A Level Mechanics
Practice Test 3: Gravitational Fields

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Advanced Gravitational Force Analysis [30 marks]

Question 1 [18 marks] Three point masses are arranged in a straight line: mass A (400 kg) at position x = 0, mass B (600 kg) at x = 4.0 m, and mass C (800 kg) at x = 10.0 m.

(a) Calculate the gravitational force on mass B due to mass A. **[3 marks]**

(b) Calculate the gravitational force on mass B due to mass C. **[3 marks]**

(c) Find the net gravitational force on mass B and state its direction. **[4 marks]**

(d) Calculate the gravitational field strength at position x = 2.0 m due to all three masses. **[8 marks]**

Question 12 [12 marks] A binary star system consists of two stars with masses 3.2×10^3 kg and 1.8×10^3 kg separated by 2.4×10^{11} m.

(a) Calculate the gravitational force between the two stars. **[4 marks]**

(b) Find the position of the center of mass of the system, measured from the more massive star. **[4 marks]**

(c) Calculate the gravitational field strength at the center of mass due to both stars. **[4 marks]**

Section B: Gravitational Fields and Potential [40 marks]

Question 2 [20 marks] Jupiter has a mass of 1.90×10^2 kg and a radius of 7.15×10 m.

(a) Calculate the gravitational field strength on Jupiter's surface. **[4 marks]**

(b) Find the gravitational potential on Jupiter's surface. **[4 marks]**

(c) Calculate the gravitational field strength at a distance of 2.0×10 m from Jupiter's center. **[4 marks]**

(d) Determine the gravitational potential at this distance. **[3 marks]**

(e) Calculate the potential difference between Jupiter's surface and the point at 2.0×10 m. **[3 marks]**

(f) A probe of mass 1500 kg moves from Jupiter's surface to this distance. Calculate the change in its potential energy. [**2 marks**]

Question 3 [**20 marks**]

(a) Explain the relationship between gravitational field strength and gravitational potential. Include the mathematical relationship. [**4 marks**]

(b) Show that the gravitational field strength inside a uniform spherical shell of mass M and radius R is zero at all points. [**6 marks**]

(c) For a uniform solid sphere of mass M and radius R, derive an expression for the gravitational field strength at distance r from the center when r ¡ R. [**6 marks**]

(d) Sketch a graph showing how gravitational field strength varies with distance from the center of a uniform solid sphere, from r = 0 to r = 3R. [**4 marks**]

Section C: Orbital Dynamics and Energy [45 marks]

Question 4 [**25 marks**] A satellite is initially in a circular orbit at height 300 km above Earth's surface. It is then transferred to a higher circular orbit at height 800 km.

(a) Calculate the orbital velocity in the initial orbit. [**4 marks**]

(b) Find the orbital period in the initial orbit. [**4 marks**]

(c) Calculate the orbital velocity in the final orbit. [**4 marks**]

(d) Determine the orbital period in the final orbit. [**4 marks**]

(e) For a satellite of mass 2500 kg, calculate the change in:

- (i) Kinetic energy. [**3 marks**]
- (ii) Potential energy. [**3 marks**]
- (iii) Total mechanical energy. [**3 marks**]

Question 5 [**20 marks**]

(a) For a satellite in circular orbit, derive the relationship between total energy E, kinetic energy KE, and potential energy PE. [**6 marks**]

(b) Show that for circular orbits, the total energy can be written as E = -GMm/(2r). [**4 marks**]

(c) A satellite orbits Earth with total energy -2.5×10^{1} J. Calculate:

- (i) The orbital radius. [**4 marks**]
- (ii) The orbital velocity. [**3 marks**]
- (iii) The orbital period. [**3 marks**]

Section D: Planetary Motion and Kepler's Laws [35 marks]

Question 6 [**20 marks**] Saturn orbits the Sun at an average distance of 1.43×10^{12} m with a period of 29.5 years.

(a) Use Saturn's orbital data to calculate the mass of the Sun. [**6 marks**]

(b) Calculate Saturn's orbital velocity. [**4 marks**]

(c) Neptune orbits at 4.50×10^{12} m from the Sun. Use Kepler's third law to predict Neptune's orbital period in years. [**5 marks**]

(d) Calculate Neptune's orbital velocity and compare it with Saturn's. [**5 marks**]

Question 7 [**15 marks**] Europa, one of Jupiter's moons, orbits at a distance of 6.71×10 m from Jupiter's center with a period of 3.55 days.

(a) Calculate Europa's orbital velocity. [**4 marks**]

(b) Use Europa's orbital data to determine Jupiter's mass. [**5 marks**]

(c) Another moon, Ganymede, orbits Jupiter with a period of 7.15 days. Calculate Ganymede's orbital distance from Jupiter. [**6 marks**]

Section E: Escape Velocity and Interplanetary Transfer [30 marks]

Question 8 [**18 marks**]

(a) Starting from first principles and using energy conservation, derive the expression for escape velocity from a planet's surface. [**6 marks**]

(b) Calculate the escape velocities from:

(i) Venus (mass 4.87×10^2 kg, radius 6.05×10 m). [**3 marks**]

(ii) Jupiter (mass 1.90×10^2 kg, radius 7.15×10 m). [**3 marks**]

(c) A spacecraft is launched from Earth with velocity 15.0 km/s. Will it escape the solar system? Justify your answer by calculating the velocity needed to escape the Sun's gravitational influence from Earth's orbit. [**6 marks**]

Question 9 [**12 marks**] A space probe is launched from Earth's surface with initial velocity 8.0 km/s vertically upward.

(a) Use energy conservation to calculate the maximum height reached by the probe. [**6 marks**]

(b) Calculate the probe's velocity when it reaches an altitude of 3000 km. [**4 marks**]

(c) Determine the time taken to reach maximum height. [**2 marks**]

Section F: Advanced Applications and Field Superposition [40 marks]

Question 10 [**25 marks**] The Lagrange points are positions in space where the gravitational forces of two large masses balance. Consider the Earth-Sun system where the Sun has mass 1.99×10^3 kg and Earth has mass 5.97×10^2 kg, separated by 1.50×10^{11} m.

(a) The L1 Lagrange point lies on the line between Earth and Sun, closer to Earth. At this point, a satellite experiences zero net gravitational force from Earth and Sun. If the satellite is at distance d from Earth's center, write the equation for force balance. [**4 marks**]

(b) Solve this equation to find the approximate distance from Earth to the L1 point. (Hint: assume d ¡¡ Earth-Sun distance for simplification) [**8 marks**]

(c) Calculate the gravitational potential at the L1 point due to both Earth and Sun. [**6 marks**]

(d) Determine the energy required to move a 1000 kg satellite from Earth's surface to the L1 point. [**7 marks**]

Question 11 [15 marks] A space elevator is a theoretical structure extending from Earth's surface to beyond geostationary orbit.

(a) Calculate the height of geostationary orbit above Earth's surface. [**4 marks**]

(b) At geostationary altitude, calculate:

(i) The gravitational field strength. [**3 marks**]

(ii) The centripetal acceleration for an object moving with Earth's rotation. [**3 marks**]

(c) Explain why the space elevator must extend beyond geostationary orbit to remain stable. [**3 marks**]

(d) Calculate the energy saving per kilogram compared to rocket launch for placing payloads in geostationary orbit. [**2 marks**]

Physics Data and Formulae

Gravitational Force and Field:

Newton's Law: $F = \frac{Gm_1m_2}{r^2}$

Field strength: $g = \frac{F}{m} = \frac{GM}{r^2}$

Field inside uniform sphere: $g = \frac{GMr}{R^3}$ (for $r < R$)

Field-potential relation: $g = -\frac{dV}{dr}$

Gravitational Potential and Energy:

Potential: $V = -\frac{GM}{r}$

Potential energy: $U = mV = -\frac{GMm}{r}$

Work done: $W = m\Delta V = \Delta U$

Orbital Motion:

Circular orbital velocity: $v = \sqrt{\frac{GM}{r}}$

Orbital period: $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Kepler's Third Law: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{r}}$

Energy in Circular Orbits:

Kinetic energy: $E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Potential energy: $E_p = -\frac{GMm}{r}$

Total energy: $E = E_k + E_p = -\frac{GMm}{2r}$

Circular Motion:

Centripetal force: $F_c = \frac{mv^2}{r} = m\omega^2 r$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Angular velocity: $\omega = \frac{2\pi}{T} = \frac{v}{r}$

Physical Constants:

Gravitational constant: $G = 6.67 \times 10^{-11}$ Nm²/kg²

Earth's mass: $M_E = 5.97 \times 10^{24}$ kg

Earth's radius: $R_E = 6.37 \times 10^6$ m

Sun's mass: $M_S = 1.99 \times 10^{30}$ kg

Earth-Sun distance: 1.50×10^{11} m

Standard gravity: $g = 9.81$ m/s²

END OF TEST

Total marks: 220

Grade boundaries: A* 198, A 176, B 154, C 132, D 110, E 88

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