²

Trigonometric Values:
$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$
$\sin 26 = 0.438$, $\cos 26 = 0.899$, $\tan 26 = 0.488$
$\sin 37 = 0.602$, $\cos 37 = 0.799$, $\tan 37 = 0.754$
$\sin 50 = 0.766$, $\cos 50 = 0.643$, $\tan 50 = 1.192$

END OF TEST

Total marks: 290

Grade boundaries: A* 261, A 232, B 203, C 174, D 145, E 116

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