# A Level Mechanics Practice Test 3: Elasticity and Springs

#### **Instructions:**

Answer all questions. Show your working clearly. Calculators may be used unless stated otherwise. Draw clear, labelled diagrams where appropriate.

Time allowed: 2 hours

# Section A: Hooke's Law and Spring Systems [45 marks]

Question 1 [12 marks] A spring obeys Hooke's Law and has a natural length of 25.0 cm.

- (a) State Hooke's Law and write its mathematical expression. [3 marks]
- (b) When a 6.0 N force is applied, the spring extends to 28.5 cm. Calculate the spring constant. [4 marks]
- (c) A force-extension graph is plotted for this spring. Sketch this graph and explain what the gradient represents. [3 marks]
- (d) State what happens when the elastic limit is exceeded. [2 marks]

Question 2 [16 marks] Three identical springs, each with spring constant 120 N/m, are arranged in different configurations.

- (a) Two springs are connected in series and a 18 N force is applied.
  - (i) Calculate the extension of each spring. [3 marks]
  - (ii) Find the effective spring constant of the series combination. [3 marks]
- (b) The same two springs are now connected in parallel and the same 18 N force is applied.
  - (i) Calculate the extension of the parallel combination. [3 marks]
  - (ii) Find the effective spring constant of the parallel combination. [2 marks]
- (c) A third spring is added in parallel to the arrangement in part (b). Calculate:
  - (i) The new effective spring constant. [2 marks]
  - (ii) The extension when the same 18 N force is applied. [3 marks]

Question 3 [17 marks] A student investigates the relationship between force and extension for two different springs, A and B. The results are shown in the table below:

Force (N)	Extension of A (cm)	Extension of B (cm)
0.0	0.0	0.0
2.0	1.5	4.0
4.0	3.0	8.0
6.0	4.5	12.0
8.0	6.0	16.0
10.0	7.5	20.0

- (a) On the same axes, plot force-extension graphs for both springs A and B. [5 marks]
- (b) Calculate the spring constant for each spring. [4 marks]
- (c) Which spring is stiffer? Justify your answer. [2 marks]
- (d) If both springs were connected in series and a 5.0 N force applied, calculate the total extension. [4 marks]
- (e) State one source of uncertainty in this experiment. [2 marks]

# Section B: Elastic Potential Energy [50 marks]

## Question 4 [15 marks]

- (a) Define elastic potential energy and state the formula for the elastic potential energy stored in a spring. [4 marks]
- (b) Derive this formula from first principles, starting with the definition of work done. [6 marks]
- (c) Explain how elastic potential energy can be converted to kinetic energy, giving a practical example. [3 marks]
- (d) State the principle of conservation of energy as applied to elastic systems. [2 marks]

Question 5 [20 marks] A spring gun uses a compressed spring to fire a ball horizontally. The spring has a spring constant of 450 N/m and is compressed by 8.0 cm before firing.

- (a) Calculate the elastic potential energy stored in the compressed spring. [3 marks]
- (b) The ball has a mass of 25 g. Assuming no energy losses, calculate:
  - (i) The maximum velocity of the ball as it leaves the spring. [4 marks]
  - (ii) The kinetic energy of the ball at this velocity. [2 marks]
- (c) In practice, only 75% of the elastic potential energy is converted to kinetic energy due to friction and other losses. Calculate:
  - (i) The actual kinetic energy of the ball. [2 marks]
  - (ii) The actual velocity of the ball as it leaves the spring. [3 marks]
  - (iii) The energy lost to friction and other factors. [2 marks]
- (d) The ball is fired horizontally from a height of 1.2 m above the ground. Calculate how far horizontally the ball travels before hitting the ground. [4 marks]

Question 6 [15 marks] A bungee cord can be modeled as a spring with spring constant 50 N/m and natural length 15 m. A person of mass 60 kg jumps from a platform 40 m above the lowest safe point.

(a) Calculate the gravitational potential energy lost by the person when they reach the lowest safe point. [2 marks]

- (b) At the lowest safe point, the bungee cord is extended by 25 m from its natural length. Calculate:
  - (i) The elastic potential energy stored in the cord. [3 marks]
  - (ii) The kinetic energy of the person at this point. [3 marks]
  - (iii) The speed of the person at this point. [3 marks]
- (c) Calculate the maximum extension of the bungee cord, assuming the person comes to rest momentarily at the lowest point. [4 marks]

# Section C: Stress, Strain and Young's Modulus [60 marks]

## Question 7 [18 marks]

- (a) Define the following terms and state their SI units:
  - (i) Stress [3 marks]
  - (ii) Strain [3 marks]
  - (iii) Young's modulus [3 marks]
- (b) Write the equation that relates Young's modulus to stress and strain. [2 marks]
- (c) Explain what Young's modulus tells us about the mechanical properties of a material. [3 marks]
- (d) A material has a large Young's modulus. What does this tell us about how the material behaves when forces are applied to it? [4 marks]

Question 8 [25 marks] A steel cable in a lift system has the following specifications:

- Length: 80.0 m
- Diameter: 12.0 mm
- Young's modulus for steel:  $2.0 \times 10^{11}$  Pa
- Maximum safe stress:  $400 \times 10$  Pa
- (a) Calculate the cross-sectional area of the cable. [3 marks]
- (b) The cable supports a load of 12,000 N. Calculate:
  - (i) The tensile stress in the cable. [3 marks]
  - (ii) The tensile strain in the cable. [3 marks]
  - (iii) The extension of the cable. [4 marks]
  - (iv) The elastic potential energy stored in the cable. [4 marks]
- (c) Determine whether the cable is operating within its safe stress limit. [2 marks]
- (d) Calculate the maximum load this cable can safely support. [3 marks]
- (e) If the diameter of the cable were increased to 15.0 mm while keeping all other factors the same, calculate:
  - (i) The new extension under the original 12,000 N load. [3 marks]

Question 9 [17 marks] A student conducts an experiment to determine Young's modulus for a copper wire. The wire has:

• Length: 2.50 m

• Diameter: 0.50 mm

The following data is obtained:

Load (N)	Extension (mm)
0.0	0.0
10.0	0.65
20.0	1.30
30.0	1.95
40.0	2.60
50.0	3.25

- (a) Plot a graph of stress (y-axis) against strain (x-axis) for this data. [8 marks]
- (b) Use your graph to determine Young's modulus for copper. [5 marks]
- (c) Compare your result with the accepted value of  $1.3 \times 10^{11}$  Pa and calculate the percentage error. [2 marks]
- (d) State two factors that could contribute to the error in your result. [2 marks]

# Section D: Simple Harmonic Motion with Springs [35 marks]

**Question 10 [20 marks]** A mass of 0.75 kg is attached to a horizontal spring with spring constant 300 N/m. The mass is displaced 4.0 cm from its equilibrium position and released.

- (a) Show that the motion is simple harmonic and write the equation of motion. [4 marks]
- (b) Calculate:
  - (i) The period of oscillation. [3 marks]
  - (ii) The frequency of oscillation. [2 marks]
  - (iii) The angular frequency. [2 marks]
- (c) Find:
  - (i) The maximum velocity during the motion. [3 marks]
  - (ii) The maximum acceleration during the motion. [3 marks]
- (d) Calculate the total mechanical energy of the system. [3 marks]

Question 11 [15 marks] A vertical spring with spring constant 180 N/m supports a mass of 1.5 kg.

- (a) Calculate the extension of the spring when the mass is in equilibrium. [3 marks]
- (b) The mass is pulled down an additional 5.0 cm from equilibrium and released. Calculate:
  - (i) The amplitude of the resulting oscillation. [1 mark]
  - (ii) The period of oscillation. [3 marks]
  - (iii) The maximum speed of the mass during oscillation. [3 marks]
- (c) At what positions during the oscillation is the speed of the mass equal to half its maximum value? [5 marks]

#### Data and Formulae

#### Springs and Hooke's Law:

$$F = kx \quad \text{(Hooke's Law)}$$
 Series springs: 
$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$$
 Parallel springs: 
$$k_{eff} = k_1 + k_2 + \dots$$

# Energy:

Elastic potential energy:  $E_p = \frac{1}{2}kx^2$ Kinetic energy:  $E_k = \frac{1}{2}mv^2$ Gravitational potential energy:  $E_p = mgh$ 

## Stress and Strain:

 $\begin{array}{c} \text{Stress: } \sigma = \frac{F}{A} \\ \text{Strain: } \epsilon = \frac{\Delta L}{L} \\ \text{Young's modulus: } E = \frac{\sigma}{\epsilon} = \frac{FL}{A\Delta L} \end{array}$ 

# Simple Harmonic Motion:

$$F = -kx \quad \text{(restoring force)}$$
 
$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$
 
$$f = \frac{1}{T} \quad \text{(frequency)}$$
 
$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)}$$
 
$$v_{max} = \omega A \quad \text{(maximum velocity)}$$
 
$$a_{max} = \omega^2 A \quad \text{(maximum acceleration)}$$

## Motion equations:

$$v^2 = u^2 + 2as$$
$$s = ut + \frac{1}{2}at^2$$

## **Constants:**

$$g = 9.81 \text{ m s}^{-2}$$
  
 $\pi = 3.14$ 

#### END OF TEST

Total marks: 190

Grade boundaries: A\* 171, A 152, B 133, C 114, D 95, E 76

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