# A Level Mechanics
# Practice Test 6: Newton's Laws of Motion

**Instructions:**
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 35 minutes

## Section A: Fundamental Force Principles [30 marks]

**Question 1 [16 marks]**

(a) Explain the difference between inertial and non-inertial reference frames. Give two examples of non-inertial frames and describe the fictitious forces that appear in each. **[6 marks]**

(b) A spacecraft accelerates at 2.5 m/s$^2$ in deep space. An astronaut of mass 75 kg releases a ball inside the spacecraft. Analyze the motion of the ball from: (i) the perspective of an observer on Earth, (ii) the perspective of the astronaut. **[6 marks]**

(c) State Newton's three laws of motion and explain how they relate to the principle of conservation of linear momentum. **[4 marks]**

**Question 2 [14 marks]** A particle of mass 2.2 kg is subject to two time-varying forces: $\vec{F}_1 = (3t^2 - 5)\hat{i} + (4t + 2)\hat{j}$ N and $\vec{F}_2 = (-2t^2 + 8t)\hat{i} + (t^2 - 6)\hat{j}$ N, where t is time in seconds.

(a) Determine the resultant force and acceleration vectors as functions of time. **[4 marks]**

(b) Given that the particle starts from rest at position $(-1, 2)$ m, find its velocity at t = 3 s. **[5 marks]**

(c) Calculate the position vector at t = 3 s. **[5 marks]**

## Section B: Static Equilibrium and Moments [42 marks]

**Question 3 [26 marks]** A non-uniform plank of mass 20 kg and length 8 m has its center of mass 3.2 m from the left end. The plank is supported by three vertical strings: one at each end and one at 3 m from the left end. Point loads of 15 kg, 8 kg, and 12 kg are placed at 1.5 m, 4.5 m, and 6.5 m from the left end respectively.

(a) Set up the force equilibrium equation and the moment equilibrium equation about the left end. **[4 marks]**

(b) If the tension in the middle string is 120 N, calculate the tensions in the end strings. **[8 marks]**

(c) Determine where a 25 kg mass should be placed to make the tension in the right-end string equal to 150 N. **[7 marks]**

(d) Find the minimum additional mass that must be added at the right end to make the left-end string slack. **[4 marks]**

(e) Verify your answer to part (b) by taking moments about the middle support point. **[3 marks]**

**Question 4 [16 marks]** A force system consists of: $\vec{F_1} = 35\hat{i} - 20\hat{j}$ N at point $(2, 4)$ m, $\vec{F_2} = -28\hat{i} + 45\hat{j}$ N at point $(4, 2)$ m, $\vec{F_3} = 12\hat{i} - 30\hat{j}$ N at point $(1, 3)$ m, and $\vec{F_4} = -19\hat{i} + 5\hat{j}$ N at point $(3, 1)$ m.

(a) Show that this force system is in equilibrium. **[3 marks]**

(b) Calculate the total moment about the origin and verify moment equilibrium. **[6 marks]**

(c) If $\vec{F_1}$ and $\vec{F_4}$ are removed, determine the single force and its location needed to maintain equilibrium. **[7 marks]**

## Section C: Friction and Motion Analysis [38 marks]

**Question 5 [22 marks]** Three blocks of masses 5 kg, 7 kg, and 9 kg are placed in contact on a horizontal surface. The coefficient of friction between the 5 kg block and surface is 0.4, between the 7 kg block and surface is 0.3, and between the 9 kg block and surface is 0.2.

(a) A horizontal force F is applied to the 5 kg block. Find the maximum value of F for which all blocks move together without slipping. **[8 marks]**

(b) If F = 45 N is applied to the 7 kg block, determine whether the blocks move together or separately, and calculate their accelerations. **[8 marks]**

(c) If F = 70 N is applied to the 9 kg block, analyze the resulting motion and find all contact forces. **[6 marks]**

**Question 6 [16 marks]** A crate of mass 18 kg rests on an inclined conveyor belt at angle $35^\circ$ to the horizontal. The coefficients of friction are $\mu_s = 0.7$ and $\mu_k = 0.5$.

(a) Determine whether the crate will slide down the stationary belt. **[4 marks]**

(b) The belt moves upward at constant speed. Find the minimum coefficient of static friction needed to prevent the crate from sliding down. **[4 marks]**

(c) A force F is applied parallel to the incline to push the crate up. Calculate the minimum F required to initiate upward motion. **[4 marks]**

(d) If F = 120 N is applied at $25^\circ$ above the incline surface, find the acceleration of the crate. **[4 marks]**

## Section D: Connected Systems and Pulleys [44 marks]

**Question 7 [26 marks]** A system consists of four masses connected by inextensible strings: masses of 4 kg and 6 kg lie on a rough horizontal table ($\mu = 0.2$) connected by a string. The 6 kg mass is also connected over a smooth pulley to a 8 kg mass hanging vertically. Separately, a 3 kg mass on the table is connected over another smooth pulley to a 5 kg hanging mass.

(a) Draw clear free body diagrams for all five masses. **[8 marks]**

(b) Write down the equations of motion for each mass. **[8 marks]**

(c) Calculate the acceleration of each part of the system. **[6 marks]**

(d) Find the tension in each string. **[4 marks]**

**Question 8 [18 marks]** A locomotive of mass 180,000 kg pulls four railway cars with masses 35,000 kg, 42,000 kg, 38,000 kg, and 45,000 kg. The locomotive provides a driving force of 220,000 N, and each vehicle experiences a resistance force of 800 N per 1000 kg of mass.

(a) Calculate the acceleration of the train on level track. **[4 marks]**

(b) Determine the tension in the coupling between each pair of vehicles. **[8 marks]**

(c) When climbing a 1.8° incline, find the new acceleration and check if any coupling becomes slack. **[6 marks]**

## Section E: Circular Motion and Banking [36 marks]

**Question 9 [22 marks]** A circular race track has radius 180 m and is banked at an angle of 18°. The coefficient of friction between tires and track varies from 0.15 (wet) to 0.9 (dry).

(a) Find the speed at which a car can negotiate the curve without relying on friction. **[4 marks]**

(b) Calculate the maximum safe speed in wet conditions ($\mu = 0.15$). **[6 marks]**

(c) Determine the minimum speed in dry conditions ($\mu = 0.9$) below which a car would slip down the banking. **[6 marks]**

(d) A 1200 kg car travels at 35 m/s with $\mu = 0.6$. Calculate all forces acting on the car and verify it can safely navigate the curve. **[6 marks]**

**Question 10 [14 marks]** A mass of 0.8 kg is attached to four equal strings, each of length 1.5 m, connected to four points forming a square of side 2 m in a horizontal plane. The mass rotates in a horizontal circle 1.2 m below this plane.

(a) Calculate the radius of circular motion and the angle each string makes with the vertical. **[4 marks]**

(b) Find the tension in each string. **[5 marks]**

(c) Determine the angular velocity and period of rotation. **[5 marks]**

## Section F: Variable Force Dynamics [30 marks]

**Question 11 [18 marks]** A particle of mass 3.5 kg moves along a straight line under the influence of force $F = 18\sin(0.4t) + 12\cos(0.8t) - 6e^{-0.4t} + 15$ N, where t is in seconds.

(a) Starting from rest at the origin, derive the velocity and displacement as functions of time. **[10 marks]**

(b) Find the velocity and displacement at t = $\pi$ seconds. **[4 marks]**

(c) Determine the first time after t = 0 when the particle momentarily comes to rest. **[4 marks]**

**Question 12 [12 marks]** A high-speed elevator operates in three phases: acceleration phase with $a_1(t) = 3.5 - 0.3t$ m/s² for 0 t 6 s, constant velocity phase for 12 s, then deceleration phase with $a_3(t) = -1.8 + 0.15(t - 18)$ m/s² until it stops.

(a) Calculate the velocity profile and total distance traveled. **[7 marks]**

(b) For a 70 kg passenger, find the maximum and minimum normal forces from the floor. **[3 marks]**

(c) Sketch the variation of the passenger's apparent weight throughout the journey. **[2 marks]**

## Section G: Advanced Constraint Motion [30 marks]

**Question 13 [30 marks]** A bead of mass 0.4 kg slides along a smooth wire bent into the shape described by the parametric curve $\vec{r}(u) = 2u\hat{i} + 0.2u^2\hat{j} + 0.08u^3\hat{k}$ meters, where u is a parameter. The bead starts from rest at u = 0.

(a) Express the position vector in terms of arc length s and find the relationship between u and s. **[6 marks]**

(b) Using energy conservation, find the speed when the bead reaches s = 3 m. **[5 marks]**

(c) Calculate the tangential and normal components of acceleration when s = 3 m. **[7 marks]**

(d) Determine the magnitude of the constraint force from the wire at s = 3 m. **[6 marks]**

(e) If the wire can withstand a maximum constraint force of 35 N, find where the bead would break free from the wire. **[6 marks]**

**Physics Data and Formulae**

**Newton's Laws:**
First Law: $\sum \vec{F} = 0 \Leftrightarrow \vec{a} = 0$
Second Law: $\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$
Third Law: $\vec{F}_{AB} = -\vec{F}_{BA}$
Momentum: $\vec{p} = m\vec{v}$

**Force Analysis:**
Friction: $f_s \leq \mu_s N$, $f_k = \mu_k N$
Inclined plane: $mg\sin\theta$ (parallel), $mg\cos\theta$ (normal)
Circular motion: $F_c = \frac{mv^2}{r} = m\omega^2 r$
Centrifugal force: $F_{cf} = m\omega^2 r$ (rotating frame)

**Equilibrium Analysis:**
Force equilibrium: $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$
Moment equilibrium: $\sum \vec{M} = 0$ (about any point)
Moment: $\vec{M} = \vec{r} \times \vec{F}$

**Circular Motion:**
Banking: $\tan\theta = \frac{v^2}{rg}$ (no friction)
With friction: $\frac{mv^2}{r} = mg\tan\theta \pm \mu mg\sec\theta$
Conical pendulum: $T\cos\theta = mg$, $T\sin\theta = \frac{mv^2}{r}$

**Variable Motion:**
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
$v = \int a\,dt$, $s = \int v\,dt$
Parametric motion: $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

**Constraint Forces:**
Normal force: perpendicular to surface
Tangential acceleration: $a_t = \frac{d|\vec{v}|}{dt}$
Normal acceleration: $a_n = \frac{v^2}{\rho}$ ( = radius of curvature)

**Vector Calculus:**
$\frac{d}{dt}(e^{at}) = ae^{at}$

$$\int e^{at}dt = \frac{1}{a}e^{at} + C$$
$$\int \cos(at)dt = \frac{1}{a}\sin(at) + C$$
$$\int \sin(at)dt = -\frac{1}{a}\cos(at) + C$$

**Constants:**
Acceleration due to gravity: $g = 9.8$ m/s$^2$

**Trigonometric Values:**
$\sin 18 = 0.309$, $\cos 18 = 0.951$, $\tan 18 = 0.325$
$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$
$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$
$\sin 1.8 = 0.031$, $\cos 1.8 = 0.999$, $\tan 1.8 = 0.031$

**END OF TEST**

Total marks: 270

Grade boundaries: A* 243, A 216, B 189, C 162, D 135, E 108

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