A Level Mechanics
Practice Test 4: Statics and Dynamics of Particles

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw clear diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 40 minutes

Section A: Multi-Force Equilibrium Systems [40 marks]

Question 1 [**18 marks**] A particle is in equilibrium under the action of five forces: 40 N due east, 25 N at 60° north of east, 30 N due west, 20 N at 45° south of west, and an unknown force F.

(a) Set up a coordinate system and resolve each known force into x and y components. [**6 marks**]

(b) Calculate the sum of x-components and sum of y-components of all known forces. [**4 marks**]

(c) Find the x and y components of force F required for equilibrium. [**3 marks**]

(d) Calculate the magnitude and direction of force F. [**4 marks**]

(e) Verify your answer by checking the equilibrium conditions. [**1 mark**]

Question 2 [**22 marks**] A decorative sign of mass 25 kg is suspended by three cables. Cable A makes an angle of 20° with the vertical, cable B makes an angle of 50° with the vertical, and cable C is horizontal. All three cables meet at the same point.

(a) Draw a clear force diagram showing all forces at the junction point. [**3 marks**]

(b) Write the equilibrium equations for horizontal and vertical directions. [**3 marks**]

(c) If the tension in the horizontal cable C is 150 N, calculate:

  (i) The tension in cable A. [**5 marks**]

  (ii) The tension in cable B. [**5 marks**]

(d) Calculate the total downward force and verify this equals the weight of the sign. [**3 marks**]

(e) Determine what happens to the tension in cable C if the angle of cable A is reduced to 10°. [**3 marks**]

Section B: Friction and Inclined Plane Dynamics [45 marks]

Question 3 [**25 marks**] A delivery truck is parked on a slope inclined at 12° to the horizontal. The truck has mass 3500 kg. The coefficient of static friction between the tires and road is 0.8, and the coefficient of kinetic friction is 0.6.

(a) Calculate the component of the truck's weight acting down the slope. [**3 marks**]

(b) Find the normal reaction force between the truck and the road. [**3 marks**]

(c) Calculate the maximum static friction force available. [**3 marks**]

(d) Determine whether the truck will remain stationary on the slope without braking. [**3 marks**]

(e) If the truck's brakes fail and it starts to slide, calculate:

(i) The kinetic friction force acting on the truck. [**3 marks**]

(ii) The net force acting down the slope. [**3 marks**]

(iii) The acceleration of the truck down the slope. [**3 marks**]

(f) If the truck slides 50 m down the slope starting from rest, calculate its final velocity. [**4 marks**]

Question 4 [**20 marks**] A box of mass 12 kg rests on an inclined conveyor belt moving upward at constant speed. The belt makes an angle of 22° with the horizontal, and the coefficient of kinetic friction between the box and belt is 0.3.

(a) When the belt moves at constant speed with the box, draw a force diagram for the box. [**3 marks**]

(b) Calculate the friction force acting on the box when it moves with the belt at constant speed. [**5 marks**]

(c) The belt suddenly stops. Calculate:

(i) The component of weight down the inclined belt. [**3 marks**]

(ii) The kinetic friction force acting up the belt. [**3 marks**]

(iii) The acceleration of the box down the belt. [**3 marks**]

(d) If the box slides 8 m down the stationary belt, find its velocity at the bottom. [**3 marks**]

Section C: Advanced Connected Particle Systems [50 marks]

Question 5 [**25 marks**] Particles A, B, and C have masses 8 kg, 5 kg, and 10 kg respectively. Particle A rests on a rough horizontal table (coefficient of kinetic friction 0.25). It is connected by a light inextensible string over a smooth pulley to particle B, which hangs freely. Particle A is also connected by another string over a second smooth pulley to particle C, which rests on a smooth inclined plane at 35° to the horizontal.

(a) Draw separate force diagrams for all three particles. [**6 marks**]

(b) Calculate the component of C's weight down the inclined plane. [**3 marks**]

(c) Write the equation of motion for particle A on the horizontal table. [**3 marks**]

(d) Write the equations of motion for particles B and C. [**4 marks**]

(e) Determine the direction of motion of the system. [**3 marks**]

(f) Calculate the acceleration of the system. **[4 marks]**

(g) Find the tensions in both connecting strings. **[2 marks]**

Question 6 [25 marks] Two particles P and Q are connected by a light inextensible string passing over two smooth pulleys. Particle P (mass 6 kg) rests on a rough inclined plane at 30° to the horizontal with coefficient of friction 0.4. Particle Q (mass 4 kg) rests on a smooth inclined plane at 45° to the horizontal. The string connecting them passes over pulleys at the tops of both planes.

(a) Draw force diagrams for both particles. **[4 marks]**

(b) For particle P, calculate:

(i) The component of weight down the plane. **[2 marks]**

(ii) The normal reaction force. **[2 marks]**

(iii) The maximum friction force available. **[2 marks]**

(c) For particle Q, calculate the component of weight down the plane. **[3 marks]**

(d) Determine which direction the system will move by comparing driving forces. **[4 marks]**

(e) Write equations of motion for both particles and solve for the acceleration. **[6 marks]**

(f) Calculate the tension in the connecting string. **[2 marks]**

Section D: Forces and Motion with Resistance [30 marks]

Question 7 [18 marks] A particle of mass 1.8 kg moves in a straight line under a driving force $F = (24 - 3t^2)$ N, where t is time in seconds. The particle also experiences a constant resistance force of 6 N opposing its motion.

(a) Write an expression for the net force on the particle at time t. **[2 marks]**

(b) Calculate the acceleration when $t = 0$, $t = 2$ s, and $t = 3$ s. **[6 marks]**

(c) If the particle starts from rest, find expressions for velocity and displacement as functions of time. **[6 marks]**

(d) Calculate the velocity and displacement when $t = 4$ s. **[2 marks]**

(e) At what time does the driving force become zero? **[2 marks]**

Question 8 [12 marks] A sphere of mass 2.0 kg falls through a viscous fluid. The resistance force is given by $R = 8v$ N, where v is the velocity in m/s.

(a) Write the equation of motion for the falling sphere. **[3 marks]**

(b) Calculate the terminal velocity of the sphere. **[3 marks]**

(c) If the sphere starts from rest, derive an expression for velocity as a function of time. **[4 marks]**

(d) Calculate the time taken to reach 90

Section E: Projectile Motion Applications [35 marks]

Question 9 [**20 marks**] A cannon fires a projectile from ground level at an angle of 52° above the horizontal with initial speed 45 m/s. The target is located on a hill 150 m away horizontally and 30 m above the firing position.

(a) Calculate the initial horizontal and vertical components of velocity. [**3 marks**]

(b) Find the time taken for the projectile to travel 150 m horizontally. [**3 marks**]

(c) Calculate the height of the projectile when it has traveled 150 m horizontally. [**4 marks**]

(d) Determine whether the projectile hits the target or passes above/below it. [**2 marks**]

(e) Calculate the velocity components of the projectile at the target location. [**4 marks**]

(f) Find the angle of the velocity vector relative to the horizontal at this point. [**4 marks**]

Question 10 [**15 marks**] A stone is thrown from the edge of a cliff 75 m high. The stone is thrown horizontally with initial speed 12 m/s toward the sea below.

(a) Calculate the time taken for the stone to reach the sea. [**4 marks**]

(b) Find the horizontal distance from the base of the cliff where the stone hits the water. [**3 marks**]

(c) Calculate the vertical component of velocity just before impact. [**3 marks**]

(d) Find the magnitude and direction of the velocity just before impact with the water. [**5 marks**]

Section F: Critical Equilibrium and Optimization [25 marks]

Question 11 [**15 marks**] A particle of mass m rests on a rough inclined plane. A force P is applied to the particle at an angle above the plane to maintain equilibrium. The coefficient of static friction is .

(a) Draw a force diagram showing all forces acting on the particle. [**3 marks**]

(b) Derive expressions for the normal reaction force and friction force in terms of m, g, P, , and the angle of the inclined plane. [**6 marks**]

(c) For the case where m = 15 kg, = 25°, = 0.6, and = 20°, calculate the force P required for equilibrium on the verge of sliding up the plane. [**4 marks**]

(d) Verify that the particle is indeed on the verge of sliding. [**2 marks**]

Question 12 [**10 marks**] A particle is suspended in equilibrium by three strings. Two strings make angles of 40° and 60° with the vertical respectively, while the third string is horizontal. The particle has mass 3.5 kg.

(a) Draw a force diagram for the particle. [**2 marks**]

(b) Calculate the tension in each of the three strings. [**6 marks**]

(c) If the horizontal string can only support a maximum tension of 25 N, determine the maximum mass that can be supported in this configuration. [**2 marks**]

Physics Data and Formulae

Forces and Equilibrium:

Equilibrium conditions: $\sum F_x = 0$, $\sum F_y = 0$

Force resolution: $F_x = F\cos\theta$, $F_y = F\sin\theta$

Resultant force: $R = \sqrt{F_x^2 + F_y^2}$

Direction: $\tan\alpha = \frac{F_y}{F_x}$

Newton's Laws:

First Law: Object at rest or uniform motion when $\sum F = 0$

Second Law: $\sum F = ma$

Third Law: Action and reaction forces equal and opposite

Friction:

Static friction: $f_s \leq \mu_s N$, maximum $f_{s,max} = \mu_s N$

Kinetic friction: $f_k = \mu_k N$

At limiting equilibrium: $f = \mu_s N$

Inclined Plane Forces:

Weight component parallel to plane: $mg\sin\alpha$ (down plane)

Weight component perpendicular to plane: $mg\cos\alpha$ (into plane)

Normal reaction: $N = mg\cos\alpha$ (perpendicular forces balanced)

Kinematics:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{u+v}{2}t$

Projectile Motion:

Horizontal motion: $x = u_x t$, $v_x = u_x$ (constant)

Vertical motion: $y = u_y t - \frac{1}{2}gt^2$, $v_y = u_y - gt$

Maximum height: $h_{max} = \frac{u_y^2}{2g}$

Time to max height: $t_{up} = \frac{u_y}{g}$

Range (level ground): $R = \frac{u^2 \sin 2\theta}{g}$

Calculus for Variable Forces:

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$v = \int a\,dt + C$

$s = \int v\,dt + C$

Constants:

Acceleration due to gravity: $g = 9.81$ m/s^2

Trigonometric Values:

$\sin 12 = 0.208$, $\cos 12 = 0.978$, $\tan 12 = 0.213$

$\sin 20 = 0.342$, $\cos 20 = 0.940$, $\tan 20 = 0.364$

$\sin 22 = 0.375$, $\cos 22 = 0.927$, $\tan 22 = 0.404$

$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$

$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$

$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$

$\sin 40 = 0.643$, $\cos 40 = 0.766$, $\tan 40 = 0.839$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

$\sin 50 = 0.766$, $\cos 50 = 0.643$, $\tan 50 = 1.192$

$\sin 52 = 0.788$, $\cos 52 = 0.616$, $\tan 52 = 1.280$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 225

Grade boundaries: A* 203, A 180, B 158, C 135, D 113, E 90

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