A Level Statistics
Practice Test 5: Advanced Topics

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Fundamental Concepts [25 marks]

1. **[12 marks]** Define and explain fundamental concepts:

(a) Define continuous probability distributions and their probability density functions.

(b) Explain what is meant by "cumulative distribution function" (CDF).

(c) State the relationship between PDF and CDF for continuous distributions.

(d) Define percentiles and quantiles of a distribution.

(e) Distinguish between symmetric and skewed distributions.

(f) Explain how continuous distributions model real-world measurements.

2. **[8 marks]** Explain the importance of these concepts:

(a) Why is the normal distribution called the "universal" distribution?

(b) Explain how standardization allows comparison across different normal distributions.

(c) Describe the role of continuous distributions in statistical modeling.

(d) Explain the relationship between continuous and discrete approximations.

3. **[5 marks]** Practical and theoretical context:

(a) Explain why measurement errors often follow normal distributions.

(b) Describe the role of exponential distributions in reliability analysis.

(c) Explain how uniform distributions apply to simulation and random sampling.

Section B: Normal Distribution Theory [30 marks]

4. **[15 marks]** Properties of the normal distribution:

(a) Write the probability density function of the normal distribution.

(b) Explain the parameters and and their effects on the distribution shape.

(c) Describe the standard normal distribution and its properties.

(d) State the empirical rule (68-95-99.7 rule) and explain its significance.

(e) Explain the symmetry properties of the normal distribution.

(f) Describe how to standardize normal random variables.

5. **[15 marks]** Normal distribution applications and related distributions:

(a) Explain how to find probabilities using the standard normal table.

(b) Describe the relationship between normal and chi-square distributions.

(c) Explain the t-distribution and when it's used instead of normal.

(d) Describe the F-distribution and its applications.

(e) Explain how normal distributions arise from the Central Limit Theorem.

(f) Describe the lognormal distribution and its applications.

(g) Explain transformations of normal random variables.

(h) Describe bivariate normal distributions.

(i) Explain normal approximations to other distributions.

Section C: Continuous Distribution Applications [35 marks]

6. **[18 marks]** The heights of adult males follow a normal distribution with mean = 175 cm and standard deviation = 8 cm:

(a) Calculate the probability that a randomly selected male is taller than 180 cm.

(b) Find the probability that a male's height is between 170 cm and 185 cm.

(c) Determine the height that is exceeded by only 10

(d) Calculate the probability that a male is shorter than 160 cm.

(e) Find the interquartile range of male heights.

(f) If we select 4 males randomly, find the probability that exactly 2 are taller than 180 cm.

(g) Calculate the probability that the average height of 16 randomly selected males exceeds 177 cm.

(h) Find the 95th percentile of the height distribution.

(i) Determine the range containing the middle 80

7. **[17 marks]** The time between arrivals at a service station follows an exponential distribution with mean 4 minutes:

(a) Write the probability density function for this distribution.

(b) Calculate the parameter for this exponential distribution.

(c) Find the probability that the next arrival occurs within 2 minutes.

(d) Calculate the probability that the time between arrivals exceeds 6 minutes.

(e) Find the median time between arrivals.

(f) Calculate the probability that the time is between 3 and 7 minutes.

(g) Determine the 90th percentile of inter-arrival times.

(h) Find the probability that at least one arrival occurs in the next 5 minutes.

(i) Explain the memoryless property and verify it for this distribution.

Answer Space

Use this space for your working and answers.

Formulae and Key Concepts

Normal Distribution:

$X \sim N(\mu, \sigma^2)$

PDF: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Standardization: $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

Standard Normal Distribution:

$Z \sim N(0,1)$

$P(Z \leq z) = \Phi(z)$

$P(Z \geq z) = 1 - \Phi(z)$

$P(-z \leq Z \leq z) = 2\Phi(z) - 1$

Empirical Rule:

$P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.68$

$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 0.95$

$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 0.997$

Percentiles and Quantiles:

$x_p = \mu + \sigma \cdot z_p$ where $\Phi(z_p) = p$

Median: $x_{0.5} = \mu$

IQR: $Q_3 - Q_1 = \sigma(z_{0.75} - z_{0.25}) \approx 1.35\sigma$

Exponential Distribution:

$X \sim \text{Exponential}(\lambda)$

PDF: $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$
CDF: $F(x) = 1 - e^{-\lambda x}$
Mean: $E[X] = \frac{1}{\lambda}$
Variance: $\text{Var}(X) = \frac{1}{\lambda^2}$

Memoryless Property:
$P(X > s + t | X > s) = P(X > t)$
Equivalently: $P(X > s + t) = P(X > s) \cdot P(X > t)$

Other Continuous Distributions:
Uniform[a,b]: $f(x) = \frac{1}{b-a}$, $E[X] = \frac{a+b}{2}$
Chi-square(): $E[X] = \nu$, $\text{Var}(X) = 2\nu$
t-distribution(): Similar to N(0,1) but heavier tails

Linear Transformations:
If $X \sim N(\mu, \sigma^2)$ then $Y = aX + b \sim N(a\mu + b, a^2\sigma^2)$

Sample Mean Distribution:
If $X_i \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

Common Standard Normal Values:
$\Phi(1.645) = 0.95$, $\Phi(1.96) = 0.975$, $\Phi(2.58) = 0.995$
$z_{0.90} = 1.28$, $z_{0.95} = 1.645$, $z_{0.975} = 1.96$, $z_{0.99} = 2.33$

END OF TEST

Total marks: 90

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