A Level Pure Mathematics
Practice Test 3: Coordinate Geometry in the (x, y) Plane

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Distance and Midpoint Formulas

1. Find the distance between these pairs of points:
   (a) $A(5, 1)$ and $B(1, 7)$
   (b) $C(-4, 6)$ and $D(2, -2)$
   (c) $E(0, -5)$ and $F(-3, 3)$
   (d) $G(2a, b)$ and $H(-a, 3b)$
2. Find the midpoint of the line segment joining:
   (a) $P(4, 8)$ and $Q(10, 2)$
   (b) $R(-6, 4)$ and $S(2, -8)$
   (c) $T(5k, 2k)$ and $U(-k, 4k)$
   (d) The point $(-3h, 2h)$ and $(h, -4h)$
3. The point $M(4, 1)$ is the midpoint of the line segment EF where $E(2, -3)$.
   (a) Find the coordinates of point F
   (b) Calculate the gradient of EF
   (c) Find the equation of the perpendicular bisector of EF
4. Points $A(3, 4)$, $B(7, 1)$, and $C(0, 7)$ form a triangle.
   (a) Prove that triangle ABC is isosceles
   (b) Find the coordinates of the orthocenter
   (c) Calculate the length of the altitude from B to AC
   (d) Find the equation of the circumcircle
5. The points $P(1, 3)$, $Q(5, 6)$, $R(2, 10)$, and $S(-2, 7)$ form a quadrilateral.
   (a) Prove that $PQRS$ is a rectangle
   (b) Find the coordinates of the center of the rectangle
   (c) Calculate the length of the diagonal
   (d) Find the area of rectangle $PQRS$

Section B: Equations of Straight Lines

6. Find the equation of the straight line:
   (a) With gradient $\frac{5}{2}$ passing through $(4, -1)$
   (b) Passing through $(-1, 5)$ and $(4, -2)$
   (c) With x-intercept -3 and y-intercept 5
   (d) Perpendicular to $5x + 2y = 15$ and passing through $(3, -4)$
7. Express these equations in intercept form $\frac{x}{a} + \frac{y}{b} = 1$:
   (a) $2x + 3y = 12$
   (b) $4x - 5y = 20$
   (c) $3x + 4y - 24 = 0$
   (d) $x - 2y + 8 = 0$
8. Find the equation of the line that:
   (a) Is parallel to $2x - 3y = 6$ and passes through $(2, -1)$
   (b) Is perpendicular to $x + 3y = 9$ and has y-intercept 4
   (c) Passes through $(5, 3)$ and makes a $45°$ angle with the positive x-axis
   (d) Bisects the angle between the lines $3x + 4y = 12$ and $4x - 3y = 16$
9. Two lines have equations $L_1 : 2x - 5y + 10 = 0$ and $L_2 : 3x + 4y - 12 = 0$.
   (a) Find the point of intersection of L_1 and L_2
   (b) Find the equation of the line through the origin parallel to L_1
   (c) Calculate the distance between the parallel lines L_1 and $2x - 5y = 0$
   (d) Find the reflection of the point $(1, 2)$ in line L_1
10. A triangle has vertices at $A(3, 1)$, $B(1, 5)$, and $C(6, 4)$.
    (a) Find the equation of the median from A to side BC
    (b) Find the equation of the angle bisector from B
    (c) Find the incenter of the triangle
    (d) Calculate the inradius of triangle ABC

Section C: Angle Between Lines

11. Calculate the acute angle between these pairs of lines:
    (a) $y = \frac{1}{2}x + 3$ and $y = -3x + 1$
    (b) $3x + y = 7$ and $x - 2y = 4$
    (c) $5x - 2y + 1 = 0$ and $3x + 4y - 2 = 0$
    (d) $y = \sqrt{3}x - 2$ and the x-axis
12. A line passes through $(3, 2)$ and makes an angle of $135°$ with the positive x-axis.
    (a) Find the equation of the line
    (b) Find where this line intersects the line $x - 2y = 4$
    (c) Calculate the angle this line makes with $x - 2y = 4$
13. Two lines intersect at $(1, 3)$ at an angle of $60°$. If one line has equation $y = 2x + 1$:

(a) Find the gradients of the possible second lines
(b) Write the equations of both possible second lines
(c) Determine which line has the steeper gradient

14. Find the equations of the lines through $(2, 1)$ that make an angle of $45°$ with the line $3x+4y = 12$.
    (a) Calculate the gradient of the given line
    (b) Use the angle formula to find the two possible gradients
    (c) Write both line equations
    (d) Verify your answers by checking the angles

Section D: Equation of a Circle

15. Write the equation of the circle with:
    (a) Center $(0, 0)$ and radius $\sqrt{13}$
    (b) Center $(5, -4)$ and radius 3
    (c) Center $(-3, 2)$ and passing through $(1, 5)$
    (d) Diameter with endpoints $(3, 2)$ and $(-1, 6)$
16. Express these equations in standard form and find the center and radius:
    (a) $x^2 + y^2 - 10x + 8y + 16 = 0$
    (b) $x^2 + y^2 + 6x - 4y - 12 = 0$
    (c) $x^2 + y^2 - 2x + 6y - 15 = 0$
    (d) $4x^2 + 4y^2 - 16x + 24y - 12 = 0$
17. A circle has center $(3, -1)$ and passes through the point $(6, 3)$.
    (a) Find the equation of the circle
    (b) Find the points where the circle intersects the line $y = 2x - 7$
    (c) Find the equation of the chord joining these intersection points
    (d) Calculate the length of this chord
18. Two circles $C_1 : x^2 + y^2 - 4x + 2y - 20 = 0$ and $C_2 : x^2 + y^2 + 6x - 8y + 9 = 0$:
    (a) Find the centers and radii of both circles
    (b) Determine the relationship between the circles
    (c) Find their points of intersection
    (d) Find the equation of their common chord
19. Find the equation of the circle that passes through $(1, 2)$, $(3, 6)$, and $(5, 2)$.
    (a) Use the general form $x^2 + y^2 + 2gx + 2fy + c = 0$
    (b) Solve for g, f, and c
    (c) Convert to standard form
    (d) Verify that the triangle is not right-angled at any vertex

Section E: Parabolas

20. For parabolas with vertex at the origin:
    (a) Find the focus and directrix of $y^2 = 20x$
    (b) Find the focus and directrix of $x^2 = -8y$
    (c) Find the equation if the focus is at $(-3, 0)$
    (d) Sketch $y^2 = -12x$ showing focus and directrix

21. A parabola has vertex at $(2, 3)$ and focus at $(2, 6)$.
    (a) Find the equation of the parabola
    (b) Find the equation of the directrix
    (c) Find the points where the parabola intersects the line $y = 7$
    (d) Calculate the length of the focal chord through $(2, 7)$

22. The parabola $y = ax^2 + bx + c$ has vertex at $(2, -1)$ and passes through $(0, 7)$.
    (a) Find the values of a, b, and c
    (b) Express in the form $y = a(x - h)^2 + k$
    (c) Find where the parabola intersects the x-axis
    (d) Find the equation of the axis of symmetry

23. A parabolic bridge arch has equation $(x - 10)^2 = -20(y - 25)$ where distances are in meters.
    (a) Identify the vertex of the arch
    (b) Find the focus of the parabola
    (c) Calculate the width of the arch at ground level (y = 0)
    (d) Find the height of the arch at x = 5 meters from the center

Section F: Ellipses

24. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:
    (a) When $a = 7$ and $b = 3$, find the foci and eccentricity
    (b) If $a = 5$ and the eccentricity is $\frac{4}{5}$, find b
    (c) Find the equation if the foci are at $(\pm 2, 0)$ and vertices at $(\pm 3, 0)$
    (d) Sketch $\frac{x^2}{9} + \frac{y^2}{25} = 1$

25. An ellipse has center at the origin, passes through $(3, 0)$ and $(0, 4)$.
    (a) Find the equation of the ellipse
    (b) Calculate the eccentricity
    (c) Find the coordinates of the foci
    (d) Find the area of the ellipse

26. The ellipse $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{25} = 1$ has center at $(3, -2)$.
    (a) Find the coordinates of the vertices
    (b) Calculate the coordinates of the foci
    (c) Find the equations of the major and minor axes
    (d) Calculate the length of the latus rectum

27. An ellipse has foci at $(2, 1)$ and $(8, 1)$ and the sum of distances from any point on the ellipse to the foci is 10.
    (a) Find the center of the ellipse
    (b) Calculate the values of a and c
    (c) Find the value of b
    (d) Write the equation of the ellipse

Section G: Hyperbolas

28. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:
    (a) When $a = 5$ and $b = 12$, find the foci and eccentricity
    (b) Find the asymptotes when $a = 3$ and $b = 4$
    (c) If the eccentricity is $\frac{5}{4}$ and $a = 4$, find b
    (d) Sketch $\frac{y^2}{16} - \frac{x^2}{9} = 1$
29. A hyperbola has equation $\frac{x^2}{9} - \frac{y^2}{7} = 1$.
    (a) Find the vertices and foci
    (b) Write the equations of the asymptotes
    (c) Calculate the eccentricity
    (d) Find the length of the latus rectum
30. For the rectangular hyperbola $xy = k$:
    (a) When $k = 24$, find where it intersects the line $x + y = 10$
    (b) Find the equation of the tangent to $xy = 25$ at $(5, 5)$
    (c) Show that the product of the intercepts made by any tangent on the axes is constant
    (d) Find the equation of the normal at the point $(t, \frac{k}{t})$
31. A hyperbola has center at $(1, 2)$, one vertex at $(4, 2)$, and eccentricity $\frac{3}{2}$.
    (a) Find the values of a and c
    (b) Calculate the value of b
    (c) Write the equation of the hyperbola
    (d) Find the coordinates of both foci

Section H: Mixed Conic Sections

32. Classify these conic sections and find their key features:
    (a) $16x^2 + 9y^2 = 144$
    (b) $x^2 - 9y^2 = 36$
    (c) $y^2 = 24x$
    (d) $x^2 + y^2 - 4x + 6y - 12 = 0$
33. For the general conic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$:
    (a) Classify: $x^2 + 2xy + y^2 - 4x - 4y + 1 = 0$
    (b) Classify: $2x^2 - 3xy + 2y^2 - 6x - 2y + 4 = 0$

(c) What is the discriminant $B^2 - 4AC$ for each?
(d) Rotate axes to eliminate the xy term in the first equation

34. Find intersection points:
   (a) Line $y = x + 2$ and circle $x^2 + y^2 = 8$
   (b) Line $y = 4$ and parabola $x^2 = 8y$
   (c) Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and hyperbola $\frac{x^2}{4} - \frac{y^2}{12} = 1$
   (d) Circle $x^2 + y^2 = 13$ and rectangular hyperbola $xy = 6$

35. Find tangent equations:
   (a) To circle $(x - 2)^2 + (y + 1)^2 = 10$ at point $(3, 2)$
   (b) To parabola $x^2 = 20y$ at point $(10, 5)$
   (c) To ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ at point $(3\sqrt{2}, 2\sqrt{2})$
   (d) To hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$ at point $(5\sqrt{2}, 3)$

Section I: Applications and Problem Solving

36. An elliptical garden has major axis 24 meters and minor axis 18 meters. A sprinkler system needs to be installed.
   (a) Find the equation of the ellipse with center at origin
   (b) Calculate the locations of the foci
   (c) If sprinklers are placed at the foci, what is the maximum coverage radius needed?
   (d) Find the area of the elliptical garden

37. A parabolic solar collector has diameter 4 meters and depth 0.8 meters at the center.
   (a) Find the equation of the parabola
   (b) Calculate the focal length
   (c) Where should the receiver tube be positioned?
   (d) If the collector tracks the sun by rotating about its axis, describe the locus of the receiver

38. A hyperbolic cooling tower has its narrowest section at height 40 meters with radius 15 meters. At ground level, the radius is 25 meters, and at the top (height 100 meters), the radius is 20 meters.
   (a) Set up coordinates with the narrow section at the origin
   (b) Find the equation of the hyperbola
   (c) Calculate the radius at height 20 meters above the narrow section
   (d) Find the equation of the asymptotes

39. A planet follows an elliptical orbit with the sun at one focus. The closest distance to the sun is 150 million km (periapsis) and the farthest is 200 million km (apoapsis).
   (a) Calculate the semi-major axis a
   (b) Find the distance from center to focus c
   (c) Calculate the semi-minor axis b
   (d) Find the eccentricity of the orbit

40. A radar system uses hyperbolic navigation. Two stations A and B are 120 km apart. A ship receives signals with a time difference of 0.0004 seconds.

    (a) Calculate the difference in distances to the two stations

    (b) Set up coordinates with stations at $(\pm 60, 0)$

    (c) Find the equation of the hyperbola on which the ship lies

    (d) If a third station at $(0, 80)$ gives another time measurement, explain how to find the ship's exact position

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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