# A Level Mechanics
# Practice Test 6: Circular Motion

**Instructions:**
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

## Section A: Circular Motion Fundamentals [30 marks]

**Question 1** [**12 marks**]

(a) Define angular impulse and derive its relationship to change in angular momentum. [**4 marks**]

(b) Explain how the direction of angular velocity vector is determined using the right-hand rule. [**3 marks**]

(c) For a particle moving in a circle with time-varying radius, derive the expression for radial acceleration. [**3 marks**]

(d) Distinguish between uniform and non-uniform circular motion, giving the acceleration components for each. [**2 marks**]

**Question 2** [**13 marks**] A computer hard disk platter of radius 0.095 m spins up from rest according to $\omega(t) = 4t^2 + 2t$ rad/s for the first 10 seconds, then maintains constant angular velocity.

(a) Find the angular acceleration during the first 10 seconds. [**2 marks**]

(b) Calculate the final angular velocity and convert to rpm. [**3 marks**]

(c) Find the total angular displacement during acceleration. [**3 marks**]

(d) Calculate the number of complete revolutions during spin-up. [**2 marks**]

(e) Find the linear speed and acceleration of a point on the outer edge at $t = 8$ seconds. [**3 marks**]

**Question 3** [**5 marks**] A particle moves in a circle of radius 4.5 m such that its speed varies according to $v(t) = 6 + 2t$ m/s.

(a) Find expressions for angular velocity and centripetal acceleration as functions of time. [**3 marks**]

(b) Calculate the magnitude of total acceleration at $t = 3$ seconds. [**2 marks**]

## Section B: Centripetal Force [32 marks]

**Question 4 [14 marks]**

(a) Explain the concept of centripetal force as a net force requirement rather than a separate force. **[4 marks]**

(b) Derive the centripetal force formula using vector analysis of velocity change. **[5 marks]**

(c) Explain why objects appear to experience outward force in rotating reference frames. **[3 marks]**

(d) Give examples where each of the following provides centripetal force: gravity, tension, friction, magnetic force. **[2 marks]**

**Question 5 [18 marks]** An Olympic hammer thrower spins a 7.26 kg hammer in a circle before release. The hammer is connected by a 1.215 m wire and reaches a maximum speed of 32 m/s.

(a) Calculate the maximum tension in the wire. **[3 marks]**

(b) Find the maximum angular velocity. **[2 marks]**

(c) Calculate the maximum centripetal acceleration in terms of g. **[3 marks]**

(d) If the wire can withstand 8000 N tension, find the safety factor at maximum speed. **[2 marks]**

(e) For vertical circular motion at 28 m/s, calculate the minimum speed at the top to maintain circular motion. **[4 marks]**

(f) Find the tension at the bottom when moving at this critical speed. **[4 marks]**

## Section C: Horizontal Circular Motion [35 marks]

**Question 6 [20 marks]** A high-speed train of mass 400,000 kg takes a horizontal curve of radius 1200 m at the maximum design speed.

(a) If the rail banking provides no lateral force and the coefficient of friction is 0.15, find the maximum safe speed. **[4 marks]**

(b) Calculate the maximum centripetal acceleration. **[2 marks]**

(c) Find the total lateral force required at maximum speed. **[2 marks]**

(d) If the train travels at 60 m/s, calculate the required centripetal force. **[2 marks]**

(e) Determine the safety margin at this operational speed. **[3 marks]**

(f) Calculate the lateral acceleration experienced by passengers at 60 m/s. **[2 marks]**

(g) If improved rail technology increases friction to 0.25, find the new maximum speed. **[3 marks]**

(h) Calculate the percentage increase in maximum speed. **[2 marks]**

**Question 7 [15 marks]** A motorcycle stunt rider performs a "wall of death" act, riding horizontally around the inside of a vertical cylindrical wall of radius 6 m.

(a) If the coefficient of friction between tire and wall is 0.8, find the minimum speed to maintain horizontal circular motion. **[5 marks]**

(b) Calculate the minimum angular velocity. **[2 marks]**

(c) Find the normal force from the wall if the rider and bike have total mass 180 kg. **[3 marks]**

(d) Calculate the centripetal acceleration at minimum speed. **[2 marks]**

(e) If the rider travels at 15 m/s, find the normal and friction forces. **[3 marks]**

## Section D: Banked Curves [27 marks]

**Question 8** [**11 marks**]

(a) Explain how banking reduces the lateral force requirements on vehicle tires. [**3 marks**]

(b) For a banked curve, derive the general equation relating speed, radius, banking angle, and friction coefficient. [**8 marks**]

**Question 9** [**16 marks**] A section of the Nürburgring racing circuit has radius 150 m and is banked at 8°. Racing cars typically have tire-road friction coefficients up to 1.4.

(a) Calculate the design speed requiring no friction. [**3 marks**]

(b) A Formula 1 car of mass 740 kg travels at 55 m/s through this section. Calculate the required centripetal force. [**2 marks**]

(c) Find the normal force from the track surface. [**4 marks**]

(d) Calculate the friction force required and determine its direction. [**4 marks**]

(e) Find the theoretical maximum speed with friction coefficient 1.4. [**3 marks**]

## Section E: Vertical Circular Motion [33 marks]

**Question 10** [**15 marks**]

(a) For vertical circular motion, derive the general equation for the normal force at any angle $\theta$ measured from the bottom. [**5 marks**]

(b) Show that the difference between maximum and minimum tensions in a string during vertical circular motion is $6mg$. [**5 marks**]

(c) Derive the condition for the string to remain taut throughout the motion. [**5 marks**]

**Question 11** [**18 marks**] A test pilot flies a high-performance aircraft through a vertical loop of radius 800 m. The pilot and aircraft have combined mass 18,000 kg.

(a) Calculate the minimum speed at the top to maintain the loop trajectory. [**3 marks**]

(b) Find the minimum speed at the bottom using energy conservation. [**4 marks**]

(c) If the aircraft enters at 180 m/s, calculate the speed at the top. [**3 marks**]

(d) Find the g-force experienced by the pilot at the top for this speed. [**3 marks**]

(e) Calculate the g-force at the bottom. [**3 marks**]

(f) If the pilot can withstand maximum 9g, determine the maximum entry speed. [**2 marks**]

## Section F: Conical Pendulums [25 marks]

**Question 12** [**11 marks**]

(a) For a conical pendulum, derive the relationship between the semi-vertical angle and the square of the period. [**6 marks**]

(b) Show that the period approaches that of a simple pendulum as the angle approaches zero. [**5 marks**]

**Question 13 [14 marks]** A conical pendulum in a physics laboratory consists of a 200 g mass suspended by a 1.5 m string. The setup rotates such that the string makes 35° with the vertical.

(a) Calculate the radius of the horizontal circular path. **[2 marks]**

(b) Find the height of the mass below the pivot point. **[2 marks]**

(c) Calculate the tension in the string. **[3 marks]**

(d) Find the angular velocity, linear speed, and period. **[4 marks]**

(e) Calculate the centripetal force and verify it equals the horizontal component of tension. **[2 marks]**

(f) If the string length is increased to 2 m while maintaining the same angle, find the new period. **[1 mark]**

## Section G: Motion in a Vertical Circle - Loops [31 marks]

**Question 14 [19 marks]** A solid cylinder of mass 2 kg and radius 0.08 m rolls without slipping down a track and enters a vertical circular loop of radius 1.2 m. The moment of inertia is $I = \frac{1}{2}mr^2$.

(a) Find the minimum translational speed at the top to maintain contact. **[3 marks]**

(b) Calculate the minimum translational speed at the bottom considering rolling motion. **[6 marks]**

(c) If the cylinder starts from rest at height 3.5 m above the bottom, find its speed entering the loop. **[4 marks]**

(d) Calculate the speed at the top of the loop. **[3 marks]**

(e) Find the normal forces at the top and bottom. **[3 marks]**

**Question 12 [12 marks]** A research facility uses a vertical loop track to test the effects of high g-forces on materials. A test pod of mass 50 kg travels through a loop of radius 3 m.

(a) Find the minimum speed at the top to complete the loop. **[2 marks]**

(b) Calculate the minimum speed at the bottom. **[3 marks]**

(c) If the pod experiences 8g at the bottom, find its speed there. **[3 marks]**

(d) Calculate the corresponding speed and g-force at the top. **[4 marks]**

## Section H: Applications and Problem Solving [31 marks]

**Question 16 [17 marks]** A massive space habitat rotates to provide artificial gravity. The toroidal structure has inner radius 500 m and outer radius 600 m, rotating to simulate Earth gravity at the outer edge.

(a) Calculate the required angular velocity. **[3 marks]**

(b) Find the period of rotation in minutes. **[2 marks]**

(c) Calculate the linear speed at the outer edge. **[2 marks]**

(d) Find the artificial gravity at the inner edge. **[3 marks]**

(e) Calculate the artificial gravity at radius 550 m. **[2 marks]**

(f) For a 70 kg resident at the outer edge, find the normal force from the floor. **[2 marks]**

(g) Analyze the gravity gradient across a 2 m room spanning radially. **[3 marks]**

**Question 17 [14 marks]** A large Ferris wheel has radius 50 m and rotates with period 8 minutes. Passengers sit in gondolas that can freely pivot to remain level.

(a) Calculate the angular velocity and maximum linear speed. **[3 marks]**

(b) Find the maximum centripetal acceleration. **[2 marks]**

(c) If a passenger has mass 80 kg, calculate the variation in apparent weight during one revolution. **[4 marks]**

(d) Find the angle the gondola makes with the vertical at the highest point if it were fixed. **[3 marks]**

(e) Calculate the minimum and maximum normal forces from the seat. **[2 marks]**

## Section I: Advanced Circular Motion [29 marks]

**Question 18 [16 marks]** A double-banked curve system has an inner curve (radius 120 m, banking 20°) and outer curve (radius 160 m, banking 15°) allowing different speed lanes.

(a) Calculate the design speed for each curve (no friction). **[4 marks]**

(b) A high-speed vehicle travels at 45 m/s. Compare the friction requirements for each lane. **[6 marks]**

(c) With friction coefficient 0.7, find the maximum safe speed for each lane. **[4 marks]**

(d) Determine which lane is preferable for different speed ranges. **[2 marks]**

**Question 19 [13 marks]** A particle slides on the inside of a smooth conical funnel. The funnel makes a constant angle of 30° with the horizontal, and the particle moves in a horizontal circle.

(a) Analyze the forces and derive the relationship between speed and radius. **[6 marks]**

(b) If the particle moves at radius 0.4 m, calculate its speed. **[3 marks]**

(c) Find the normal force from the funnel surface if the particle has mass 0.3 kg. **[2 marks]**

(d) Calculate the angular velocity and period. **[2 marks]**

## Section J: Comprehensive Applications [32 marks]

**Question 20 [19 marks]** A high-g centrifuge for pilot training consists of a 15 m arm with a cockpit at the end. The system can produce up to 12g acceleration.

(a) Calculate the maximum angular velocity. **[3 marks]**

(b) Find the maximum linear speed of the cockpit. **[2 marks]**

(c) Calculate the minimum period of rotation. **[2 marks]**

(d) If a 75 kg pilot experiences 10g, find the centripetal force. **[2 marks]**

(e) Calculate the corresponding angular velocity. **[2 marks]**

(f) Find the apparent weight of the pilot at 10g. **[2 marks]**

(g) If the centrifuge accelerates uniformly from rest to 10g in 45 seconds, calculate the angular acceleration. **[3 marks]**

(h) Find the total number of revolutions during this acceleration phase. [**3 marks**]

**Question 21** [**13 marks**] Three particles with masses 1 kg, 2 kg, and 3 kg are positioned at the vertices of an equilateral triangle with side length 2 m. The system rotates about an axis through the center of mass perpendicular to the plane.

(a) Find the center of mass of the system. [**4 marks**]

(b) Calculate the distance of each mass from the rotation axis. [**3 marks**]

(c) If the system rotates at 5 rad/s, find the centripetal force on each mass. [**3 marks**]

(d) Calculate the total moment of inertia and kinetic energy. [**3 marks**]

## Physics Data and Formulae

### Circular Motion:

Angular velocity: $\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$

Angular acceleration: $\alpha = \frac{d\omega}{dt}$

Tangential acceleration: $a_t = r\alpha$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Total acceleration: $a = \sqrt{a_t^2 + a_c^2}$

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

### Rotational Kinematics:

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

### Vertical Circular Motion:

At top: $T + mg = \frac{mv^2}{r}$ or $N + mg = \frac{mv^2}{r}$

At bottom: $T - mg = \frac{mv^2}{r}$ or $N - mg = \frac{mv^2}{r}$

At angle $\theta$ from bottom: $N - mg\cos\theta = \frac{mv^2}{r}$

Minimum speed at top: $v_{min} = \sqrt{gr}$

Maximum tension difference: $T_{max} - T_{min} = 6mg$

### Banking:

No friction: $\tan\theta = \frac{v^2}{rg}$

With friction up slope: $\tan\theta = \frac{v^2/rg - \mu}{1 + \mu v^2/rg}$

With friction down slope: $\tan\theta = \frac{v^2/rg + \mu}{1 - \mu v^2/rg}$

### Conical Pendulum:

$\cos\theta = \frac{g}{\omega^2 l}$, $\tan\theta = \frac{\omega^2 r}{g}$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 r$

Period: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

### Energy Conservation:

$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

Rolling motion: $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling constraint: $v = r\omega$

### Artificial Gravity:

$g_{artificial} = \omega^2 r$

For Earth gravity: $\omega = \sqrt{\frac{g}{r}}$

**Constants:**
Acceleration due to gravity: $g = 9.8$ m/s²

**Trigonometric Values:**
$\sin 8 = 0.139$, $\cos 8 = 0.990$, $\tan 8 = 0.141$
$\sin 20 = 0.342$, $\cos 20 = 0.940$, $\tan 20 = 0.364$
$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$
$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$

**END OF TEST**

Total marks: 295

Grade boundaries: A* 266, A 236, B 207, C 177, D 148, E 118

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