GCSE Higher Mathematics
Practice Test 8: Ratio, Proportion, and Rates of Change

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Ratios

1. Express these ratios in their simplest form:
   (a) $8.4 : 5.6 : 2.8$
   (b) $\frac{7}{12} : \frac{5}{16} : \frac{3}{32}$
   (c) $0.54 : 0.36 : 0.18$
   (d) $2\frac{3}{4} : 1\frac{5}{8} : 3\frac{1}{8}$

2. A technology startup allocates equity among founders, employees, investors, and advisors in the ratio $8 : 5 : 11 : 1$. If the total equity is worth £200,000:
   (a) How much equity does each group receive?
   (b) If a bonus equity pool of £50,000 is distributed in the same ratio, how much extra does each group get?
   (c) What percentage of the total equity do the investors receive?

3. The exterior angles of an irregular heptagon are in the ratio $3 : 4 : 5 : 6 : 7 : 8 : 12$. Find each exterior angle.

4. A smoothie blend mixes apple juice, orange juice, and mango juice in the ratio $7 : 9 : 4$ by volume.
   (a) How much of each juice is needed for 400ml of smoothie?
   (b) If 135ml of orange juice is used, find the volume of apple and mango juice needed
   (c) What percentage of the smoothie is apple juice?

5. Distribute £2700 among three research projects so that Project A gets £300 more than Project B, and Project B gets £150 more than Project C.

Section B: Direct Proportion

6. y is directly proportional to x. When $x = 42$, $y = 35$.
   (a) Find the equation connecting y and x
   (b) Calculate y when $x = 54$

(c) Calculate x when $y = 45$

(d) Sketch the graph of y against x

7. P varies directly as the square of Q. When $Q = 9$, $P = 243$.

   (a) Express P in terms of Q

   (b) Find P when $Q = 12$

   (c) Find Q when $P = 507$

   (d) What happens to P when Q is increased by 200%?

8. The cost of landscaping varies directly as the area of the garden. A 60 m^2 garden costs £1800.

   (a) Find the cost per m^2

   (b) How much does an 85 m^2 garden cost?

   (c) What area can be landscaped for £2250?

   (d) Express the cost C in terms of area A

9. The kinetic energy of a moving object varies directly as the square of its velocity. At 20 m/s, an object has 800 joules of kinetic energy.

   (a) Find the kinetic energy at 30 m/s

   (b) At what velocity does the object have 1250 joules?

   (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x. When $x = 18$, $y = 24$.

    (a) Find the equation connecting y and x

    (b) Calculate y when $x = 27$

    (c) Calculate x when $y = 32$

    (d) Sketch the graph of y against x

11. The time to render a video is inversely proportional to the computer's processing power. With 8 cores, it takes 45 minutes to render.

    (a) How long with 12 cores?

    (b) How many cores are needed to render in 30 minutes?

    (c) Express time T in terms of cores C

    (d) If the computer loses 2 cores due to overheating, how much longer does rendering take?

12. V is inversely proportional to the square of W. When $W = 10$, $V = 36$.

    (a) Express V in terms of W

    (b) Find V when $W = 15$

    (c) Find W when $V = 25$

    (d) What happens to V when W is increased by 50%?

13. The atmospheric pressure is inversely proportional to the square of the altitude above sea level. At 2 km altitude, the pressure is 81 kPa.

    (a) Find the pressure at 3 km altitude

    (b) At what altitude is the pressure 64 kPa?

    (c) Write the relationship as an equation

Section D: Combined Proportion

14. n varies directly as p and inversely as q. When $p = 36$ and $q = 24$, $n = 45$.

    (a) Express n in terms of p and q

    (b) Find n when $p = 48$ and $q = 18$

    (c) Find q when $p = 54$ and $n = 40.5$

    (d) What happens to n if p is increased by 25% and q is decreased by 20%?

15. The tidal force F on a moon varies directly as the planet's mass M and inversely as the cube of the distance d between them. When $M = 8$ units and $d = 2$ units, $F = 4$ units.

    (a) Express F in terms of M and d

    (b) Find F when $M = 27$ units and $d = 3$ units

    (c) Find d when $F = 16$ units and $M = 64$ units

    (d) What happens to F if M is doubled and d is tripled?

16. The capacitance C of a parallel plate capacitor varies directly as the area A of the plates and inversely as the distance d between them. When $A = 12$ cm^2 and $d = 3$ mm, $C = 8$ picofarads.

    (a) Express C in terms of A and d

    (b) Find C when $A = 18$ cm^2 and $d = 2$ mm

    (c) Find d when $C = 15$ picofarads and $A = 9$ cm^2

17. u varies directly as v^3 and inversely as w^2. When $v = 4$ and $w = 6$, $u = 32$.

    (a) Find the constant of proportionality

    (b) Express u in terms of v and w

    (c) Calculate u when $v = 6$ and $w = 4$

Section E: Speed, Distance, and Time

18. A submarine travels 285 km in 3 hours 48 minutes. Calculate:

    (a) The average speed in km/h

    (b) The average speed in m/s

    (c) How far it travels in 2 hours 30 minutes at this speed

    (d) How long it takes to travel 225 km at this speed

19. Convert these speeds:

    (a) 34 m/s to km/h

    (b) 117 km/h to m/s

    (c) 52 mph to km/h (use 1 mile = 1.6 km)

    (d) 104 km/h to mph

20. A pilot flies from base to destination at 240 km/h and returns at 360 km/h. The total flight time is 2.5 hours.

    (a) Find the distance from base to destination

    (b) Calculate the average speed for the whole flight

    (c) How much time was saved on the return flight?

21. Two drones start simultaneously from points 420 km apart and fly towards each other. One flies at 105 km/h and the other at 70 km/h.

    (a) When do they meet?

    (b) How far from each starting point do they meet?

    (c) What is their combined approach speed?

22. A hiker completes an 18 km trail. For the first 11 km, they hike at 4.4 km/h. For the remaining distance, they slow to 3.5 km/h.

    (a) Calculate the total time taken

    (b) Find the average speed for the whole hike

    (c) How much faster would they need to hike the second part to achieve an overall average of 4.2 km/h?

Section F: Density and Flow Rates

23. A polymer sample has volume 350 cm^3 and mass 455 g.

    (a) Calculate its density in g/cm^3

    (b) Calculate its density in kg/m^3

    (c) What mass of this polymer would have volume 560 cm^3?

    (d) What volume would 728 g of this polymer occupy?

24. Different composites have these densities:

    - Carbon fiber: 1.6 g/cm^3
    - Fiberglass: 2.0 g/cm^3
    - Kevlar: 1.4 g/cm^3

    (a) Compare the masses of 125 cm^3 of each composite

    (b) What volume of carbon fiber has the same mass as 175 cm^3 of kevlar?

    (c) A component weighs 420 g and has volume 210 cm^3. Which composite is it most likely made from?

25. Chemical solution flows through a reactor at a rate of 8.4 liters per minute.

    (a) How much solution flows in 28 minutes?

    (b) How long to process 504 liters of solution?

    (c) Express the flow rate in cm^3 per second

    (d) If the reactor efficiency drops by 15%, what happens to the flow rate?

26. An electronics assembly line operates at different rates during shifts:

    - Morning shift (6 hours): 480 units/hour
    - Afternoon shift (6 hours): 420 units/hour
    - Evening shift (4 hours): 360 units/hour

    (a) Calculate total daily production

    (b) Find the average production rate

    (c) How long to produce 3600 units at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar nonagons have corresponding sides in the ratio 6:13.

    (a) If the smaller nonagon has area 72 cm^2, find the area of the larger nonagon

    (b) If the larger nonagon has perimeter 169 cm, find the perimeter of the smaller nonagon

    (c) Find the ratio of their areas

28. A scale model of a skyscraper is built to scale 1:250. The real skyscraper is 375 m tall and 75 m wide.

    (a) Find the dimensions of the model

    (b) If the model requires 0.72 m^2 of cladding material, how much does the real skyscraper need?

    (c) The model weighs 12 kg. Estimate the mass of the real skyscraper if made from the same material

29. A military map has scale 1:75000. Two checkpoints are 26.4 cm apart on the map.

    (a) Calculate the actual distance in km

    (b) What map distance represents 12 km?

    (c) A restricted zone covers 18.75 cm^2 on the map. Find its actual area in km^2

30. Two similar octahedra have face area ratio 9:25.

    (a) Find the ratio of their edge lengths

    (b) Find the ratio of their volumes

    (c) If the smaller octahedron has volume 216 cm^3, what is the volume of the larger octahedron?

    (d) If the larger octahedron has surface area 600 cm^2, what is the surface area of the smaller octahedron?

Section H: Advanced Rate Problems

31. A social media post's reach increases by 40% every 3 hours. Starting with 200 views:

    (a) Write an expression for views after t hours

    (b) How many views after 12 hours?

    (c) When will the post reach 5000 views?

    (d) What is the rate of increase after 9 hours (views per hour)?

32. A brewery has four input pipes and three output valves. The pipes deliver 32, 28, 24, and 36 liters/minute respectively. The valves release 26, 34, and 18 liters/minute respectively.

    (a) What is the net rate of filling when all systems operate?

    (b) How long to fill a 2280-liter brewing tank from empty?

    (c) If the largest input pipe is blocked, what is the new net rate?

    (d) What capacity would a fourth valve need to exactly balance all inputs?

33. Currency exchange rates:

    - £1 = \$1.35
    - £1 = €1.21
    - \$1 = ¥118

(a) Convert £560 to dollars

(b) Convert €242 to pounds

(c) Convert \$189 to yen

(d) Find the exchange rate from euros to dollars

34. A renewable energy plant operates at variable output:

- Startup phase: 1.5 hours at 45% output (full output is 300 MW/hour)
- Peak generation: 8 hours at full output
- Maintenance phase: 0.5 hours at 25% output

(a) Calculate total energy generated

(b) Find the average generation rate over the whole cycle

(c) How long would it take to generate the same energy at 80% output?

Section I: Problem Solving and Applications

35. A pasta sauce recipe for 8 servings uses:

- 640g tomatoes
- 240ml olive oil
- 32g garlic
- 16g herbs

(a) Adapt the recipe for 14 servings

(b) How much of each ingredient for 5 servings?

(c) If you have 1.6 kg tomatoes, what's the maximum number of servings you can make?

36. A music festival has 5400 tickets. The ratio of general admission to VIP to backstage passes is 15:4:1. Due to demand, 270 general admission tickets are converted to VIP tickets.

(a) How many tickets of each type were there originally?

(b) What is the new ratio of general admission to VIP to backstage passes?

(c) How many backstage passes would need to be created to restore the original ratio?

37. The data transfer rate of a wireless connection varies inversely as the square of the distance from the router. At 5 meters, the rate is 72 Mbps.

(a) Find the transfer rate at 8 meters

(b) At what distance is the rate 32 Mbps?

(c) Compare transfer rates at 3 meters and 9 meters

38. An aviation company's fuel cost F (in thousands) varies as the square of flight hours H up to 1000 hours, then increases at a constant rate. When $H = 20$, $F = 80$. When $H = 1200$, $F = 5200$.

(a) Find the cost function for $H \leq 1000$

(b) Find the cost function for $H > 1000$

(c) What number of flight hours minimizes cost per hour for $H \leq 1000$?

(d) What is the constant rate of increase for hours beyond 1000?

39. A precision instrument has a gear system with nine gears. Gear 1 has 36 teeth and rotates at 180 rpm. The subsequent gears have 24, 54, 18, 72, 12, 48, 16, and 60 teeth respectively, connected in sequence.

    (a) Find the rotation speed of each gear

    (b) What is the overall gear ratio from gear 1 to gear 9?

    (c) If gear 1's speed increases to 270 rpm, find the new speed of gear 9

    (d) How would the final speed change if gear 7 had 36 teeth instead of 48?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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