A Level Statistics
Practice Test 4: Advanced Topics

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Fundamental Concepts [25 marks]

1. [**12 marks**] Define and explain fundamental concepts:

(a) Define probability distributions and explain their role in modeling uncertainty.

(b) Explain what is meant by "discrete" and "continuous" probability distributions.

(c) State the key properties that define a valid probability distribution.

(d) Define expected value (mean) and variance of a random variable.

(e) Distinguish between population parameters and sample statistics.

(f) Explain how probability distributions relate to real-world phenomena.

2. [**8 marks**] Explain the importance of these concepts:

(a) Why are probability models essential for statistical inference?

(b) Explain how distributions help in making predictions and assessments.

(c) Describe the role of parameters in characterizing distributions.

(d) Explain the relationship between theoretical distributions and observed data.

3. [**5 marks**] Practical and theoretical context:

(a) Explain why different distributions are suited to different types of data.

(b) Describe the role of probability distributions in risk assessment.

(c) Explain how distributions apply to quality control and reliability engineering.

Section B: Discrete Probability Distributions [30 marks]

4. **[15 marks]** Binomial and Poisson distributions:

(a) State the conditions for a binomial distribution and write its probability mass function.

(b) Explain the parameters n and p in the binomial distribution.

(c) Derive the mean and variance of the binomial distribution.

(d) State the conditions for a Poisson distribution and write its probability mass function.

(e) Explain the parameter in the Poisson distribution.

(f) Describe when the Poisson approximation to binomial is appropriate.

5. **[15 marks]** Properties and applications of discrete distributions:

(a) Explain the relationship between binomial and Poisson distributions.

(b) Describe the normal approximation to the binomial distribution.

(c) State the continuity correction and explain when to use it.

(d) Explain the memoryless property of geometric distributions.

(e) Describe the negative binomial distribution and its applications.

(f) Explain hypergeometric distributions and sampling without replacement.

(g) Describe Poisson processes and their properties.

(h) Explain compound Poisson distributions.

(i) Describe applications in reliability and queuing theory.

Section C: Distribution Applications [35 marks]

6. **[18 marks]** A manufacturing process produces defective items with probability p = 0.08. In a sample of 25 items:

(a) State the appropriate probability distribution and its parameters.

(b) Calculate the probability of exactly 3 defective items.

(c) Calculate the probability of at most 2 defective items.

(d) Find the expected number and variance of defective items.

(e) Calculate the probability of more than 5 defective items.

(f) Use the normal approximation with continuity correction to find P(X 2).

(g) Compare the exact binomial probability with the normal approximation.

(h) Determine the most likely number of defective items.

(i) Calculate the standard deviation of the number of defective items.

7. **[17 marks]** Calls arrive at a call center at an average rate of 3.5 per minute:

(a) State the appropriate distribution for the number of calls in one minute.

(b) Calculate the probability of exactly 4 calls in one minute.

(c) Calculate the probability of no calls in one minute.

(d) Find the probability of more than 6 calls in one minute.

(e) Calculate the probability of fewer than 3 calls in a 2-minute period.

(f) Find the expected number and variance of calls in a 5-minute period.

(g) Calculate the probability of exactly 10 calls in a 3-minute period.

(h) Use the normal approximation to find P(X ¿ 20) for a 5-minute period.

(i) Explain the assumptions underlying the Poisson model for this scenario.

Answer Space

Use this space for your working and answers.

Formulae and Key Concepts

Binomial Distribution:

$$X \sim \text{Binomial}(n, p)$$
PMF: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

Mean: $E[X] = np$

Variance: $\text{Var}(X) = np(1-p)$

Poisson Distribution:

$$X \sim \text{Poisson}(\lambda)$$
PMF: $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

Mean: $E[X] = \lambda$

Variance: $\text{Var}(X) = \lambda$

Normal Approximations:

Binomial to Normal: When $np \geq 5$ and $n(1-p) \geq 5$

$$X \sim N(np, np(1-p))$$
Poisson to Normal: When $\lambda \geq 5$

$$X \sim N(\lambda, \lambda)$$

Continuity Correction:

$$P(X = k) \approx P(k - 0.5 < Y < k + 0.5)$$
$$P(X \leq k) \approx P(Y \leq k + 0.5)$$
$$P(X \geq k) \approx P(Y \geq k - 0.5)$$
$$P(X < k) \approx P(Y < k - 0.5)$$
$$P(X > k) \approx P(Y > k + 0.5)$$

Other Discrete Distributions:
Geometric: $P(X = k) = (1 - p)^{k-1}p$, $E[X] = \frac{1}{p}$
Negative Binomial: $P(X = k) = \binom{k-1}{r-1}p^r(1 - p)^{k-r}$
Hypergeometric: $P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$

Poisson Process:
For rate λ events per unit time:
Number in time t: $X \sim \text{Poisson}(\lambda t)$
Time between events: $T \sim \text{Exponential}(\lambda)$

Standard Normal:
$Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$
$\Phi(z) = P(Z \leq z)$
$\Phi(-z) = 1 - \Phi(z)$

Approximation Conditions:
Poisson approximation to Binomial: $n \geq 20$, $p \leq 0.05$, $np < 5$
Use $\lambda = np$
Normal approximation guidelines: Check variance conditions above

END OF TEST

Total marks: 90

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