

A Level Mechanics

Practice Test 3: Newton's Laws of Motion

Instructions:

Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 25 minutes

Section A: Theoretical Foundations [30 marks]

Question 1 [18 marks]

- (a) Explain the concept of "fictitious forces" that appear in non-inertial reference frames. Give an example of such a force. [5 marks]
- (b) A train accelerates forward at 2 m/s^2 . Describe the motion of a pendulum hanging from the train ceiling as observed by: (i) a passenger on the train, (ii) an observer on the platform. [6 marks]
- (c) State the conditions under which Newton's laws are valid and explain why they break down in certain reference frames. [4 marks]
- (d) Discuss the relationship between Newton's laws and Galilean relativity. [3 marks]

Question 2 [12 marks] A force $\vec{F} = (3t^2 + 5)\hat{i} + (4t - 2)\hat{j}$ N acts on a particle of mass 2 kg, where t is time in seconds.

- (a) Calculate the acceleration vector at $t = 3$ seconds. [3 marks]
- (b) If the particle starts from rest at the origin, find its velocity vector at $t = 2$ seconds. [5 marks]
- (c) Determine the position vector at $t = 2$ seconds. [4 marks]

Section B: Advanced Equilibrium Systems [35 marks]

Question 3 [20 marks] A uniform rod of mass 8 kg and length 3 m is suspended horizontally by two vertical cables. One cable is attached 0.5 m from the left end, and the other is attached 0.8 m from the right end. A load of mass 12 kg is attached to the rod.

- (a) If the load is placed 1 m from the left end, calculate the tension in each cable. [6 marks]
- (b) Determine the position where the load must be placed to make the tension in the left cable twice that in the right cable. [6 marks]
- (c) Find the maximum mass that can be attached at the right end without the left cable becoming slack. [5 marks]
- (d) Verify your answers using both force and moment equilibrium conditions. [3 marks]

Question 4 [15 marks] A system of forces acts on a rigid body: $\vec{F}_1 = 20\hat{i} + 15\hat{j}$ N at position $(2, 1)$ m, $\vec{F}_2 = -10\hat{i} + 25\hat{j}$ N at position $(0, 3)$ m, and $\vec{F}_3 = -5\hat{i} - 30\hat{j}$ N at position $(3, 0)$ m.

- (a) Calculate the resultant force. [3 marks]
- (b) Find the resultant moment about the origin. [5 marks]
- (c) Determine the position where a single equivalent force would need to act. [4 marks]
- (d) Verify that this equivalent force produces the same moment about any point. [3 marks]

Section C: Complex Friction Systems [32 marks]

Question 5 [18 marks] Two blocks with masses 8 kg and 5 kg are stacked on a rough horizontal surface. The coefficient of friction between the blocks is 0.4, between the bottom block and surface is 0.3, and between the top block and surface is 0.6.

- (a) A horizontal force F is applied to the bottom block. Find the maximum value of F before the blocks slide relative to each other. [6 marks]
- (b) If $F = 50$ N is applied to the bottom block, determine the motion of both blocks. [6 marks]
- (c) Now apply the same force to the top block instead. Analyze the resulting motion. [6 marks]

Question 6 [14 marks] A block of mass 6 kg is placed on an inclined plane of angle 35° . The coefficient of static friction is 0.8 and kinetic friction is 0.6.

- (a) Determine if the block will slide down the plane. [4 marks]
- (b) A horizontal force is applied to push the block up the plane. Calculate the minimum force needed to start motion up the plane. [5 marks]
- (c) If this minimum force is maintained, find the acceleration up the plane. [5 marks]

Section D: Multi-Body Dynamics [40 marks]

Question 7 [22 marks] A system consists of three masses: 4 kg and 6 kg connected by a string on a rough horizontal table (coefficient of friction 0.25 for both), with the 6 kg mass also connected over a pulley to a hanging 8 kg mass.

- (a) Draw comprehensive free body diagrams for all three masses. [6 marks]
- (b) Calculate the acceleration of the system. [8 marks]
- (c) Find the tension in each string segment. [6 marks]
- (d) Determine the minimum coefficient of friction needed to prevent any sliding. [2 marks]

Question 8 [18 marks] A locomotive of mass 80,000 kg pulls three carriages of masses 20,000 kg, 25,000 kg, and 30,000 kg. The locomotive exerts a driving force of 120,000 N, and each vehicle experiences resistance proportional to its mass, with constant of proportionality 0.8 N/kg.

- (a) Calculate the acceleration of the train. [4 marks]
- (b) Find the tension in each coupling between vehicles. [8 marks]
- (c) If the train travels up a gradient of 3° , recalculate the acceleration and coupling tensions. [6 marks]

Section E: Advanced Circular Motion [28 marks]

Question 9 [16 marks] A car travels around a banked circular track of radius 120 m. The banking angle is 15° and the coefficient of friction is 0.7.

- (a) Calculate the speed at which the car can travel without relying on friction. [4 marks]
- (b) Determine the maximum safe speed with friction. [6 marks]
- (c) Find the minimum speed below which the car would slide down the banking. [6 marks]

Question 10 [12 marks] A particle of mass 0.8 kg is attached to two strings of lengths 1.2 m and 0.9 m. The strings are attached to two points in the same horizontal plane 1.5 m apart. The particle moves in a horizontal circle.

- (a) If the plane of the circle is 0.6 m below the attachment points, find the radius of the circular motion. [4 marks]
- (b) Calculate the tension in each string. [5 marks]
- (c) Determine the speed of the particle. [3 marks]

Section F: Non-Uniform Motion Analysis [25 marks]

Question 11 [15 marks] A particle of mass 3 kg moves along a straight line under the action of a force $F = 12 + 6t - 2t^2$ N, where t is time in seconds.

- (a) If the particle starts from rest, find expressions for velocity and displacement as functions of time. [6 marks]
- (b) Calculate the velocity and displacement at $t = 4$ seconds. [3 marks]
- (c) Determine when the particle momentarily comes to rest again. [3 marks]
- (d) Find the maximum displacement from the starting point. [3 marks]

Question 12 [10 marks] A lift accelerates upward from rest with acceleration $a = 2 - 0.4t$ m/s² for the first 5 seconds, then moves at constant velocity.

- (a) Find the velocity at $t = 5$ seconds. [3 marks]
- (b) Calculate the displacement during the acceleration phase. [4 marks]
- (c) If a 60 kg person is in the lift, determine the variation of apparent weight during acceleration. [3 marks]

Section G: Constraint Force Analysis [20 marks]

Question 13 [20 marks] A bead of mass 0.2 kg slides on a smooth circular wire of radius 0.8 m in a vertical plane. The bead is given an initial speed at the bottom of the circle.

- (a) If the initial speed is 5 m/s, calculate the normal force from the wire when the bead is at the bottom. [3 marks]
- (b) Find the speed and normal force when the bead reaches the top of the circle. [5 marks]
- (c) Determine the minimum initial speed needed for the bead to complete the circular motion. [4 marks]
- (d) If the initial speed is 3 m/s, find the angle at which the bead leaves the wire. [5 marks]

- (e) Calculate the normal force just before the bead leaves the wire. **[3 marks]**

Physics Data and Formulae

Newton's Laws:

First Law: $\sum \vec{F} = 0 \Rightarrow \vec{a} = 0$

Second Law: $\vec{F}_{net} = m\vec{a}$ or $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

Third Law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Force Analysis:

Friction: $f_s \leq \mu_s N$, $f_k = \mu_k N$

Weight components on incline: $mg \sin \theta$ (down), $mg \cos \theta$ (normal)

Centripetal force: $F_c = \frac{mv^2}{r} = m\omega^2 r$

Equilibrium Conditions:

Force equilibrium: $\sum F_x = 0$, $\sum F_y = 0$

Moment equilibrium: $\sum M = 0$ (about any point)

Moment: $M = Fd$ (force \times perpendicular distance)

Circular Motion:

Banking without friction: $\tan \theta = \frac{v^2}{rg}$

Centripetal acceleration: $a_c = \frac{v^2}{r}$

Conical pendulum: $\tan \theta = \frac{v^2}{rg}$

Kinematics:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

For variable acceleration: $a = \frac{dv}{dt}$, $v = \frac{ds}{dt}$

Vector Operations:

Magnitude: $|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

Cross product for moment: $\vec{M} = \vec{r} \times \vec{F}$

Unit vector: $\hat{n} = \frac{\vec{F}}{|\vec{F}|}$

Constants:

Acceleration due to gravity: $g = 9.8 \text{ m/s}^2$

Trigonometric Values:

$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$

$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$

$\sin 3 = 0.052$, $\cos 3 = 0.999$, $\tan 3 = 0.052$

END OF TEST

Total marks: 210

Grade boundaries: A* 189, A 168, B 147, C 126, D 105, E 84

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stepupmaths.co.uk**