A Level Pure Mathematics
Practice Test 6: Vectors

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Vector Basics and Notation

1. Given vectors $\mathbf{w} = \begin{pmatrix} 8 \\ -5 \\ 3 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$, calculate:
   (a) $\mathbf{w} + \mathbf{x}$
   (b) $\mathbf{w} - \mathbf{x}$
   (c) $4\mathbf{w} + 3\mathbf{x}$
   (d) $6\mathbf{w} - 2\mathbf{x}$
   (e) $|\mathbf{w}|$ and $|\mathbf{x}|$
   (f) A unit vector in the direction of $\mathbf{x}$

2. Express these vectors in component form:
   (a) $\overrightarrow{AB}$ where $A(6, 3, -1)$ and $B(2, 7, 4)$
   (b) $\overrightarrow{ST}$ where $S(-3, 1, 5)$ and $T(4, -2, 3)$
   (c) The position vector of point Z if $\overrightarrow{OZ} = 6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$
   (d) $\overrightarrow{BA}$ where $A(4, -3, 2)$ and $B(7, 1, -4)$

3. Given $\mathbf{g} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{h} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$:
   (a) Find $|\mathbf{g}|$ and $|\mathbf{h}|$
   (b) Calculate $\mathbf{g} + \mathbf{h}$ and $\mathbf{g} - \mathbf{h}$
   (c) Find scalars m and n such that $m\mathbf{g} + n\mathbf{h} = \begin{pmatrix} 5 \\ -9 \\ 14 \end{pmatrix}$
   (d) Determine if $\mathbf{g}$ and $\mathbf{h}$ are parallel

4. Points R, S, and T have position vectors $\mathbf{r} = \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$, $\mathbf{s} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, and $\mathbf{t} = \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix}$.
   (a) Find vectors $\overrightarrow{RS}$ and $\overrightarrow{RT}$
   (b) Calculate the lengths $|RS|$ and $|RT|$

(c) Find the position vector of the midpoint of ST

(d) Determine if triangle RST is isosceles

5. Find the values of p for which these vectors are perpendicular:

(a) $\mathbf{u} = \begin{pmatrix} 5 \\ p \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} p \\ 2 \\ -5 \end{pmatrix}$

(b) $\mathbf{a} = \begin{pmatrix} 3 \\ 4p \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ p \end{pmatrix}$

(c) $\mathbf{c} = p\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ and $\mathbf{d} = 5\mathbf{i} + p\mathbf{j} + 4\mathbf{k}$

Section B: Dot Product (Scalar Product)

6. Calculate the dot product of these vectors:

(a) $\mathbf{e} = \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix}$ and $\mathbf{f} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$

(b) $\mathbf{g} = 6\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{h} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

(c) $\mathbf{i} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{j} = \begin{pmatrix} 3 \\ 7 \\ -2 \end{pmatrix}$

(d) $\mathbf{k} = 5\mathbf{i} + 4\mathbf{j}$ and $\mathbf{l} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$

7. Find the angle between these pairs of vectors:

(a) $\mathbf{m} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(b) $\mathbf{o} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

(c) $\mathbf{q} = 4\mathbf{i} + 6\mathbf{j}$ and $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

(d) $\mathbf{s} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix}$

8. Use the dot product to verify these properties:

(a) $\mathbf{m} \cdot \mathbf{n} = \mathbf{n} \cdot \mathbf{m}$ (commutative)

(b) $\mathbf{m} \cdot (\mathbf{n} + \mathbf{o}) = \mathbf{m} \cdot \mathbf{n} + \mathbf{m} \cdot \mathbf{o}$ (distributive)

(c) $(k\mathbf{m}) \cdot \mathbf{n} = k(\mathbf{m} \cdot \mathbf{n})$ for scalar k

(d) $\mathbf{m} \cdot \mathbf{m} = |\mathbf{m}|^2$

9. Given vectors $\mathbf{p} = \begin{pmatrix} 6 \\ 5 \\ -2 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 5 \\ -6 \\ 2 \end{pmatrix}$, and $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$:

(a) Show that $\mathbf{p}$ and $\mathbf{q}$ are perpendicular

(b) Find the component of $\mathbf{r}$ in the direction of $\mathbf{p}$

(c) Calculate $|\mathbf{p} + \mathbf{q} + \mathbf{r}|$

(d) Find the angle between $\mathbf{p} + \mathbf{q}$ and $\mathbf{r}$

10. A triangle has vertices at $J(6, 3, 2)$, $K(4, 7, 1)$, and $L(5, 4, 6)$.

(a) Find the vectors $\overrightarrow{JK}$ and $\overrightarrow{JL}$
(b) Calculate the angle $\angle KJL$
(c) Find the area of triangle JKL
(d) Determine if the triangle is right-angled

Section C: Cross Product (Vector Product)

11. Calculate the cross product of these vectors:

(a) $\mathbf{s} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$
(b) $\mathbf{u} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$
(c) $\mathbf{w} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$
(d) $\mathbf{y} = 6\mathbf{i} + 4\mathbf{j}$ and $\mathbf{z} = 2\mathbf{i} + 5\mathbf{k}$

12. Verify these properties of the cross product:

(a) $\mathbf{r} \times \mathbf{s} = -(\mathbf{s} \times \mathbf{r})$ (anti-commutative)
(b) $\mathbf{r} \times (\mathbf{s} + \mathbf{t}) = \mathbf{r} \times \mathbf{s} + \mathbf{r} \times \mathbf{t}$ (distributive)
(c) $\mathbf{r} \times \mathbf{r} = \mathbf{0}$
(d) $|\mathbf{r} \times \mathbf{s}|^2 = |\mathbf{r}|^2|\mathbf{s}|^2 - (\mathbf{r} \cdot \mathbf{s})^2$

13. Find the area of the parallelogram spanned by:

(a) $\mathbf{a} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$
(b) $\mathbf{c} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ and $\mathbf{d} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$
(c) Vectors from origin to points $(3, 6, 2)$ and $(5, 2, 4)$
(d) $\overrightarrow{MN}$ and $\overrightarrow{MO}$ where $M(6, 2, 4)$, $N(3, 5, 2)$, $O(5, 3, 7)$

14. Given $\mathbf{u} = \begin{pmatrix} 7 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 6 \\ -5 \end{pmatrix}$:

(a) Calculate $\mathbf{u} \times \mathbf{v}$
(b) Verify that $\mathbf{u} \times \mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$
(c) Find a unit vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$
(d) Calculate the area of triangle with sides $\mathbf{u}$ and $\mathbf{v}$

15. Use the scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ to find:

(a) The volume of parallelepiped with edges $\mathbf{u} = \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} 4 \\ 3 \\ 6 \end{pmatrix}$
(b) Whether points $P(6, 2, 4)$, $Q(3, 7, 2)$, $R(5, 4, 3)$, $S(4, 6, 5)$ are coplanar
(c) The volume of tetrahedron with vertices at $(0, 0, 0)$, $(6, 2, 4)$, $(3, 5, 2)$, $(4, 3, 6)$

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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