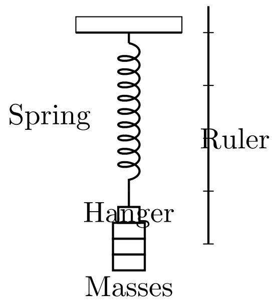
A Level Mechanics
Practice Test 6: Elasticity and Springs

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw clear, labelled diagrams where appropriate.
Time allowed: 2 hours 20 minutes

Section A: Fundamental Spring Behavior [40 marks]

Question 1 [16 marks] A student investigates the elastic properties of a spring using the setup shown below:



The natural length of the spring is 25.0 cm. The following measurements are taken:

Number of 100g masses	Total mass (kg)	Total length (cm)
0	0.05	26.5
1	0.15	28.5
2	0.25	30.5
3	0.35	32.5
4	0.45	34.5
5	0.55	36.5

(a) Calculate the weight of each total mass and complete a force-extension table. **[4 marks]**

(b) Plot a graph of force (y-axis) against extension (x-axis). **[4 marks]**

(c) Use your graph to determine the spring constant. **[3 marks]**

(d) State what the graph shows about the relationship between force and extension for this spring. **[2 marks]**

(e) Calculate the extension when a total force of 7.5 N is applied. **[3 marks]**

Question 2 [14 marks] Two springs A and B have spring constants of 180 N/m and 120 N/m respectively.

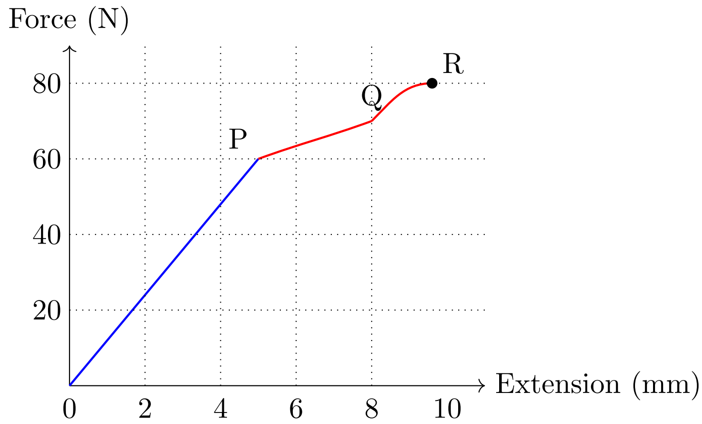
(a) Define the term "spring constant" and state its SI unit. [**3 marks**]

(b) The springs are connected in series. When a force of 36 N is applied:

   (i) Calculate the extension of spring A. [**2 marks**]
   (ii) Calculate the extension of spring B. [**2 marks**]
   (iii) Find the total extension. [**1 mark**]
   (iv) Calculate the effective spring constant. [**3 marks**]

(c) The same springs are now arranged in parallel with the same 36 N force applied. Calculate the extension of this arrangement. [**3 marks**]

Question 3 [**10 marks**] The force-extension graph below shows the behavior of an elastic material being stretched until it breaks:



(a) Describe what is happening to the material in each region: OP, PQ, QR. [**6 marks**]

(b) Calculate the spring constant in the elastic region. [**2 marks**]

(c) At which point does the material stop obeying Hooke's Law? [**2 marks**]

Section B: Elastic Potential Energy [50 marks]

Question 4 [**15 marks**]

(a) Define elastic potential energy. [**2 marks**]

(b) Derive the formula $E_p = \frac{1}{2}kx^2$ for elastic potential energy, starting from the definition of work done. [**5 marks**]

(c) A spring with spring constant 400 N/m is compressed by 8.0 cm. Calculate:

   (i) The elastic potential energy stored. [**3 marks**]
   (ii) The force required to compress the spring by this amount. [**2 marks**]

(d) Explain the relationship between the area under a force-extension graph and elastic potential energy. [**3 marks**]

Question 5 [**20 marks**] A spring-loaded projectile launcher is used to fire a ball horizontally. The spring has a spring constant of 600 N/m and the ball has a mass of 80 g.

(a) The spring is compressed by 10.0 cm before firing. Calculate:

   (i) The elastic potential energy stored in the compressed spring. [**3 marks**]

(ii) The maximum possible velocity of the ball (assuming 100% energy transfer). [**4 marks**]

(b) In practice, only 70% of the elastic potential energy is transferred to the ball due to energy losses. Calculate:

(i) The actual kinetic energy of the ball as it leaves the launcher. [**2 marks**]

(ii) The actual velocity of the ball. [**3 marks**]

(iii) The energy lost due to friction and other factors. [**2 marks**]

(c) The ball is fired horizontally from a height of 1.5 m above the ground. Calculate:

(i) The time taken to reach the ground. [**3 marks**]

(ii) The horizontal distance traveled before hitting the ground. [**3 marks**]

Question 6 [**15 marks**] A bungee jumper of mass 70 kg jumps from a platform. The bungee cord behaves like a spring with spring constant 40 N/m and has a natural length of 20 m. The jumper falls a total distance of 50 m before coming to rest momentarily.

(a) Calculate the gravitational potential energy lost by the jumper. [**3 marks**]

(b) Find the extension of the bungee cord when the jumper comes to rest. [**2 marks**]

(c) Calculate the elastic potential energy stored in the cord at maximum extension. [**3 marks**]

(d) Use conservation of energy to find the kinetic energy of the jumper when the cord just becomes taut (after falling 20 m). [**4 marks**]

(e) Calculate the speed of the jumper at this point. [**3 marks**]

Section C: Stress, Strain and Young's Modulus [55 marks]

Question 7 [**18 marks**]

(a) Define the following terms and state their SI units:

(i) Tensile stress. [**3 marks**]

(ii) Tensile strain. [**3 marks**]

(iii) Young's modulus. [**3 marks**]

(b) Write equations linking:

(i) Young's modulus to stress and strain. [**2 marks**]

(ii) Young's modulus to force, area, length and extension. [**3 marks**]

(c) Explain what a large value of Young's modulus tells us about a material's properties. [**4 marks**]

Question 8 [**22 marks**] A steel supporting cable in a crane has the following specifications:

- Length: 40.0 m
- Diameter: 20.0 mm
- Young's modulus for steel: 2.1×10^{11} Pa
- Density of steel: 7800 kg/m^3

(a) Calculate:

(i) The cross-sectional area of the cable. [**3 marks**]

(ii) The volume of the cable. [**2 marks**]

(iii) The mass of the cable. [**2 marks**]

(iv) The weight of the cable. [**2 marks**]

(b) The cable supports a load of 20,000 N. Calculate:

(i) The total force at the top of the cable (including its own weight). [**2 marks**]

(ii) The tensile stress in the cable. [**3 marks**]

(iii) The tensile strain in the cable. [**3 marks**]

(iv) The extension of the cable. [**3 marks**]

(c) Calculate the elastic potential energy stored in the stretched cable. [**2 marks**]

Question 9 [**15 marks**] A student tests wire samples of three different materials. Each wire has length 1.5 m and diameter 0.6 mm. The following data is collected under a 50 N load:

Material	Extension (mm)	Young's Modulus ($\times 10^{11}$ Pa)
Steel	0.30	
Brass	0.48	
Copper	0.58	

(a) Calculate the cross-sectional area of each wire. [**2 marks**]

(b) For each material, calculate:

(i) The stress when the 50 N load is applied. [**3 marks**]

(ii) The strain when the 50 N load is applied. [**3 marks**]

(iii) Young's modulus (complete the table). [**3 marks**]

(c) Rank the materials in order of stiffness, with the stiffest first. [**2 marks**]

(d) Which material would be most suitable for piano wire? Justify your answer. [**2 marks**]

Section D: Simple Harmonic Motion with Springs [50 marks]

Question 10 [**25 marks**] A horizontal mass-spring system consists of a 0.8 kg mass attached to a spring with spring constant 320 N/m.

(a) Explain why this system exhibits simple harmonic motion when displaced from equilibrium. [**4 marks**]

(b) The mass is displaced 4.0 cm from equilibrium and released. Calculate:

(i) The period of oscillation. [**3 marks**]

(ii) The frequency of oscillation. [**2 marks**]

(iii) The angular frequency. [**2 marks**]

(c) For this motion, determine:

(i) The amplitude of oscillation. [**1 mark**]

(ii) The maximum velocity. [**3 marks**]

(iii) The maximum acceleration. [**3 marks**]

(iv) The velocity when the displacement is 2.5 cm from equilibrium. [**4 marks**]

(d) Calculate the total mechanical energy of the system. [**3 marks**]

Question 11 [25 marks] A vertical spring with spring constant 150 N/m supports a mass of 1.2 kg in equilibrium.

(a) Calculate the compression of the spring when the mass is in equilibrium. **[3 marks]**

(b) The mass is pushed down an additional 5.0 cm from equilibrium and released.

- (i) State the amplitude of the resulting oscillation. **[1 mark]**
- (ii) Calculate the period of oscillation. **[3 marks]**
- (iii) Find the maximum speed during oscillation. **[3 marks]**

(c) During the oscillation, calculate the elastic potential energy stored in the spring when the mass is:

- (i) At its lowest position. **[4 marks]**
- (ii) At its highest position. **[4 marks]**
- (iii) Passing through the equilibrium position. **[3 marks]**

(d) At what position during the oscillation is the kinetic energy equal to half the total mechanical energy? **[4 marks]**

Physics Data and Formulae

Springs and Elasticity:

Hooke's Law: $F = kx$

Springs in series: $\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + ...$

Springs in parallel: $k_{total} = k_1 + k_2 + ...$

Elastic potential energy: $E_p = \frac{1}{2}kx^2$

Work done on spring: $W = \frac{1}{2}Fx$

Material Properties:

Stress: $\sigma = \frac{F}{A}$

Strain: $\epsilon = \frac{\Delta L}{L_0}$

Young's modulus: $E = \frac{\sigma}{\epsilon} = \frac{F \cdot L_0}{A \cdot \Delta L}$

Energy:

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Gravitational potential energy: $E_p = mgh$

Conservation of energy: $E_{total} = E_k + E_p =$ constant

Work-energy theorem: $W = \Delta E_k$

Simple Harmonic Motion:

Restoring force: $F = -kx$

Period: $T = 2\pi\sqrt{\frac{m}{k}}$

Frequency: $f = \frac{1}{T}$

Angular frequency: $\omega = 2\pi f = \sqrt{\frac{k}{m}}$

Displacement: $x = A\cos(\omega t + \phi)$

Velocity: $v = \pm\omega\sqrt{A^2 - x^2}$

Acceleration: $a = -\omega^2 x$

Maximum velocity: $v_{max} = \omega A$

Maximum acceleration: $a_{max} = \omega^2 A$

Kinematics:

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Geometry:

Area of circle: $A = \pi r^2$

Volume of cylinder: $V = \pi r^2 h$

Physical Constants:

Acceleration due to gravity: $g = 9.81$ m s^{-2}

$\pi = 3.14159...$

END OF TEST

Total marks: 195

Grade boundaries: A* 176, A 156, B 137, C 117, D 98, E 78

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stepupmaths.co.uk