A Level Mechanics
Practice Test 3: Rotational Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Rotational Motion Fundamentals [25 marks]

Question 1 [**9 marks**]

(a) Define the fundamental quantities in rotational motion: angular displacement, angular velocity, and angular acceleration, stating their units and symbols. [**3 marks**]

(b) Derive the relationships between linear and angular quantities for a particle moving in circular motion. [**4 marks**]

(c) Write the three rotational kinematic equations and identify the linear motion analogue for each. [**2 marks**]

Question 2 [**12 marks**] A turbine blade rotates with angular velocity described by $\omega(t) = 3t^3 - 9t^2 + 12t$ rad/s.

(a) Find the angular acceleration function $\alpha(t)$. [**2 marks**]

(b) Calculate the angular velocity at $t = 2$ seconds. [**2 marks**]

(c) Determine the angular acceleration at $t = 1.5$ seconds. [**2 marks**]

(d) Find all times when the blade momentarily stops rotating. [**3 marks**]

(e) Calculate the angular displacement from $t = 1$ to $t = 2.5$ seconds. [**3 marks**]

Question 3 [**4 marks**] A point on the edge of a rotating wheel has tangential speed 18 m/s and centripetal acceleration 450 m/s^2. The wheel has angular acceleration 3.5 rad/s^2.

(a) Calculate the radius of the wheel and its angular velocity. [**2 marks**]

(b) Find the magnitude of the total acceleration of the point. [**2 marks**]

Section B: Moment of Inertia - Point Masses [29 marks]

Question 4 [**7 marks**]

(a) Define moment of inertia and explain why it is sometimes called "rotational inertia." [**3 marks**]

(b) State the parallel axis theorem and derive it for a simple case. [**4 marks**]

Question 5 [22 marks] Calculate moments of inertia for the following configurations:

(a) Seven point masses: 1.5 kg at (2,3), 2.8 kg at (-2,1), 3.2 kg at (1,-3), 2.1 kg at (-3,-2), 1.8 kg at (4,1), 2.5 kg at (-1,4), and 3.6 kg at (3,-1) meters. Find the moment of inertia about the z-axis. **[6 marks]**

(b) Five masses of 4 kg each are positioned at the vertices of a regular pentagon with side length 2 m. Calculate the moment of inertia about the central axis perpendicular to the plane. **[5 marks]**

(c) Three point masses of 6 kg, 9 kg, and 15 kg are arranged along a straight line with the 9 kg mass at the origin, the 6 kg mass at x = -1.5 m, and the 15 kg mass at x = 2.5 m. Find the moment of inertia about: (i) the origin, (ii) the center of mass. **[6 marks]**

(d) A square framework has four point masses of 5 kg each at its corners. The square has side length 1.6 m. Calculate the moment of inertia about an axis through the center parallel to one side. **[3 marks]**

(e) Verify one calculation using the parallel axis theorem. **[2 marks]**

Section C: Moment of Inertia - Extended Bodies [35 marks]

Question 6 [11 marks]

(a) State the moment of inertia formulas for: uniform rod about center, uniform rod about end, solid disc, thin ring, solid sphere, and hollow sphere. **[6 marks]**

(b) Derive the moment of inertia of a solid disc of mass M and radius R about its central axis using integration. **[4 marks]**

(c) Explain the physical meaning of why a thin ring has moment of inertia $I = MR^2$ while a solid disc has $I = \frac{1}{2}MR^2$. **[1 mark]**

Question 7 [24 marks] Calculate moments of inertia for composite objects:

(a) A solid disc of mass 12 kg and radius 0.45 m has two concentric holes: one of radius 0.15 m and another of radius 0.08 m. Find the moment of inertia about the central axis. **[5 marks]**

(b) An L-shaped object consists of two uniform rods: one horizontal (mass 5 kg, length 3 m) and one vertical (mass 4 kg, length 2.5 m) joined at the end of the horizontal rod. Calculate the moment of inertia about an axis through the junction perpendicular to both rods. **[6 marks]**

(c) A complex system has three solid spheres (each mass 8 kg, radius 0.15 m) connected by two uniform rods (each mass 3 kg, length 1.8 m) in a straight line with sphere centers at the rod ends and middle. Find the moment of inertia about the center perpendicular to the line. **[8 marks]**

(d) A turbine consists of a solid cylinder hub (mass 20 kg, radius 0.3 m) with twelve uniform blade-spokes (each mass 2.5 kg, length 0.8 m) extending radially. Calculate the total moment of inertia about the central axis. **[5 marks]**

Section D: Torque and Rotational Dynamics [27 marks]

Question 8 [7 marks]

(a) Define torque both mathematically and conceptually, giving its SI unit. **[3 marks]**

(b) Explain how the direction of the torque vector is determined and what this represents physically. **[2 marks]**

(c) State the rotational equation of motion and compare it with Newton's second law for linear motion. **[2 marks]**

Question 9 [20 marks] A compound disc system consists of two discs rigidly connected: a larger disc (mass 15 kg, radius 0.8 m) and a smaller disc (mass 8 kg, radius 0.4 m) sharing the same central axis.

(a) Calculate the total moment of inertia of the system. **[3 marks]**

(b) Three forces are applied simultaneously: 35 N tangentially at the edge of the large disc, 20 N at 45° to the radius at 0.6 m from center on the large disc, and 18 N tangentially at the edge of the small disc in the opposite direction. Calculate the net torque. **[6 marks]**

(c) Find the angular acceleration of the system. **[2 marks]**

(d) Starting from rest, calculate the angular velocity after 4 seconds. **[2 marks]**

(e) Determine the number of complete revolutions in 4 seconds. **[2 marks]**

(f) Calculate the total work done and the final rotational kinetic energy. **[3 marks]**

(g) Find the average power during the 4-second interval. **[2 marks]**

Section E: Rotational Energy [31 marks]

Question 10 [11 marks]

(a) Starting from the kinetic energy of individual particles, derive the general expression for rotational kinetic energy of a rigid body. **[4 marks]**

(b) Define work done by a torque and establish the work-energy theorem for rotational motion. **[4 marks]**

(c) Derive the relationship between power, torque, and angular velocity. **[3 marks]**

Question 11 [20 marks] A solid cylinder of mass 25 kg and radius 0.35 m rolls without slipping down a curved ramp that drops 3.5 m vertically over a horizontal distance of 8 m.

(a) Calculate the moment of inertia of the cylinder. **[2 marks]**

(b) Write the rolling constraint equation and explain its physical meaning. **[3 marks]**

(c) Use conservation of energy to find the linear speed at the bottom of the ramp. **[5 marks]**

(d) Calculate the angular velocity at the bottom. **[2 marks]**

(e) If the cylinder then rolls up an incline of angle 20°, find the maximum distance it travels up the incline. **[4 marks]**

(f) Calculate the translational and rotational kinetic energies at the bottom of the initial ramp. **[2 marks]**

(g) What fraction of the total kinetic energy is rotational? Compare this with the fraction for a solid sphere. **[2 marks]**

Section F: Angular Momentum [23 marks]

Question 12 [5 marks]

(a) Define angular momentum for a rigid body and relate it to the rotational variables. [**2 marks**]

(b) State the conservation of angular momentum principle and specify the conditions under which it applies. [**3 marks**]

Question 13 [18 marks] A diver performs a somersault, initially in an extended position with moment of inertia 12 kg·m^2 and angular velocity 2.2 rad/s.

(a) Calculate the initial angular momentum. [**2 marks**]

(b) The diver tucks into a compact position, reducing the moment of inertia to 3.5 kg·m^2. Find the new angular velocity. [**3 marks**]

(c) Calculate the change in rotational kinetic energy and explain where this energy comes from. [**5 marks**]

(d) If the position change takes 0.6 seconds, calculate the average torque applied by the diver's muscles. [**3 marks**]

(e) During the dive, a 2 kg camera (moving horizontally at 5 m/s) collides and sticks to the diver's hand at 1.2 m from the rotation axis while the diver is in the extended position. Find the new angular velocity. [**5 marks**]

Section G: Conservation in Rotational Systems [24 marks]

Question 14 [15 marks] A large horizontal disc (uniform, mass 300 kg, radius 2.5 m) rotates at 0.8 rad/s. Three people jump onto it: a 70 kg person at the rim, a 60 kg person at 1.8 m from center, and a 55 kg person at 1.2 m from center.

(a) Calculate the initial angular momentum of the disc alone. [**3 marks**]

(b) Find the total moment of inertia after all three people are on the disc. [**4 marks**]

(c) Determine the final angular velocity using conservation of angular momentum. [**3 marks**]

(d) Calculate the kinetic energy before and after, and account for the energy difference. [**3 marks**]

(e) If the 70 kg person runs tangentially onto the disc at 6 m/s while the others step on gently, find the final angular velocity. [**2 marks**]

Question 15 [9 marks] Two identical discs (each with moment of inertia 2.4 kg·m^2) can rotate about the same vertical axis. Initially, disc A rotates at 12 rad/s clockwise and disc B rotates at 8 rad/s counterclockwise.

(a) Calculate the net angular momentum before the discs are coupled. [**3 marks**]

(b) Find the common angular velocity after they are brought together. [**3 marks**]

(c) Calculate the energy dissipated during the coupling process and explain where it goes. [**3 marks**]

Section H: Combined Motion [21 marks]

Question 16 [**13 marks**] A uniform solid cylinder (mass 8 kg, radius 0.15 m) with string wound around its circumference is released from rest and falls vertically while unwinding.

(a) Calculate the moment of inertia about the central axis. [**2 marks**]

(b) Apply Newton's second law for both translation and rotation to find the linear acceleration. [**6 marks**]

(c) Calculate the tension in the string. [**2 marks**]

(d) Compare the acceleration to that of free fall and explain the difference. [**2 marks**]

(e) If released from a height of 2 m, find the linear and angular velocities just before hitting the ground. [**1 mark**]

Question 17 [**8 marks**] A thin ring of mass 4 kg and radius 0.3 m rolls up an inclined plane (angle 35°) with initial linear velocity 12 m/s.

(a) Calculate the initial total kinetic energy. [**3 marks**]

(b) Find the maximum height reached up the incline. [**3 marks**]

(c) Determine the distance traveled along the incline before momentarily stopping. [**2 marks**]

Section I: Complex Systems [25 marks]

Question 18 [**15 marks**] A pulley system consists of two wheels with different radii rigidly joined together. The larger wheel (radius 0.7 m) supports a 35 kg mass, while the smaller wheel (radius 0.4 m) supports a 20 kg mass hanging on the opposite side. The combined moment of inertia of both wheels is 6.5 kg·m^2.

(a) Draw detailed free body diagrams for both masses and the wheel system. [**4 marks**]

(b) Set up the complete system of equations of motion. [**5 marks**]

(c) Solve for the angular acceleration of the wheels. [**3 marks**]

(d) Calculate the tensions in both cables. [**3 marks**]

Question 19 [**10 marks**] A uniform rod of mass 6 kg and length 3 m is pivoted at a point 1.2 m from one end and released from rest when inclined at 60° below the horizontal.

(a) Calculate the moment of inertia about the pivot point. [**3 marks**]

(b) Find the initial angular acceleration when released. [**4 marks**]

(c) Using energy conservation, calculate the angular velocity when the rod swings to the vertical position. [**3 marks**]

Section J: Advanced Applications [22 marks]

Question 20 [**14 marks**] A space telescope consists of a main cylindrical body (mass 1200 kg, radius 1.8 m, length 4 m) with three instrument modules. Each module (mass 150 kg) can be modeled as a point mass located 2.5 m from the telescope's central axis.

(a) Calculate the moment of inertia of the main body about its central axis. [**2 marks**]

(b) Find the total moment of inertia of the complete telescope. [**3 marks**]

(c) If the telescope rotates at 0.05 rad/s, calculate its angular momentum. [**2 marks**]

(d) Gyroscopic stabilizers apply a torque of 95 N·m for 60 seconds. Find the change in angular velocity. [**3 marks**]

(e) During a malfunction, one instrument module separates from the telescope. Calculate the new angular velocity of the remaining system. [**4 marks**]

Question 21 [**8 marks**] A bench grinder has a grinding wheel (solid disc, mass 18 kg, radius 0.25 m) initially rotating at 240 rpm. When a workpiece is pressed against it, the friction force is 120 N at the rim.

(a) Calculate the initial angular momentum of the wheel. [**2 marks**]

(b) Find the angular deceleration due to the friction force. [**3 marks**]

(c) Calculate the time required for the wheel to reduce its speed to 180 rpm. [**3 marks**]

Physics Data and Formulae

Rotational Kinematics:

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $v = r\omega$, $a_t = r\alpha$, $a_c = \frac{v^2}{r} = \omega^2 r$

Moment of Inertia:

Point mass: $I = mr^2$

Parallel axis theorem: $I = I_{cm} + md^2$

Uniform rod (center): $I = \frac{1}{12}ML^2$, (end): $I = \frac{1}{3}ML^2$

Solid disc/cylinder: $I = \frac{1}{2}MR^2$, Thin ring: $I = MR^2$

Solid sphere: $I = \frac{2}{5}MR^2$, Hollow sphere: $I = \frac{2}{3}MR^2$

Rotational Dynamics:

$\tau = I\alpha$ (equation of rotational motion)

$\tau = \vec{r} \times \vec{F}$ (torque from force)

Work: $W = \tau\theta$, Power: $P = \tau\omega$

Rotational Energy:

$KE_{rot} = \frac{1}{2}I\omega^2$

Rolling motion: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling constraint: $v = r\omega$ (no slipping)

Angular Momentum:

$L = I\omega$ (rigid body), $L = \vec{r} \times \vec{p}$ (point particle)

Conservation: $L_i = L_f$ (when $\sum \tau_{ext} = 0$)

Constants:

Acceleration due to gravity: $g = 9.8$ m/s²

Trigonometric Values:

$\sin 20 = 0.342$, $\cos 20 = 0.940$, $\tan 20 = 0.364$

$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 282

Grade boundaries: A* 254, A 226, B 197, C 169, D 141, E 113

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