GCSE Higher Mathematics
Practice Test 1: Geometry and Measures

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Pythagoras' Theorem

1. Use Pythagoras' theorem to find the missing lengths:
   (a) A right-angled triangle with legs 5 cm and 12 cm. Find the hypotenuse.
   (b) A right-angled triangle with hypotenuse 15 cm and one leg 9 cm. Find the other leg.
   (c) A right-angled triangle with legs 7.5 cm and 10 cm. Find the hypotenuse.
   (d) A right-angled triangle with hypotenuse 20 cm and one leg 16 cm. Find the other leg.
2. Find the distance between these pairs of coordinate points:
   (a) A(2, 3) and B(8, 11)
   (b) C(-4, 1) and D(5, -7)
   (c) E(-3, -2) and F(4, 6)
   (d) G(0, 5) and H(-8, -1)
3. A rectangle has length 15 cm and diagonal 17 cm. Find:
   (a) The width of the rectangle
   (b) The area of the rectangle
   (c) The perimeter of the rectangle
4. A ladder 6.5 m long leans against a vertical wall. The foot of the ladder is 2.5 m from the wall.
   (a) How high up the wall does the ladder reach?
   (b) If the ladder slips so its foot is now 3.5 m from the wall, how far does the top slide down?
5. Determine whether these triangles are right-angled:
   (a) Sides 9 cm, 12 cm, 15 cm
   (b) Sides 8 cm, 15 cm, 17 cm
   (c) Sides 5 cm, 13 cm, 14 cm
   (d) Sides 20 cm, 21 cm, 29 cm

Section B: Basic Trigonometry

6. In a right-angled triangle with hypotenuse 10 cm and one leg 6 cm:
   (a) Find the other leg using Pythagoras
   (b) Calculate $\sin A$, $\cos A$, and $\tan A$ where A is the angle opposite the 6 cm side
   (c) Find angle A to 1 decimal place
7. Use trigonometry to find the missing sides (give answers to 3 significant figures):
   (a) Right-angled triangle: angle 35°, adjacent side 8 cm, find opposite side
   (b) Right-angled triangle: angle 52°, hypotenuse 12 cm, find opposite side
   (c) Right-angled triangle: angle 28°, opposite side 7 cm, find adjacent side
   (d) Right-angled triangle: angle 67°, adjacent side 9 cm, find hypotenuse
8. Find the missing angles (to 1 decimal place):
   (a) Right-angled triangle: opposite 5 cm, adjacent 8 cm
   (b) Right-angled triangle: opposite 7.2 cm, hypotenuse 11.5 cm
   (c) Right-angled triangle: adjacent 6.8 cm, hypotenuse 9.3 cm
   (d) Right-angled triangle: opposite 12 cm, adjacent 9 cm
9. A 15 m ladder makes an angle of 72° with the ground.
   (a) How high up the wall does it reach?
   (b) How far is the foot of the ladder from the wall?
   (c) What angle would the ladder make if its foot was 8 m from the wall?
10. From the top of a 25 m cliff, the angle of depression to a boat is 18°.
    (a) Draw a diagram
    (b) Find the horizontal distance from the cliff to the boat
    (c) Find the direct distance from the top of the cliff to the boat

Section C: Advanced Trigonometry - Sine and Cosine Rules

11. Use the sine rule to find the missing parts:
    (a) Triangle ABC: $a = 8$ cm, $A = 35°$, $B = 67°$. Find b and c.
    (b) Triangle PQR: $p = 12$ cm, $q = 15$ cm, $P = 42°$. Find angle Q.
    (c) Triangle XYZ: $x = 7.2$ cm, $y = 9.8$ cm, $X = 38°$. Find angle Y.
12. Use the cosine rule to find the missing parts:
    (a) Triangle ABC: $a = 9$ cm, $b = 7$ cm, $c = 5$ cm. Find angle C.
    (b) Triangle DEF: $d = 11$ cm, $e = 8$ cm, $F = 52°$. Find side f.
    (c) Triangle GHI: $g = 6.5$ cm, $h = 8.2$ cm, $i = 4.7$ cm. Find the largest angle.
13. A triangle has sides 13 cm, 14 cm, and 15 cm.
    (a) Use the cosine rule to find the largest angle
    (b) Use the sine rule to find another angle
    (c) Find the area using the formula Area $= \frac{1}{2}ab\sin C$

14. Two ships leave port at the same time. Ship A travels 25 km on a bearing of 040°. Ship B travels 18 km on a bearing of 110°.

    (a) Find the angle between their paths

    (b) Calculate the distance between the ships

    (c) Find the bearing of ship B from ship A

15. A parallelogram has sides 8 cm and 12 cm, with an included angle of 65°.

    (a) Find the length of the diagonals

    (b) Find the area of the parallelogram

    (c) Find the other angles of the parallelogram

Section D: Circle Theorems

16. Apply circle theorems to find the missing angles:

    (a) Angle at center is 80°. Find the angle at circumference subtending the same arc.

    (b) Angle at circumference is 35°. Find the angle at center subtending the same arc.

    (c) Two angles at circumference subtend the same arc. One is 42°. Find the other.

    (d) Angle in semicircle ABC where AB is diameter. Find angle ACB.

17. In a circle with center O:

    (a) Chord AB subtends angle 46° at circumference point C. Find angle AOB.

    (b) Tangent PT touches circle at T. If angle PTO = 28°, find angle TPO.

    (c) Two chords AB and CD intersect at P inside the circle. If angles APC = 85°, find angle APD.

    (d) Tangents from external point P touch circle at A and B. If angle APB = 60°, find angle AOB.

18. Prove these circle theorems (provide written proof):

    (a) Angle in semicircle is 90°

    (b) Angles in same segment are equal

    (c) Angle at center is twice angle at circumference

    (d) Tangent is perpendicular to radius at point of contact

19. A circle has center O and radius 8 cm. Chord AB has length 12 cm.

    (a) Find the perpendicular distance from O to chord AB

    (b) Find angle AOB

    (c) Find the area of sector AOB

    (d) Find the area of triangle AOB

Section E: Geometrical Proof and Reasoning

20. Prove that the sum of angles in any triangle is 180°.

21. Prove that the exterior angle of a triangle equals the sum of the two interior opposite angles.

22. In triangle ABC, D is the midpoint of BC. Prove that AD is a median and that the three medians of a triangle meet at a single point.

23. ABCD is a parallelogram. Prove that:
    (a) Opposite sides are equal
    (b) Opposite angles are equal
    (c) Diagonals bisect each other
24. Prove that if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.
25. In triangle ABC, the perpendicular bisectors of sides AB and BC meet at point P. Prove that P is equidistant from A, B, and C.
26. ABCD is a cyclic quadrilateral (vertices lie on a circle). Prove that opposite angles sum to $180°$.
27. Prove that the angle bisectors of a triangle meet at a single point (the incenter).

Section F: Surface Area and Volume

28. Calculate the surface area and volume of these shapes:
    (a) Cuboid: length 8 cm, width 6 cm, height 5 cm
    (b) Cylinder: radius 4 cm, height 10 cm
    (c) Sphere: radius 6 cm
    (d) Cone: radius 5 cm, height 12 cm
29. A triangular prism has:
    - Triangular cross-section with base 6 cm and height 4 cm
    - Length 15 cm

    Calculate:
    (a) The volume
    (b) The surface area
    (c) The total edge length
30. A square-based pyramid has:
    - Base edge 8 cm
    - Vertical height 6 cm

    Find:
    (a) The volume
    (b) The slant height
    (c) The surface area
31. A hemisphere sits on top of a cylinder. Both have radius 3 cm and the cylinder has height 8 cm.
    (a) Find the total volume
    (b) Find the total surface area
    (c) If this shape is a water tank, how much water does it hold?
32. A cone has base radius 4 cm and slant height 10 cm.
    (a) Find the vertical height
    (b) Calculate the volume
    (c) Calculate the curved surface area
    (d) Calculate the total surface area

Section G: Complex 3D Problems

33. A rectangular swimming pool is 25 m long, 12 m wide, and 2 m deep.
    (a) Calculate the volume of water needed to fill it
    (b) Find the surface area of the pool (5 faces - no top)
    (c) How much does it cost to tile the pool at £35 per m^2?
    (d) If water is added at 500 liters per minute, how long to fill the pool?

34. A solid consists of a cylinder with radius 6 cm and height 10 cm, with a cone of radius 6 cm and height 8 cm on top.
    (a) Calculate the total volume
    (b) Calculate the total surface area
    (c) If the solid is made of metal with density 7.8 g/cm^3, find its mass

35. A spherical ball has radius 15 cm. A cylindrical hole of radius 4 cm is drilled through the center.
    (a) Calculate the volume of material removed
    (b) Calculate the remaining volume
    (c) Calculate the new surface area (including the cylindrical hole)

36. A water tank is a horizontal cylinder with length 5 m and radius 1.2 m.
    (a) Calculate its capacity in liters
    (b) If it's half full, find the depth of water
    (c) Calculate the surface area of water

Section H: Problem Solving and Applications

37. A regular hexagon is inscribed in a circle of radius 10 cm.
    (a) Find the side length of the hexagon
    (b) Calculate the area of the hexagon
    (c) Find the area between the circle and hexagon

38. From point A, the angle of elevation to the top of a tower is 25°. From point B, 100 m closer to the tower, the angle of elevation is 40°.
    (a) Draw a diagram
    (b) Find the height of the tower
    (c) Find the distances from A and B to the base of the tower

39. A field is in the shape of a triangle with sides 120 m, 150 m, and 200 m.
    (a) Find the area of the field using Heron's formula
    (b) Find the area using trigonometry
    (c) A path 2 m wide runs around the perimeter. Find the area of the path

40. A church spire consists of a square-based pyramid on top of a cuboid. The cuboid is 8 m × 8 m × 15 m high. The pyramid has height 12 m.
    (a) Calculate the total volume
    (b) Calculate the total surface area

(c) Find the angle the pyramid faces make with the horizontal

41. Two circles with radii 8 cm and 6 cm have centers 20 cm apart.

    (a) Do the circles intersect? Justify your answer.

    (b) Find the length of their common external tangent

    (c) Calculate the area of overlap if they moved closer together

42. A satellite orbits Earth at height 300 km above the surface. Earth's radius is 6400 km.

    (a) Calculate the satellite's orbital radius

    (b) Find the maximum distance the satellite can see on Earth's surface

    (c) Calculate the area of Earth visible from the satellite

43. Prove that in any triangle, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

44. A frustum (truncated cone) has top radius 3 cm, bottom radius 8 cm, and height 6 cm.

    (a) Find the height of the complete cone

    (b) Calculate the volume of the frustum

    (c) Calculate the curved surface area of the frustum

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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