

GCSE Higher Mathematics

Practice Test 9: Number

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:

- (a) 12^2
- (b) 8^{-2}
- (c) 29^0
- (d) $(-12)^2$
- (e) $(-8)^3$
- (f) 18^{-2}

2. Write these as single powers:

- (a) $11^8 \times 11^{13}$
- (b) $21^{12} \div 21^6$
- (c) $(8^{10})^7$
- (d) $17^{-3} \times 17^{15}$
- (e) $\frac{23^{11}}{23^{-10}}$
- (f) $(13^8)^{-5}$

3. Evaluate these fractional indices:

- (a) $196^{\frac{1}{2}}$
- (b) $1000^{\frac{1}{3}}$
- (c) $4194304^{\frac{3}{11}}$
- (d) $1073741824^{\frac{2}{10}}$
- (e) $6859^{\frac{2}{3}}$
- (f) $8192^{-\frac{3}{13}}$

4. Simplify these expressions:

- (a) $\sqrt{324}$
- (b) $\sqrt[3]{1728}$

- (c) $\sqrt[4]{10000}$
- (d) $\sqrt{4.41}$
- (e) $\sqrt[3]{-1331}$
- (f) $\sqrt[5]{1024000}$

5. Express in index form:

- (a) $\sqrt{37}$
- (b) $\sqrt[3]{29}$
- (c) $\frac{1}{\sqrt{23}}$
- (d) $\sqrt[4]{x^{19}}$
- (e) $\frac{1}{\sqrt[3]{y^{14}}}$
- (f) $\sqrt{x} \times \sqrt[3]{x}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

- (a) $11^9 \times 11^{-12} \times 11^{13}$
- (b) $\frac{16^{14} \times 16^{-10}}{16^{-9}}$
- (c) $(13^7)^{-9} \times 13^{24}$
- (d) $\frac{17^{-10} \times 17^{21}}{17^9}$

7. Evaluate these expressions:

- (a) $6859^{\frac{2}{3}}$
- (b) $4194304^{-\frac{3}{11}}$
- (c) $10000000^{\frac{4}{7}}$
- (d) $1073741824^{-\frac{6}{30}}$
- (e) $19683^{\frac{1}{3}}$
- (f) $100000000^{-\frac{4}{8}}$

8. Simplify these expressions:

- (a) $x^{\frac{11}{12}} \times x^{\frac{1}{7}}$
- (b) $\frac{c^{\frac{13}{6}}}{c^{\frac{1}{8}}}$
- (c) $(y^{\frac{1}{10}})^{20}$
- (d) $\sqrt{x} \times x^{\frac{1}{11}}$
- (e) $\frac{\sqrt[3]{d^{14}}}{\sqrt{d}}$
- (f) $(e^{-\frac{1}{10}})^{-20}$

9. Write these in the form a^n where a and n are rational:

- (a) $\sqrt{29} \times 29^{10}$
- (b) $\frac{31^8}{\sqrt[3]{31}}$
- (c) $\sqrt[4]{18^{19}} \times 18^{-\frac{7}{8}}$
- (d) $\frac{\sqrt{37}}{\sqrt[3]{37^{14}}}$

Section C: Surds

10. Simplify these surds:

- (a) $\sqrt{116}$
- (b) $\sqrt{176}$
- (c) $\sqrt{188}$
- (d) $\sqrt{333}$
- (e) $\sqrt{363}$
- (f) $\sqrt{1200}$

11. Simplify these expressions:

- (a) $12\sqrt{29} + 19\sqrt{29}$
- (b) $23\sqrt{11} - 11\sqrt{11}$
- (c) $\sqrt{92} + \sqrt{207}$
- (d) $\sqrt{333} - \sqrt{252}$
- (e) $11\sqrt{36} + 15\sqrt{81}$
- (f) $\sqrt{333} - \sqrt{192} + \sqrt{75}$

12. Multiply and simplify:

- (a) $\sqrt{17} \times \sqrt{68}$
- (b) $\sqrt{36} \times \sqrt{144}$
- (c) $13\sqrt{8} \times 9\sqrt{32}$
- (d) $\sqrt{24} \times \sqrt{96}$
- (e) $\sqrt{11} \times \sqrt{44} \times \sqrt{176}$
- (f) $15\sqrt{10} \times 9\sqrt{40}$

13. Expand and simplify:

- (a) $(10 + \sqrt{23})(7 - \sqrt{23})$
- (b) $(9 + \sqrt{31})(6 + 11\sqrt{31})$
- (c) $(12 - \sqrt{29})^2$
- (d) $(\sqrt{37} + \sqrt{17})(\sqrt{37} - \sqrt{17})$
- (e) $(11\sqrt{23} + 1)(11\sqrt{23} - 1)$
- (f) $(\sqrt{31} + 10)^2$

14. Rationalize the denominators:

- (a) $\frac{1}{\sqrt{31}}$
- (b) $\frac{23}{\sqrt{37}}$
- (c) $\frac{\sqrt{23}}{\sqrt{92}}$
- (d) $\frac{20}{10\sqrt{2}}$
- (e) $\frac{1}{9+\sqrt{23}}$
- (f) $\frac{15}{1-\sqrt{31}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

(a) $\frac{17}{10+\sqrt{29}}$

(b) $\frac{29}{9-\sqrt{41}}$

(c) $\frac{\sqrt{23}}{1+\sqrt{23}}$

(d) $\frac{11\sqrt{31}}{9+\sqrt{31}}$

(e) $\frac{1}{\sqrt{30}-\sqrt{23}}$

(f) $\frac{\sqrt{31}+9}{\sqrt{31}-10}$

16. Simplify these expressions completely:

(a) $\frac{\sqrt{44}+\sqrt{99}}{\sqrt{11}}$

(b) $\frac{\sqrt{125}-\sqrt{100}}{\sqrt{5}}$

(c) $\sqrt{(11+\sqrt{31})(11-\sqrt{31})}$

(d) $\sqrt{188}-10\sqrt{47}+\sqrt{147}$

(e) $(\sqrt{23}+\sqrt{92})^2$

(f) $\frac{\sqrt{176}}{\sqrt{23}}+\frac{\sqrt{138}}{\sqrt{23}}$

17. Prove that:

(a) $(\sqrt{g}+\sqrt{h})(\sqrt{g}-\sqrt{h})=g-h$

(b) $\frac{1}{\sqrt{j}+\sqrt{k}}=\frac{\sqrt{j}-\sqrt{k}}{j-k}$

(c) $(j+k\sqrt{v})^2=j^2+2jk\sqrt{v}+k^2v$

Section E: Standard Form

18. Write these numbers in standard form:

(a) 1145000

(b) 0.000151

(c) 9810000000

(d) 0.0000000739

(e) 1485.7

(f) 0.01513

19. Write these in ordinary form:

(a) 1.3×10^{-2}

(b) 1.014×10^{-12}

(c) 1.4857×10^{-15}

(d) 9.81×10^{18}

(e) 1.51×10^{-10}

(f) 1.145×10^{13}

20. Calculate, giving answers in standard form:

- (a) $(15 \times 10^{12}) \times (17 \times 10^{14})$
- (b) $(11 \times 10^{-10}) \times (19 \times 10^{15})$
- (c) $(30 \times 10^{13}) \div (10 \times 10^{-8})$
- (d) $(27 \times 10^{-12}) \div (18 \times 10^{-15})$
- (e) $(15 \times 10^{11})^2$
- (f) $\sqrt{121 \times 10^{24}}$

21. Calculate these more complex expressions:

- (a) $(10.8 \times 10^{11}) \times (4.25 \times 10^{-13})$
- (b) $\frac{21.6 \times 10^{14}}{5.4 \times 10^{-11}}$
- (c) $(11.4 \times 10^{-10}) + (1.29 \times 10^{-9})$
- (d) $(14.8 \times 10^{13}) - (10.9 \times 10^{12})$
- (e) $\frac{(10.5 \times 10^9) \times (9.0 \times 10^{-10})}{15.75 \times 10^{-8}}$
- (f) $(1.3225 \times 10^{22})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

- (a) $\frac{11}{19} + \frac{15}{38}$
- (b) $\frac{29}{32} - \frac{17}{48}$
- (c) $\frac{22}{23} \times \frac{46}{33}$
- (d) $\frac{20}{37} \div \frac{30}{48}$
- (e) $\frac{17}{18} - \frac{13}{27} + \frac{23}{54}$
- (f) $(\frac{11}{19})^{-2}$

23. Convert these recurring decimals to fractions:

- (a) $0.\bar{2}$
- (b) $0.\bar{84}$
- (c) $0.9\bar{5}$
- (d) $0.\overline{857142}$
- (e) $8.3\bar{7}$
- (f) $0.84\bar{2}$

24. Work out these percentage calculations:

- (a) Increase 1080 by 95%
- (b) Decrease 1320 by 48%
- (c) Find 52.5% of 1480
- (d) What percentage is 217 out of 280?
- (e) If 115% of a number is 299, find the number
- (f) A price increases from £165 to £181.5. Find the percentage increase

25. Solve these percentage problems:

- (a) After a 100% increase, a price is £398. Find the original price
- (b) After a 95% decrease, a quantity is 180. Find the original quantity
- (c) The value of a car decreases by 60% each year. If it's worth £4320 now, what was it worth 2 years ago?
- (d) An investment grows by 15% per year. After 2 years it's worth £2645.25. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

(a) $11^{-2} + 17^0 - 18^{-1}$

(b) $\sqrt{144} \times 1728^{\frac{1}{3}} - 11^{-2}$

(c) $\frac{6859^{\frac{2}{3}} - 4194304^{\frac{3}{11}}}{169^{\frac{1}{2}}}$

(d) $100^{-\frac{1}{2}} + 169^{\frac{1}{2}} \times 9^{-1}$

27. Calculate exactly (leave surds in your answer):

(a) $\frac{17}{\sqrt{23}} + \frac{13}{\sqrt{92}}$

(b) $\sqrt{81} \times \sqrt{144} - \sqrt{324}$

(c) $\frac{\sqrt{294} + \sqrt{192}}{\sqrt{98}}$

(d) $(11\sqrt{3} - 9)^2$

28. Work with standard form in context:

(a) The mass of a lithium atom is 1.152×10^{-26} kg. Find the mass of 6.02×10^{23} lithium atoms

(b) Gamma rays travel at 3×10^8 m/s. How far do they travel in one century (use 1 century = 3.154×10^9 seconds)?

(c) The diameter of a protein molecule is approximately 5×10^{-9} m. How many protein molecules would fit across a distance of 9 mm?

(d) A supercomputer processes 1.152×10^{17} operations per second. How many operations in 60 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{23}$ is irrational. (Use proof by contradiction: assume $\sqrt{23} = \frac{a}{b}$ where a and b are integers with no common factors)

30. The number η satisfies $\eta^2 = 8\eta - 6$.

(a) Show that $\eta = 4 + \sqrt{10}$

(b) Calculate η to 4 decimal places

(c) Find $\frac{1}{\eta}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{19} + \sqrt{23} + \sqrt{29}}$. (Hint: First rationalize using $(\sqrt{19} + \sqrt{23}) - \sqrt{29}$)

32. A rectangle has sides of length $(10 + \sqrt{23})$ cm and $(10 - \sqrt{23})$ cm.

(a) Find the exact area

(b) Find the exact perimeter

(c) Show that the area is rational but the perimeter is irrational

33. The population of viruses decuples every 16 hours. If there are initially 17×10^2 viruses:

(a) How many viruses after 64 hours?

(b) Express your answer in standard form

(c) After how many hours will there be more than 5×10^{13} viruses?

34. Show that $\frac{1}{\sqrt{g} + \sqrt{h}} + \frac{1}{\sqrt{g} - \sqrt{h}} = \frac{2\sqrt{g}}{g - h}$

35. A cylinder has volume $V = \pi r^2 h$. If the volume is $864\pi \text{ cm}^3$ and $h = 24 \text{ cm}$:

- (a) Find the radius in surd form
- (b) Find the surface area (use $A = 2\pi r^2 + 2\pi r h$)
- (c) Express both answers exactly

36. The equation $x^2 - 20x + 1 = 0$ has solutions $x = 10 \pm 3\sqrt{11}$.

- (a) Verify this by substitution
- (b) Find $\frac{1}{10+3\sqrt{11}} + \frac{1}{10-3\sqrt{11}}$ without using a calculator
- (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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