GCSE Higher Mathematics
Practice Test 1: Ratio, Proportion, and Rates of Change

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Ratios

1. Express these ratios in their simplest form:
   (a) $2.4 : 3.6 : 1.2$
   (b) $\frac{2}{3} : \frac{3}{4} : \frac{1}{2}$
   (c) $0.25 : 0.375 : 0.125$
   (d) $1\frac{1}{4} : 2\frac{1}{2} : 3\frac{3}{4}$
2. Three business partners invest in the ratio $4 : 6 : 5$. If the total investment is £75,000:
   (a) How much does each partner invest?
   (b) If the profit is £22,500, how much does each partner receive?
   (c) What percentage of the total investment does the largest investor contribute?
3. The angles of a quadrilateral are in the ratio $3 : 4 : 5 : 6$. Find each angle.
4. A concrete mixture uses cement, sand, and gravel in the ratio $2 : 3 : 5$ by mass.
   (a) How much of each material is needed for 400kg of concrete?
   (b) If 75kg of sand is used, find the mass of cement and gravel needed
   (c) What fraction of the mixture is cement?
5. Divide £840 between three people so that the first gets £20 more than the second, and the second gets £40 more than the third.

Section B: Direct Proportion

6. y is directly proportional to x. When $x = 12$, $y = 8$.
   (a) Find the equation connecting y and x
   (b) Calculate y when $x = 15$
   (c) Calculate x when $y = 20$
   (d) Sketch the graph of y against x
7. P varies directly as the square of Q. When $Q = 4$, $P = 32$.

(a) Express P in terms of Q

(b) Find P when $Q = 6$

(c) Find Q when $P = 200$

(d) What happens to P when Q is doubled?

8. The cost of copper wire varies directly as its length. 15 meters cost £24.

(a) Find the cost per meter

(b) How much does 22.5 meters cost?

(c) What length can be bought for £40?

(d) Express the cost C in terms of length L

9. The energy stored in a spring varies directly as the square of its compression. When compressed by 3cm, it stores 18 joules.

(a) Find the energy stored when compressed by 5cm

(b) What compression stores 50 joules?

(c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x. When $x = 5$, $y = 12$.

(a) Find the equation connecting y and x

(b) Calculate y when $x = 8$

(c) Calculate x when $y = 15$

(d) Sketch the graph of y against x

11. The time taken to complete a journey is inversely proportional to the average speed. At 60 km/h, the journey takes 2.5 hours.

(a) How long does the journey take at 75 km/h?

(b) What speed is needed to complete the journey in 2 hours?

(c) What is the distance of the journey?

(d) Express time T in terms of speed S

12. P is inversely proportional to the square of Q. When $Q = 2$, $P = 18$.

(a) Express P in terms of Q

(b) Find P when $Q = 3$

(c) Find Q when $P = 2$

(d) What happens to P when Q is halved?

13. The intensity of light is inversely proportional to the square of the distance from the source. At 3 meters, the intensity is 200 units.

(a) Find the intensity at 5 meters

(b) At what distance is the intensity 50 units?

(c) Write the relationship as an equation

Section D: Combined Proportion

14. z varies directly as x and inversely as y. When $x = 6$ and $y = 4$, $z = 9$.

    (a) Express z in terms of x and y

    (b) Find z when $x = 8$ and $y = 3$

    (c) Find y when $x = 10$ and $z = 5$

    (d) What happens to z if both x and y are doubled?

15. The volume V of a cylinder varies directly as its height h and as the square of its radius r. When $h = 10$ and $r = 3$, $V = 90\pi$.

    (a) Express V in terms of h and r

    (b) Find V when $h = 15$ and $r = 4$

    (c) Find r when $h = 20$ and $V = 500\pi$

    (d) What happens to V if h is doubled and r is halved?

16. The resistance R of a wire varies directly as its length L and inversely as the square of its diameter d. When $L = 100$ and $d = 2$, $R = 25$.

    (a) Express R in terms of L and d

    (b) Find R when $L = 150$ and $d = 1.5$

    (c) Find L when $R = 40$ and $d = 2.5$

17. y varies directly as x^2 and inversely as z^3. When $x = 2$ and $z = 3$, $y = 8$.

    (a) Find the constant of proportionality

    (b) Express y in terms of x and z

    (c) Calculate y when $x = 3$ and $z = 2$

Section E: Speed, Distance, and Time

18. A car travels 240 km in 3 hours 20 minutes. Calculate:

    (a) The average speed in km/h

    (b) The average speed in m/s

    (c) How far it travels in 2 hours 45 minutes at this speed

    (d) How long it takes to travel 180 km at this speed

19. Convert these speeds:

    (a) 25 m/s to km/h

    (b) 108 km/h to m/s

    (c) 45 mph to km/h (use 1 mile = 1.6 km)

    (d) 80 km/h to mph

20. A cyclist travels from A to B at 20 km/h and returns at 30 km/h. The total journey takes 5 hours.

    (a) Find the distance from A to B

    (b) Calculate the average speed for the whole journey

    (c) How much time was saved on the return journey?

21. Two trains start simultaneously from stations 300 km apart and travel towards each other. One travels at 80 km/h and the other at 70 km/h.

    (a) When do they meet?

    (b) How far from each starting point do they meet?

    (c) What is their relative speed of approach?

22. A runner completes a 10 km race. For the first 6 km, they maintain a speed of 12 km/h. For the remaining 4 km, they slow to 8 km/h.

    (a) Calculate the total time taken

    (b) Find the average speed for the whole race

    (c) How much faster would they need to run the second part to achieve an overall average of 10 km/h?

Section F: Density and Flow Rates

23. A metal block has volume 250 cm^3 and mass 2.25 kg.

    (a) Calculate its density in g/cm^3

    (b) Calculate its density in kg/m^3

    (c) What mass of this metal would have volume 400 cm^3?

    (d) What volume would 3.6 kg of this metal occupy?

24. Different substances have these densities:

    - Aluminum: 2.7 g/cm^3
    - Iron: 7.9 g/cm^3
    - Gold: 19.3 g/cm^3

    (a) Compare the masses of 1 cm^3 of each metal

    (b) What volume of iron has the same mass as 50 cm^3 of aluminum?

    (c) A crown weighs 500g and has volume 30 cm^3. Is it pure gold?

25. Water flows through a pipe at a rate of 1.5 liters per second.

    (a) How much water flows in 5 minutes?

    (b) How long to fill a 600-liter tank?

    (c) Express the flow rate in m^3/hour

    (d) If the pipe diameter halves, what happens to the flow speed?

26. A factory produces items at a rate that varies throughout the day:

    - Morning (4 hours): 150 items/hour
    - Afternoon (5 hours): 120 items/hour
    - Evening (3 hours): 100 items/hour

    (a) Calculate total daily production

    (b) Find the average production rate

    (c) How long to produce 1000 items at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar triangles have corresponding sides in the ratio 3:5.

    (a) If the smaller triangle has area 18 cm^2, find the area of the larger triangle

    (b) If the larger triangle has perimeter 40 cm, find the perimeter of the smaller triangle

    (c) The volume ratio of similar pyramids based on these triangles

28. A model car is built to scale 1:24. The real car is 4.2 m long and 1.8 m wide.

    (a) Find the dimensions of the model

    (b) If the model uses 15 cm^2 of paint, how much paint does the real car need?

    (c) The model weighs 200g. Estimate the mass of the real car if made from the same material

29. A map has scale 1:25000. Two towns are 8.4 cm apart on the map.

    (a) Calculate the actual distance in km

    (b) What map distance represents 5 km?

    (c) A lake covers 2.4 cm^2 on the map. Find its actual area in hectares (1 hectare = 10000 m^2)

30. Two similar cylinders have radius ratio 2:3.

    (a) Find the ratio of their surface areas

    (b) Find the ratio of their volumes

    (c) If the smaller cylinder holds 400 ml, what does the larger one hold?

    (d) If the larger cylinder uses 180 cm^2 of material, how much does the smaller one use?

Section H: Advanced Rate Problems

31. The population of bacteria doubles every 3 hours. Starting with 500 bacteria:

    (a) Write an expression for population after t hours

    (b) How many bacteria after 12 hours?

    (c) When will the population reach 32000?

    (d) What is the rate of increase after 6 hours (bacteria per hour)?

32. A water tank is being filled and drained simultaneously. The inlet pipe fills at 20 liters/minute and the outlet drains at 12 liters/minute.

    (a) What is the net rate of filling?

    (b) How long to fill a 600-liter tank from empty?

    (c) If the tank starts half full (300 liters), when will it be full?

    (d) What size outlet pipe would balance a 25 liter/minute inlet?

33. Currency exchange rates:

    - £1 = \$1.25
    - £1 = €1.15
    - \$1 = ¥110

    (a) Convert £400 to dollars

    (b) Convert €230 to pounds

    (c) Convert \$275 to yen

(d) Find the exchange rate from euros to dollars

34. A photocopier can produce 45 copies per minute. During a busy period:

- Morning: runs for 2.5 hours at 80% capacity
- Afternoon: runs for 3 hours at full capacity
- Evening: runs for 1.5 hours at 60% capacity

(a) Calculate total copies produced
(b) Find the average rate over the whole day
(c) How long would it take to produce the same number of copies at full capacity?

Section I: Problem Solving and Applications

35. A recipe for 6 people uses:

- 450g flour
- 300ml milk
- 3 eggs
- 75g butter

(a) Adapt the recipe for 10 people
(b) How much of each ingredient for 4 people?
(c) If you have 1kg flour, what's the maximum number of people you can serve?

36. A school has 1200 students. The ratio of boys to girls is 7:5. Due to new admissions, 60 more girls join but no new boys.

(a) How many boys and girls were there originally?
(b) What is the new ratio of boys to girls?
(c) How many more girls would need to join to make the ratio 1:1?

37. The fuel consumption of a car varies inversely as the square of its speed for speeds over 30 mph. At 60 mph, it uses 8 liters per 100 km.

(a) Find the consumption at 40 mph
(b) At what speed does it use 12.8 liters per 100 km?
(c) Compare the fuel costs for a 200 km journey at 50 mph vs 80 mph

38. A company's profit P (in thousands) varies as the square of advertising spend A (in thousands) up to £10k, then inversely thereafter. When $A = 5$, $P = 100$. When $A = 20$, $P = 125$.

(a) Find the profit function for $A \leq 10$
(b) Find the profit function for $A > 10$
(c) What advertising spend maximizes profit?
(d) What is the maximum profit?

39. Two gears are connected. The smaller gear has 20 teeth and rotates at 300 rpm. The larger gear has 75 teeth.

(a) Find the rotation speed of the larger gear
(b) What is the gear ratio?
(c) If a third gear with 45 teeth is connected to the larger gear, find its speed
(d) Calculate the overall gear ratio from the smallest to largest gear

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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