A Level Statistics
Practice Test 3: Hypothesis Testing

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Non-Parametric Tests [25 marks]

1. [**12 marks**] Define and explain non-parametric hypothesis tests:

(a) Explain when non-parametric tests should be used instead of parametric tests.

(b) Define the Mann-Whitney U test and state its purpose.

(c) Explain the Wilcoxon signed-rank test and when it's appropriate.

(d) Describe the Kruskal-Wallis test as a non-parametric alternative to ANOVA.

(e) Explain the advantages and disadvantages of non-parametric tests.

(f) Define rank correlation and explain Spearman's rank correlation coefficient.

2. [**8 marks**] Explain the concept of ranking in non-parametric tests:

(a) Describe how to assign ranks to data values.

(b) Explain how to handle tied values when ranking.

(c) Define the concept of rank sum and its use in hypothesis testing.

(d) Explain why ranks are used instead of original data values.

3. [**5 marks**] Compare parametric and non-parametric approaches:

(a) Compare the assumptions required for t-tests versus Mann-Whitney U tests.

(b) Explain the concept of statistical efficiency in non-parametric tests.

(c) Describe when you might choose a non-parametric test despite having normal data.

Section B: Correlation and Regression Testing [30 marks]

4. **[15 marks]** Testing correlation coefficients:

   (a) Write the test statistic for testing whether a correlation coefficient is significantly different from zero.

   (b) Explain how to test whether a correlation coefficient equals a specific value.

   (c) Describe Fisher's z-transformation and its purpose.

   (d) Explain how to compare two correlation coefficients from independent samples.

   (e) State the assumptions required for testing correlation coefficients.

   (f) Describe Spearman's rank correlation test and its interpretation.

5. **[15 marks]** A study examines the relationship between study hours (x) and exam scores (y) for 12 students:

Student	1	2	3	4	5	6	7	8	9	10	11	12
Hours (x)	15	20	12	25	18	22	10	28	16	24	14	30
Score (y)	65	78	58	85	72	80	55	90	68	82	62	95

Given: r = 0.94, $\sum x = 234$, $\sum y = 890$, $\sum x^2 = 4896$, $\sum y^2 = 67254$, $\sum xy = 18365$

   (a) Test whether the correlation coefficient is significantly different from zero at $= 0.05$.

   (b) Calculate the t-statistic for this test.

   (c) Find the p-value and state your conclusion.

   (d) Test whether the correlation coefficient is significantly different from 0.8.

   (e) Calculate Spearman's rank correlation coefficient for this data.

   (f) Compare the Pearson and Spearman correlation coefficients and explain any differences.

   (g) Calculate a 95

   (h) Interpret the practical significance of this correlation in the context of the study.

Section C: Advanced Testing Scenarios [35 marks]

6. **[18 marks]** A manufacturing company tests four different production methods for defect rates. The number of defects per 100 items produced is recorded:

   Method A: 3, 5, 4, 6, 2, 4, 5, 3 **Method B:** 7, 9, 8, 10, 6, 8, 9, 7 **Method C:** 4, 6, 5, 7, 3, 5, 6, 4 **Method D:** 2, 4, 3, 5, 1, 3, 4, 2

   (a) Calculate the mean and standard deviation for each method.

   (b) Test the assumption of equal variances using Levene's test concept.

   (c) Perform a Kruskal-Wallis test to compare the four methods.

   (d) Calculate the test statistic and degrees of freedom.

   (e) Find the critical value at $= 0.05$ and make a conclusion.

   (f) Rank all observations and calculate rank sums for each group.

   (g) Perform pairwise Mann-Whitney U tests between methods with significant differences.

(h) Apply appropriate multiple testing corrections.

(i) Compare the conclusions from parametric ANOVA versus non-parametric Kruskal-Wallis.

7. **[17 marks]** A medical researcher studies the effectiveness of a new treatment using before-and-after measurements:

Patient	1	2	3	4	5	6	7	8	9	10
Before	85	92	78	95	88	90	82	97	86	91
After	78	85	72	88	84	86	79	89	83	87

(a) Calculate the differences (Before - After) for each patient.

(b) Test the normality of the differences using appropriate methods.

(c) Perform a paired t-test assuming normality.

(d) Perform a Wilcoxon signed-rank test as a non-parametric alternative.

(e) Calculate the test statistics for both tests.

(f) Compare the p-values and conclusions from both approaches.

(g) Calculate effect sizes for both parametric and non-parametric tests.

(h) Determine which test is more appropriate for this data.

(i) Calculate a 95

(j) Discuss the clinical significance of the treatment effect.

Answer Space

Use this space for your working and answers.

Formulae and Key Concepts

Correlation Tests:

Test for r = 0: $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ with df = n-2

Fisher's z-transformation: $z' = \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right)$

SE of z': $SE_{z'} = \frac{1}{\sqrt{n-3}}$

Spearman's Rank Correlation:

$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$ where d_i = difference in ranks

For large n: $t = \frac{r_s\sqrt{n-2}}{\sqrt{1-r_s^2}}$

Mann-Whitney U Test:
$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$
$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$
Test statistic: U = min(U, U)

Wilcoxon Signed-Rank Test:
Rank absolute differences, sum ranks of positive differences
Test statistic: W = smaller of positive or negative rank sums
For large n: $z = \frac{W-\mu_W}{\sigma_W}$ where $\mu_W = \frac{n(n+1)}{4}$, $\sigma_W = \sqrt{\frac{n(n+1)(2n+1)}{24}}$

Kruskal-Wallis Test:
$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$
where N = total sample size, k = number of groups, R_i = rank sum for group i
df = k-1, approximately ² distributed

Tied Ranks:
Assign average of positions: e.g., values 5,5,5 in positions 3,4,5 get rank 4
Correction factor for ties in Kruskal-Wallis and other tests

Effect Size (Non-parametric):
r = $\frac{z}{\sqrt{n}}$ for Mann-Whitney and Wilcoxon tests
Eta squared: $\eta^2 = \frac{H-k+1}{N-k}$ for Kruskal-Wallis

Critical Values:
Mann-Whitney: Use tables or normal approximation for large samples
Wilcoxon: Use tables or normal approximation for n ¿ 20
Spearman: Use tables or t-distribution for large samples

Assumptions:
Non-parametric: Independent observations, similar distributions shapes
Parametric: Normality, independence, equal variances (where applicable)
Correlation: Bivariate normality, linear relationship

END OF TEST

Total marks: 90

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