A Level Mechanics
Practice Test 1: Rotational Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Rotational Motion Fundamentals [28 marks]

Question 1 [10 marks]

(a) Define angular displacement, angular velocity, and angular acceleration, stating their SI units. **[3 marks]**

(b) Create a comparison table showing the analogies between linear and rotational motion quantities. **[3 marks]**

(c) Derive the relationship between linear velocity and angular velocity for a point on a rotating object. **[2 marks]**

(d) State the rotational equivalent of Newton's second law of motion. **[2 marks]**

Question 2 [12 marks] A flywheel rotates with angular velocity given by $\omega(t) = 5t^2 - 8t + 3$ rad/s.

(a) Find the angular acceleration as a function of time. **[2 marks]**

(b) Calculate the angular velocity at $t = 2.5$ seconds. **[2 marks]**

(c) Determine the angular acceleration at $t = 1.5$ seconds. **[2 marks]**

(d) Find all times when the flywheel momentarily stops rotating. **[3 marks]**

(e) Calculate the angular displacement between $t = 0.5$ and $t = 3$ seconds. **[3 marks]**

Question 3 [6 marks] A point P on the rim of a rotating disc moves with tangential speed 12 m/s. The disc has radius 0.6 m and angular acceleration 2.5 rad/s².

(a) Calculate the angular velocity of the disc. **[1 mark]**

(b) Find the tangential acceleration of point P. **[1 mark]**

(c) Calculate the centripetal acceleration of point P. **[2 marks]**

(d) Determine the magnitude and direction of the total acceleration of point P. **[2 marks]**

Section B: Moment of Inertia - Point Masses [32 marks]

Question 4 [**8 marks**]

(a) Define moment of inertia and explain why it is called the "rotational inertia." [**3 marks**]

(b) Write the mathematical expression for moment of inertia of a system of point masses. [**2 marks**]

(c) State the parallel axis theorem and explain its significance. [**3 marks**]

Question 5 [**24 marks**] Calculate moments of inertia for the following systems:

(a) Five point masses: 3 kg at (4,0), 2 kg at (0,3), 4 kg at (-2,0), 1 kg at (0,-5), and 2.5 kg at (3,4) meters. Find the moment of inertia about the z-axis. [**5 marks**]

(b) Four masses of 6 kg each are positioned at the corners of a square with side length 2.4 m. Calculate the moment of inertia about an axis through the center perpendicular to the plane. [**4 marks**]

(c) Two masses of 10 kg and 15 kg are connected by a rigid rod of negligible mass and length 2 m. Find the moment of inertia about an axis perpendicular to the rod passing through: (i) the center of mass, (ii) the 10 kg mass. [**8 marks**]

(d) A uniform rod of mass 8 kg and length 2.5 m has point masses of 3 kg attached at each end. Calculate the moment of inertia about the center of the rod. [**4 marks**]

(e) Use the parallel axis theorem to verify one of your previous calculations. [**3 marks**]

Section C: Moment of Inertia - Extended Bodies [36 marks]

Question 6 [**12 marks**]

(a) State the moments of inertia formulas for: uniform rod about center, uniform rod about end, solid disc, solid sphere, and hollow sphere. [**5 marks**]

(b) Derive the moment of inertia of a uniform rod of mass M and length L about an axis through its center perpendicular to its length. [**4 marks**]

(c) Explain why a hollow cylinder has a larger moment of inertia than a solid cylinder of the same mass and outer radius. [**3 marks**]

Question 7 [**24 marks**] Calculate moments of inertia for composite objects:

(a) A uniform disc of mass 6 kg and radius 0.4 m has a concentric circular hole of radius 0.15 m cut out. Find its moment of inertia about the center. [**5 marks**]

(b) A system consists of a uniform rod (mass 3 kg, length 1.8 m) with a solid disc (mass 4 kg, radius 0.25 m) attached at one end with its center coinciding with the rod's end. Calculate the moment of inertia about the other end of the rod. [**6 marks**]

(c) A compound object consists of two solid spheres (each mass 7 kg, radius 0.2 m) connected by a thin uniform rod (mass 2 kg, length 1.2 m, with sphere centers at the rod ends). Find the moment of inertia about the center of the rod. [**7 marks**]

(d) A wheel consists of a solid disc (mass 12 kg, radius 0.5 m) with six uniform spokes, each modeled as a rod (mass 0.8 kg, length 0.5 m) extending radially. Calculate the total moment of inertia about the axle. [**6 marks**]

Section D: Torque and Rotational Dynamics [30 marks]

Question 8 [**10 marks**]

(a) Define torque and state its SI unit. Write the vector expression for torque. [**3 marks**]

(b) State the rotational form of Newton's second law and explain each term. [**3 marks**]

(c) Explain how the direction of torque is determined using the right-hand rule. [**2 marks**]

(d) Describe the relationship between net torque and angular acceleration. [**2 marks**]

Question 9 [**20 marks**] A uniform disc of mass 8 kg and radius 0.75 m can rotate freely about a fixed horizontal axis through its center.

(a) Calculate the moment of inertia of the disc. [**2 marks**]

(b) A tangential force of 20 N is applied at the rim of the disc. Find the angular acceleration. [**3 marks**]

(c) Starting from rest, calculate the angular velocity after 5 seconds. [**2 marks**]

(d) Determine the angular displacement during these 5 seconds. [**3 marks**]

(e) Calculate the work done by the applied force during this time. [**4 marks**]

(f) Find the rotational kinetic energy after 5 seconds and verify it equals the work done. [**3 marks**]

(g) Calculate the power delivered by the force at t = 5 seconds. [**3 marks**]

Section E: Rotational Energy [34 marks]

Question 10 [**12 marks**]

(a) Derive the formula for rotational kinetic energy starting from the kinetic energy of individual particles. [**4 marks**]

(b) Define the work done by a torque and write its mathematical expression. [**3 marks**]

(c) State the work-energy theorem for rotational motion. [**2 marks**]

(d) Write the expression for power in rotational systems and explain each term. [**3 marks**]

Question 11 [**22 marks**] A solid cylinder of mass 15 kg and radius 0.3 m rolls without slipping down an inclined plane of angle 25°.

(a) Calculate the moment of inertia of the cylinder about its axis. [**2 marks**]

(b) Write the constraint equation for rolling without slipping. [**2 marks**]

(c) Use energy conservation to derive the linear acceleration down the plane. [**6 marks**]

(d) Calculate the angular acceleration of the cylinder. [**2 marks**]

(e) If the cylinder starts from rest and rolls 4 m down the plane, find its final linear and angular velocities. [**4 marks**]

(f) Calculate the total kinetic energy at the bottom and verify using energy conservation. [**3 marks**]

(g) Determine what fraction of the total kinetic energy is rotational. [**3 marks**]

Section F: Angular Momentum [26 marks]

Question 12 [**8 marks**]

(a) Define angular momentum for a rotating rigid body and state its SI unit. [**2 marks**]

(b) Write the relationship between angular momentum, moment of inertia, and angular velocity. [**2 marks**]

(c) State the principle of conservation of angular momentum and the conditions under which it applies. [**4 marks**]

Question 13 [**18 marks**] A figure skater is spinning with arms extended, having moment of inertia 4.5 kg·m² and angular velocity 2.5 rad/s.

(a) Calculate the initial angular momentum of the skater. [**2 marks**]

(b) When the skater pulls in their arms, the moment of inertia reduces to 1.6 kg·m². Find the new angular velocity using conservation of angular momentum. [**3 marks**]

(c) Calculate the initial and final rotational kinetic energies. [**4 marks**]

(d) Explain the change in kinetic energy and identify its source. [**3 marks**]

(e) If the skater takes 1.5 seconds to pull in their arms, calculate the average rate of change of angular momentum. [**3 marks**]

(f) A 0.5 kg object moving tangentially at 8 m/s collides and sticks to the skater's hand at 0.8 m from the axis. Find the final angular velocity if this occurs when the skater's arms are extended. [**3 marks**]

Section G: Conservation in Rotational Systems [28 marks]

Question 14 [**18 marks**] A horizontal turntable (modeled as a uniform disc) has mass 180 kg and radius 2 m. It rotates at 1.2 rad/s when a 60 kg person walks onto it and stands at the edge.

(a) Calculate the initial angular momentum of the turntable. [**3 marks**]

(b) Find the moment of inertia of the system after the person steps on. [**3 marks**]

(c) Determine the final angular velocity using conservation of angular momentum. [**3 marks**]

(d) Calculate the change in kinetic energy and explain what happened to the "lost" energy. [**4 marks**]

(e) If instead the person runs onto the turntable tangentially with speed 4 m/s, find the final angular velocity. [**3 marks**]

(f) Compare the energy changes in parts (d) and (e) and explain the difference. [**2 marks**]

Question 15 [**10 marks**] Two horizontal discs can rotate about the same vertical axis. Disc A (moment of inertia 0.8 kg·m²) rotates at 6 rad/s clockwise, while disc B (moment of inertia 0.5 kg·m²) rotates at 4 rad/s counterclockwise.

(a) Calculate the total angular momentum before the discs are coupled together. [**3 marks**]

(b) When the discs are connected and rotate together, find their common angular velocity. [**3 marks**]

(c) Calculate the kinetic energy before and after coupling. [**2 marks**]

(d) Account for the energy difference and explain where the energy went. [**2 marks**]

Section H: Combined Motion [20 marks]

Question 16 [**12 marks**] A yo-yo consists of two solid discs of mass 0.2 kg each and radius 0.05 m, connected by a thin axle of radius 0.008 m and negligible mass.

(a) Calculate the moment of inertia of the yo-yo about its center. [**3 marks**]

(b) When the yo-yo unwinds under gravity, derive its linear acceleration using rotational dynamics. [**4 marks**]

(c) Calculate the tension in the string. [**3 marks**]

(d) Compare the yo-yo's acceleration with free fall acceleration and explain the difference. [**2 marks**]

Question 17 [**8 marks**] A solid sphere of mass 2.5 kg and radius 0.25 m rolls up an inclined plane with initial velocity 8 m/s.

(a) Calculate the initial total kinetic energy (translational plus rotational). [**3 marks**]

(b) Find the maximum height reached using energy conservation. [**3 marks**]

(c) Calculate the linear velocity when the sphere returns to its starting position. [**2 marks**]

Section I: Complex Systems [24 marks]

Question 18 [**15 marks**] A compound pulley consists of two wheels rigidly connected: a larger wheel (radius 0.5 m, moment of inertia 3.0 kg·m^2) and a smaller wheel (radius 0.3 m). A 25 kg mass hangs from the larger wheel, and a 12 kg mass hangs from the smaller wheel.

(a) Draw a clear free body diagram for the system. [**3 marks**]

(b) Write the equations of motion for both masses and the wheel system. [**4 marks**]

(c) Calculate the angular acceleration of the wheels. [**4 marks**]

(d) Find the tensions in both strings. [**4 marks**]

Question 19 [**9 marks**] A uniform rod of mass 4 kg and length 2 m is pivoted at one end and released from rest in a horizontal position.

(a) Calculate the moment of inertia about the pivot point. [**2 marks**]

(b) Find the initial angular acceleration when released. [**3 marks**]

(c) Use energy conservation to find the angular velocity when the rod reaches the vertical position. [**4 marks**]

Section J: Advanced Applications [22 marks]

Question 20 [**14 marks**] A space station consists of a central hub (mass 800 kg, radius 1.5 m, modeled as a solid cylinder) with four identical modules attached. Each module (mass 200 kg) can be modeled as a point mass located 8 m from the station's center.

(a) Calculate the total moment of inertia of the space station about its central axis. [**4 marks**]

(b) If the station rotates at 0.15 rad/s, find its angular momentum. [**2 marks**]

(c) Thrusters provide a constant torque of 120 N·m for 45 seconds. Calculate the change in angular velocity. [**4 marks**]

(d) If one module detaches during operation, find the new angular velocity of the remaining system. **[4 marks]**

Question 21 [8 marks] A potter's wheel (solid disc, mass 50 kg, radius 0.6 m) spins at 45 rpm. A 3 kg ball of clay is dropped onto the wheel at a distance of 0.4 m from the center.

(a) Calculate the initial angular momentum of the wheel. **[2 marks]**

(b) Find the angular velocity after the clay is added. **[3 marks]**

(c) Calculate the percentage change in kinetic energy. **[3 marks]**

Physics Data and Formulae

Rotational Kinematics:
$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $v = r\omega$, $a_t = r\alpha$, $a_c = \frac{v^2}{r} = \omega^2 r$

Moment of Inertia:
Point mass: $I = mr^2$
Parallel axis theorem: $I = I_{cm} + md^2$
Uniform rod (center): $I = \frac{1}{12}ML^2$, (end): $I = \frac{1}{3}ML^2$
Solid disc/cylinder: $I = \frac{1}{2}MR^2$
Solid sphere: $I = \frac{2}{5}MR^2$, Hollow sphere: $I = \frac{2}{3}MR^2$

Rotational Dynamics:
$\tau = I\alpha$ (equation of rotational motion)
$\tau = \vec{r} \times \vec{F}$ (torque from force)
Work: $W = \tau\theta$, Power: $P = \tau\omega$

Rotational Energy:
$KE_{rot} = \frac{1}{2}I\omega^2$
Rolling motion: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
Rolling constraint: $v = r\omega$ (no slipping)

Angular Momentum:
$L = I\omega$ (rigid body), $L = \vec{r} \times \vec{p}$ (point particle)
Conservation: $L_i = L_f$ (when $\sum \tau_{ext} = 0$)

Constants:
Acceleration due to gravity: $g = 9.8$ m/s²

Trigonometric Values:
$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$
¡/center

END OF TEST

Total marks: 280

Grade boundaries: A* 252, A 224, B 196, C 168, D 140, E 112

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