GCSE Higher Mathematics
Practice Test 7: Further Algebra

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Function Notation and Evaluation

1. Given $f(x) = 9x + 6$ and $g(x) = x^2 - 8x$, find:
   (a) $f(4)$
   (b) $g(-3)$
   (c) $f(0)$
   (d) $g(6a)$
   (e) $f(x + 6)$
   (f) $g(x - 5)$

2. For the function $h(x) = 6x^2 - 7x + 4$, calculate:
   (a) $h(1)$
   (b) $h(-2)$
   (c) $h(a - 3)$
   (d) $h(6t)$
   (e) The value(s) of x when $h(x) = 12$
   (f) The value(s) of x when $h(x) = 0$

3. Given $f(x) = \frac{8x-7}{x+6}$ where $x \neq -6$:
   (a) Find $f(2)$
   (b) Find $f(-3)$
   (c) For what value of x is $f(x) = 6$?
   (d) For what value of x is $f(x) = 0$?
   (e) Explain why $x = -6$ is excluded from the domain
   (f) Find the range of values that $f(x)$ cannot take

4. A function is defined as $p(x) = x^3 - 18x + 12$.
   (a) Calculate $p(0)$, $p(1)$, $p(2)$, and $p(-1)$
   (b) Use your results to sketch the graph of $y = p(x)$
   (c) Estimate the roots of $p(x) = 0$
   (d) For what values of k does $p(x) = k$ have three real solutions?

Section B: Composite Functions

5. Given $f(x) = 6x - 5$ and $g(x) = x^2 + 6$, find:
   (a) $f(g(2))$
   (b) $g(f(2))$
   (c) $f(g(x))$
   (d) $g(f(x))$
   (e) $(f \circ g)(x)$
   (f) $(g \circ f)(x)$
6. For $h(x) = 9x + 4$ and $k(x) = \frac{x-4}{9}$:
   (a) Find $h(k(x))$
   (b) Find $k(h(x))$
   (c) What do you notice about your answers?
   (d) Verify that $h(k(22)) = 22$
   (e) Explain the relationship between functions h and k
7. Given $f(x) = x^2 - 5$ and $g(x) = \sqrt{x+5}$ where $x \geq -5$:
   (a) Find the domain of $g(x)$
   (b) Calculate $f(g(4))$
   (c) Calculate $g(f(3))$
   (d) Find $f(g(x))$ and simplify
   (e) Find $g(f(x))$ and state its domain
   (f) Solve $f(g(x)) = 31$
8. If $f(x) = x - 6$, $g(x) = 8x$, and $h(x) = x^2$:
   (a) Find $f(g(h(x)))$
   (b) Find $h(g(f(x)))$
   (c) Find $g(h(f(x)))$
   (d) Calculate $f(g(h(4)))$
   (e) Solve $g(h(f(x))) = 200$

Section C: Inverse Functions

9. Find the inverse function for each of the following:
   (a) $f(x) = 9x + 7$
   (b) $g(x) = \frac{x-11}{7}$
   (c) $h(x) = 6x - 13$
   (d) $k(x) = \frac{8x+3}{10}$
10. For the function $f(x) = \frac{8x+5}{x-7}$ where $x \neq 7$:
   (a) Find $f^{-1}(x)$
   (b) State the domain and range of $f^{-1}(x)$
   (c) Verify that $f(f^{-1}(x)) = x$

(d) Verify that $f^{-1}(f(x)) = x$
(e) Solve $f(x) = f^{-1}(x)$

11. Given $g(x) = x^2 - 6$ for $x \geq 0$:
    (a) Explain why the domain restriction is necessary
    (b) Find $g^{-1}(x)$
    (c) State the domain and range of $g^{-1}(x)$
    (d) Sketch both $g(x)$ and $g^{-1}(x)$ on the same axes
    (e) Find the point of intersection of $y = g(x)$ and $y = g^{-1}(x)$

12. A function f has the property that $f(5) = 26$, $f(8) = 35$, and $f(x) = 3x + 11$.
    (a) Verify that the given points satisfy $f(x) = 3x + 11$
    (b) Find $f^{-1}(x)$
    (c) Calculate $f^{-1}(26)$ and $f^{-1}(35)$
    (d) What do you notice about these values?
    (e) If $f(a) = b$, what is $f^{-1}(b)$?

Section D: Function Transformations

13. Given the function $f(x) = x^2$, describe the transformation and sketch:
    (a) $y = f(x) + 9$
    (b) $y = f(x) - 7$
    (c) $y = f(x + 7)$
    (d) $y = f(x - 8)$
    (e) $y = 8f(x)$
    (f) $y = \frac{1}{8}f(x)$

14. The graph of $y = f(x)$ passes through the points $(0, 7)$, $(5, 4)$, and $(8, 10)$. Find the coordinates of these points on:
    (a) $y = f(x) + 10$
    (b) $y = f(x - 7)$
    (c) $y = 7f(x)$
    (d) $y = f(7x)$
    (e) $y = -f(x)$
    (f) $y = f(-x)$

15. Given $f(x) = (x + 3)^2 - 6$:
    (a) Describe the transformations applied to $y = x^2$
    (b) State the vertex of the parabola
    (c) Find $f(x - 5)$ and describe its transformation
    (d) Find $7f(x) + 4$ and describe its transformation
    (e) Sketch all four graphs on the same axes

16. The function $g(x) = |x|$ is transformed to $h(x) = 8|x - 5| + 1$.
    (a) Describe each transformation step by step
    (b) State the vertex of $h(x)$
    (c) Find the range of $h(x)$
    (d) Solve $h(x) = 17$
    (e) Sketch both $g(x)$ and $h(x)$

Section E: Exponential Functions - Basics

17. Evaluate these exponential expressions:
    (a) 8^3
    (b) 9^{-2}
    (c) $64^{0.5}$
    (d) $216^{-1.5}$
    (e) $(\frac{1}{8})^{-2}$
    (f) $512^{\frac{2}{3}}$

18. Sketch the graphs of these exponential functions:
    (a) $y = 8^x$
    (b) $y = 9^x$
    (c) $y = (\frac{1}{8})^x$
    (d) $y = (\frac{1}{9})^x$
    (e) $y = 8^x + 7$
    (f) $y = 8^{x-7}$

19. For the function $f(x) = 8^x$:
    (a) Calculate $f(0)$, $f(1)$, $f(2)$, $f(-1)$, $f(-2)$
    (b) State the domain and range of $f(x)$
    (c) Find the y-intercept
    (d) Describe the behavior as $x \to \infty$ and $x \to -\infty$
    (e) Solve $8^x = 512$
    (f) Solve $8^x = \frac{1}{64}$

20. Compare the graphs of $y = 8^x$ and $y = (\frac{1}{8})^x$:
    (a) What transformation relates these functions?
    (b) Where do they intersect?
    (c) Which grows faster for $x > 0$?
    (d) Which approaches zero faster as $x \to \infty$?
    (e) Express $(\frac{1}{8})^x$ in the form $8^{g(x)}$

Section F: Exponential Growth and Decay

21. A population of protozoa triples every 45 minutes. Initially, there are 720 protozoa.
    (a) Write a function $P(t)$ for the population after t minutes
    (b) Calculate the population after 90 minutes
    (c) Calculate the population after 2.25 hours
    (d) When will the population reach 52488?
    (e) What is the growth rate per minute?
    (f) How long for the population to increase by 800%?

22. A radioactive isotope has a half-life of 8 years. Initially, there are 640g of the isotope.

(a) Write a function $A(t)$ for the amount after t years
(b) How much remains after 16 years?
(c) How much remains after 24 years?
(d) When will only 40g remain?
(e) What percentage remains after one half-life?
(f) Calculate the decay rate per year

23. An investment of £30000 grows at 8.5% per year compound interest.
    (a) Write a function $V(t)$ for the value after t years
    (b) Calculate the value after 3 years
    (c) Calculate the value after 6 years
    (d) When will the investment double?
    (e) When will it reach £75000?
    (f) Compare with simple interest of 8.5% per year

24. The temperature of cooling tea follows Newton's law of cooling: $T(t) = 21 + 79e^{-0.25t}$ where T is temperature in °C and t is time in minutes.
    (a) What is the initial temperature?
    (b) What is the room temperature?
    (c) Find the temperature after 4 minutes
    (d) When will the temperature be 30°C?
    (e) Sketch the graph of $T(t)$
    (f) What happens as $t \to \infty$?

Section G: Advanced Exponential Applications

25. A tablet depreciates in value according to $V(t) = 32000 \times 0.65^t$ where V is value in pounds and t is age in years.
    (a) What was the original value?
    (b) What is the annual depreciation rate?
    (c) Calculate the value after 3 years
    (d) When will the tablet be worth £9000?
    (e) After how many years will it lose half its value?
    (f) What percentage of value is retained each year?

26. The spread of a challenge video follows $N(t) = 560 \times 2.1^t$ where N is views (in thousands) and t is days since posting.
    (a) How many views after 2 days?
    (b) How many views after 5 days?
    (c) When will it reach 25 million views?
    (d) What is the daily growth rate?
    (e) If the growth rate drops to 45% per day after 3 days, model the new function

27. A wetland habitat decreases due to drainage. The area A (in hectares) after t years is $A(t) = 22000 \times 0.82^t$.

(a) What is the initial wetland habitat area?
(b) What percentage is lost each year?
(c) Calculate the area after 7 years
(d) When will half the habitat be gone?
(e) If restoration efforts reduce the loss to 9% per year, how does this change the model?
(f) Compare the areas after 15 years under both scenarios

28. A caffeine concentration in bloodstream follows $C(t) = 150e^{-0.35t}$ where C is concentration (mg/L) and t is hours after consumption.

(a) What is the initial concentration?
(b) Find the concentration after 1 hour
(c) When will the concentration drop to 30 mg/L?
(d) What is the half-life of caffeine?
(e) A second dose is consumed when concentration drops to 20 mg/L. When should this be?
(f) Sketch the concentration curve

Section H: Problem Solving and Integration

29. A function f is defined by $f(x) = ax + b$ where a and b are constants. Given that $f(6) = 29$ and $f(-4) = -1$:

(a) Find the values of a and b
(b) Write down $f(x)$
(c) Find $f^{-1}(x)$
(d) Solve $f(x) = f^{-1}(x)$
(e) If $g(x) = x^2$, find $f(g(x))$ and $g(f(x))$

30. Two exponential functions $p(x) = 8^x$ and $q(x) = 9^x$ intersect at the point where $x = 0$.

(a) Verify this intersection point
(b) For what values of x is $p(x) > q(x)$?
(c) Find the function $r(x) = \frac{q(x)}{p(x)}$
(d) Simplify $r(x)$ and identify what type of function it is
(e) Sketch all three functions on the same axes

31. A population model combines growth and limiting factors: $P(t) = \frac{1800}{1+17e^{-1.2t}}$ where P is population and t is time in years.

(a) Find the initial population $P(0)$
(b) Calculate $P(1)$ and $P(2)$
(c) What happens to $P(t)$ as $t \to \infty$?
(d) When will the population reach 900?
(e) Sketch the graph and describe its shape
(f) How does this differ from unlimited exponential growth?

32. A transformation maps the function $f(x) = 8^x$ to $g(x) = 7 \times 8^{x-5} + 10$.

(a) Identify each transformation in the correct order
(b) Find the y-intercept of $g(x)$

(c) Find the horizontal asymptote of $g(x)$

(d) Solve $g(x) = 66$

(e) Find $g^{-1}(x)$

(f) Verify that $g(g^{-1}(66)) = 66$

33. A savings account earns compound interest. After 1 year, £10000 becomes £10700. After 2 years, it becomes £11449.

    (a) Verify this follows exponential growth

    (b) Find the annual interest rate

    (c) Write the exponential function $A(t)$ for any initial amount P

    (d) How long to triple an investment?

    (e) Compare with quarterly compounding at the same annual rate

    (f) What continuous compound rate gives the same result?

34. Design a real-world scenario that can be modeled by an exponential function:

    (a) Describe your scenario clearly

    (b) Define variables and state assumptions

    (c) Write the exponential function

    (d) Calculate specific values and time periods

    (e) Discuss limitations of the model

    (f) Suggest modifications for greater realism

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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stepupmaths.co.uk