

# A Level Mechanics

## Practice Test 3: Work, Energy, and Power

### Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Draw diagrams where appropriate to illustrate your solutions.

Time allowed: 3 hours

### Section A: Work and Energy Fundamentals [27 marks]

#### Question 1 [8 marks]

- (a) Define work done by a force and state when work is: (i) maximum, (ii) zero, (iii) negative. [4 marks]
- (b) Explain why work is classified as a scalar quantity despite being calculated from two vector quantities. [2 marks]
- (c) State the relationship between work and energy, giving the fundamental principle. [2 marks]

**Question 2 [12 marks]** Three forces act on an object:  $\vec{F}_1 = (8\hat{i} + 6\hat{j})$  N,  $\vec{F}_2 = (-4\hat{i} + 12\hat{j})$  N, and  $\vec{F}_3 = (10\hat{i} - 8\hat{j})$  N. The object moves from position (2, 1) m to (7, 5) m.

- (a) Calculate the resultant force. [2 marks]
- (b) Find the displacement vector. [1 mark]
- (c) Calculate the work done by each individual force. [4 marks]
- (d) Find the total work done by all forces. [2 marks]
- (e) Verify your answer using the resultant force method. [3 marks]

**Question 3 [7 marks]** A 6 kg object initially at rest has a constant force of 36 N applied to it for 4 seconds at an angle of  $30^\circ$  to its direction of motion.

- (a) Calculate the acceleration of the object. [2 marks]
- (b) Find the distance traveled and the work done by the force. [3 marks]
- (c) Calculate the final kinetic energy and verify the work-energy theorem. [2 marks]

**Section B: Work Done by Constant Forces [26 marks]**

**Question 4 [14 marks]** A package of mass 18 kg is dragged 25 m across a rough horizontal floor by a rope inclined at  $28^\circ$  to the horizontal. The tension in the rope is 85 N and the coefficient of kinetic friction between the package and floor is 0.15.

- (a) Draw a complete force diagram for the package. [2 marks]
- (b) Calculate the normal force between the package and floor. [3 marks]
- (c) Find the friction force acting on the package. [2 marks]
- (d) Calculate the work done by: (i) the tension force, (ii) friction, (iii) gravity. [4 marks]
- (e) Determine the net work done and the final speed of the package. [3 marks]

**Question 5 [12 marks]** A train of mass 150,000 kg accelerates from 15 m/s to 45 m/s over a distance of 800 m while traveling up a  $2^\circ$  incline. The train experiences a total resistance force of 12,000 N.

- (a) Calculate the change in kinetic energy of the train. [3 marks]
- (b) Find the work done against gravity. [3 marks]
- (c) Calculate the work done against resistance forces. [2 marks]
- (d) Determine the total work done by the engine. [2 marks]
- (e) Find the average engine force during this acceleration. [2 marks]

**Section C: Variable Force and Work-Energy Theorem [29 marks]**

**Question 6 [15 marks]** A particle of mass 3 kg moves along the x-axis under a variable force  $F = 15 - 4x + x^2$  N, where  $x$  is measured in metres.

- (a) Calculate the work done as the particle moves from  $x = 1$  m to  $x = 4$  m. [4 marks]
- (b) Find the work done as the particle moves from  $x = 2$  m to  $x = 5$  m. [4 marks]
- (c) If the particle has speed 2 m/s at  $x = 1$  m, find its speed at  $x = 3$  m. [3 marks]
- (d) Determine where the force is zero and classify these points. [2 marks]
- (e) Find the position where the force is minimum in the interval  $[0, 6]$ . [2 marks]

**Question 7 [14 marks]**

- (a) Prove the work-energy theorem starting from Newton's second law for variable force. [4 marks]
- (b) A ball of mass 1.2 kg is thrown at  $50^\circ$  above horizontal with initial speed 22 m/s. Calculate the speed when it is 12 m above the launch point. [4 marks]
- (c) A 20 kg crate slides down a rough ramp (angle  $30^\circ$ ) for 10 m. With coefficient of friction 0.22, find the final speed from rest. [4 marks]
- (d) An object of mass 8 kg experiences a time-varying force  $F(t) = 30 - 5t$  N for 5 seconds. Starting from rest, calculate the final kinetic energy. [2 marks]

**Section D: Gravitational Potential Energy [25 marks]****Question 8 [7 marks]**

- (a) Define a conservative force and explain why gravitational force is conservative. [3 marks]
- (b) Show mathematically that the work done by gravity depends only on the change in height. [2 marks]
- (c) Explain the significance of choosing different reference levels for potential energy. [2 marks]

**Question 9 [18 marks]** A golf ball of mass 0.05 kg is hit from ground level with initial velocity 35 m/s at  $42^\circ$  to the horizontal.

- (a) Calculate the initial kinetic and potential energies. [3 marks]
- (b) Find the total mechanical energy. [1 mark]
- (c) Use energy conservation to determine the maximum height reached. [3 marks]
- (d) Calculate the speed when the ball is at  $2/3$  of maximum height. [4 marks]
- (e) Find the speed when the ball returns to ground level. [2 marks]
- (f) At what height above ground is the potential energy twice the kinetic energy? [3 marks]
- (g) Determine the horizontal and vertical components of velocity at maximum height. [2 marks]

**Section E: Elastic Potential Energy [23 marks]**

**Question 10 [13 marks]** A spring with spring constant  $k = 1200 \text{ N/m}$  is compressed by 0.25 m from its equilibrium position.

- (a) Calculate the elastic potential energy stored in the spring. [2 marks]
- (b) A mass of 2 kg is attached to the spring and released. Find the maximum speed. [3 marks]
- (c) At what displacement from equilibrium is the speed 4 m/s? [3 marks]
- (d) Calculate the acceleration when the compression is 0.18 m. [2 marks]
- (e) Find the displacement where kinetic energy is maximum. [1 mark]
- (f) Determine the ratio of kinetic to potential energy when displacement is 0.1 m. [2 marks]

**Question 11 [10 marks]** A 4 kg mass hangs from a vertical spring with spring constant 800 N/m. The mass is pulled down 0.4 m from its equilibrium position and released.

- (a) Calculate the equilibrium extension of the spring when the mass hangs freely. [2 marks]
- (b) Find the total mechanical energy of the oscillating system. [2 marks]
- (c) Determine the maximum speed during oscillation. [2 marks]
- (d) Calculate the speed when the mass is 0.2 m below equilibrium. [2 marks]
- (e) Find the maximum height the mass reaches above its release point. [2 marks]

## Section F: Conservation of Mechanical Energy [27 marks]

**Question 12 [15 marks]** A pendulum consists of a 2 kg bob attached to a 2.5 m string. It is released from rest when the string makes  $55^\circ$  with the vertical.

- (a) Calculate the initial height of the bob above its lowest position. [2 marks]
- (b) Find the speed at the bottom of the swing using conservation of energy. [3 marks]
- (c) Calculate the tension in the string at the lowest point. [3 marks]
- (d) Determine the speed when the string makes  $30^\circ$  with the vertical. [3 marks]
- (e) Find the minimum release angle for the bob to just reach the top of a complete circle. [2 marks]
- (f) Calculate the speed at the top of the circle for this minimum case. [2 marks]

**Question 13 [12 marks]** A 1000 kg roller coaster car starts from rest at height 35 m and travels on a track with varying friction.

- (a) Find the speed at the bottom (ground level) assuming no friction. [2 marks]
- (b) The car then climbs to 20 m height. Calculate the speed at this point with no friction. [3 marks]
- (c) If friction causes 30,000 J energy loss over the entire journey, find the actual speed at 20 m height. [3 marks]
- (d) For a circular loop of radius 6 m at the bottom, find the minimum starting height needed to complete the loop with this friction. [4 marks]

## Section G: Power Calculations [24 marks]

**Question 14 [9 marks]**

- (a) Define power and derive the relationship between power, force, and velocity. [4 marks]
- (b) Explain the difference between mechanical power and electrical power in motors. [3 marks]
- (c) Why is efficiency important in power transmission systems? [2 marks]

**Question 15 [15 marks]** A construction crane lifts materials with varying loads and speeds.

- (a) Calculate the power needed to lift 1500 kg at constant speed 1.8 m/s. [2 marks]
- (b) If the motor has 78% efficiency, find the electrical power input. [2 marks]
- (c) The crane accelerates a 1200 kg load upward at  $0.8 \text{ m/s}^2$  from rest. Find the motor force. [3 marks]
- (d) Calculate the power output after 5 seconds of this acceleration. [3 marks]
- (e) Determine the total work done during the first 8 seconds of acceleration. [3 marks]
- (f) If the crane then operates at constant speed for 15 seconds, calculate the total energy consumed. [2 marks]

## Section H: Power in Vehicle Motion [21 marks]

**Question 16 [13 marks]** A sports car engine delivers constant power of 120 kW. The car has mass 1600 kg and faces air resistance  $F_r = 400 + 1.5v^2$  N, where  $v$  is speed in m/s.

- (a) Find the maximum speed by equating engine force to resistance. [3 marks]
- (b) Calculate the acceleration when the car travels at 25 m/s. [4 marks]
- (c) Determine the speed when acceleration equals  $0.5 \text{ m/s}^2$ . [3 marks]
- (d) Estimate the time required to accelerate from 20 m/s to 35 m/s. [3 marks]

**Question 17 [8 marks]** A delivery truck (mass 3500 kg including load) maintains constant power of 85 kW.

- (a) On level road with 800 N resistance, find the maximum speed. [2 marks]
- (b) When climbing a  $6^\circ$  gradient at steady speed, calculate the new maximum speed. [3 marks]
- (c) What power would be required to maintain 15 m/s up this gradient? [3 marks]

## Section I: Energy in Collisions and Springs [23 marks]

**Question 18 [13 marks]** A 5 kg object moving at 12 m/s collides elastically with a 7 kg object initially at rest.

- (a) Use conservation laws to find both final velocities. [5 marks]
- (b) Calculate kinetic energies before and after collision to verify energy conservation. [3 marks]
- (c) Find the change in kinetic energy of each object. [2 marks]
- (d) Calculate the impulse experienced by each object. [3 marks]

**Question 19 [10 marks]** A 0.25 kg ball moving at 18 m/s hits a wall perpendicularly and rebounds at 14 m/s.

- (a) Calculate the change in momentum and impulse. [3 marks]
- (b) Find the kinetic energies before and after impact. [2 marks]
- (c) Determine the energy lost and coefficient of restitution. [3 marks]
- (d) If contact time is 0.012 seconds, calculate the average impact force. [2 marks]

## Section J: Comprehensive Applications [30 marks]

**Question 20 [18 marks]** A 7 kg block slides down a frictionless track from height 6 m, then compresses a spring ( $k = 2200 \text{ N/m}$ ) on a horizontal surface with friction coefficient 0.18.

- (a) Calculate the speed just before hitting the spring. [2 marks]
- (b) If the surface were frictionless, find the maximum compression. [3 marks]
- (c) With friction present, write the energy equation for maximum compression. [3 marks]
- (d) Calculate the actual maximum compression. [3 marks]
- (e) Find the speed when the block leaves the spring on its return. [3 marks]
- (f) Determine the maximum height reached on the return journey up the track. [2 marks]

- (g) Calculate the total energy dissipated by friction. [2 marks]

**Question 21 [12 marks]** A wind farm generates electricity using turbines in a steady 12 m/s wind. Each turbine has blade diameter 80 m and converts 35% of the wind's kinetic energy to electrical energy.

- (a) Calculate the mass of air passing through one turbine per second (air density = 1.2 kg/m<sup>3</sup>). [3 marks]
- (b) Find the kinetic energy in this air mass per second. [2 marks]
- (c) Calculate the electrical power output from one turbine. [2 marks]
- (d) If the wind farm has 50 turbines, find the total power output. [1 mark]
- (e) How many households can this supply if each uses 4 kW average power? [2 marks]
- (f) Compare the environmental impact with equivalent fossil fuel generation. [2 marks]

### Physics Data and Formulae

#### Work and Energy:

$$\text{Work done: } W = \vec{F} \cdot \vec{s} = Fs \cos \theta$$

$$\text{Variable force: } W = \int_{s_1}^{s_2} F(s) ds$$

$$\text{Kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{Gravitational potential energy: } PE_g = mgh$$

$$\text{Elastic potential energy: } PE_e = \frac{1}{2}kx^2$$

#### Work-Energy Theorem:

$$W_{net} = \Delta KE = KE_f - KE_i$$

#### Conservation of Energy:

$$E_{total} = KE + PE = \text{constant (conservative forces)}$$

$$KE_i + PE_i = KE_f + PE_f + \text{work done against friction}$$

#### Power:

$$P = \frac{W}{t} \text{ (average), } P = \frac{dW}{dt} \text{ (instantaneous)}$$

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

$$\text{Efficiency: } \eta = \frac{P_{output}}{P_{input}} \times 100\%$$

#### Springs and Oscillations:

$$\text{Hooke's Law: } F = -kx$$

$$\text{Equilibrium position: } x_0 = \frac{mg}{k} \text{ (vertical spring)}$$

$$\text{Total energy: } E = \frac{1}{2}kA^2 \text{ (amplitude } A)$$

#### Collisions:

$$\text{Coefficient of restitution: } e = \frac{\text{relative speed of separation}}{\text{relative speed of approach}}$$

$$\text{Elastic collision: } e = 1, \text{ KE conserved}$$

$$\text{Perfectly inelastic: } e = 0, \text{ objects stick together}$$

#### Constants:

$$\text{Acceleration due to gravity: } g = 9.8 \text{ m/s}^2$$

$$\text{Air density (standard): } \rho_{air} = 1.2 \text{ kg/m}^3$$

#### Trigonometric Values:

$$\sin 2 = 0.035, \cos 2 = 0.999, \tan 2 = 0.035$$

$\sin 6 = 0.105$ ,  $\cos 6 = 0.995$ ,  $\tan 6 = 0.105$   
 $\sin 28 = 0.469$ ,  $\cos 28 = 0.883$ ,  $\tan 28 = 0.532$   
 $\sin 30 = 0.500$ ,  $\cos 30 = 0.866$ ,  $\tan 30 = 0.577$   
 $\sin 42 = 0.669$ ,  $\cos 42 = 0.743$ ,  $\tan 42 = 0.900$   
 $\sin 50 = 0.766$ ,  $\cos 50 = 0.643$ ,  $\tan 50 = 1.192$   
 $\sin 55 = 0.819$ ,  $\cos 55 = 0.574$ ,  $\tan 55 = 1.428$

**END OF TEST**

Total marks: 275

Grade boundaries: A\* 248, A 220, B 193, C 165, D 138, E 110

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