A Level Pure Mathematics
Practice Test 2: Coordinate Geometry in the (x, y) Plane

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Distance and Midpoint Formulas

1. Find the distance between these pairs of points:
   (a) $A(2, 6)$ and $B(8, -2)$
   (b) $C(-3, 4)$ and $D(5, -1)$
   (c) $E(-4, -2)$ and $F(3, 6)$
   (d) $G(p, q)$ and $H(3p, 2q)$

2. Find the midpoint of the line segment joining:
   (a) $P(1, 9)$ and $Q(7, 5)$
   (b) $R(-5, 3)$ and $S(3, -7)$
   (c) $T(2m, 3n)$ and $U(-3m, n)$
   (d) The point $(4k, 6k)$ and the origin

3. The point $M(3, 4)$ is the midpoint of the line segment CD where $C(-1, 2)$.
   (a) Find the coordinates of point D
   (b) Calculate the length of CD
   (c) Find the equation of the line through M perpendicular to CD

4. Points $A(2, 5)$, $B(6, 2)$, and $C(1, 8)$ form a triangle.
   (a) Calculate the length of each side
   (b) Determine what type of triangle ABC is
   (c) Find the coordinates of the centroid
   (d) Calculate the perimeter of triangle ABC

5. The points $P(-1, 2)$, $Q(3, 5)$, $R(6, 1)$, and $S(2, -2)$ form a quadrilateral.
   (a) Show that $PQRS$ is a rhombus
   (b) Find the lengths of the diagonals
   (c) Calculate the area of rhombus $PQRS$
   (d) Verify that the diagonals bisect each other at right angles

Section B: Equations of Straight Lines

6. Find the equation of the straight line:
   (a) With gradient -2 passing through $(3, 1)$
   (b) Passing through $(2, 7)$ and $(5, 1)$
   (c) With x-intercept 4 and y-intercept -3
   (d) Perpendicular to $3x - 4y = 12$ and passing through $(2, 5)$

7. Express these equations in the form $ax + by + c = 0$:
   (a) $y = 2x - 5$
   (b) $y = -\frac{3}{4}x + 2$
   (c) $\frac{x}{3} + \frac{y}{4} = 1$
   (d) $y - 3 = 2(x + 1)$

8. Find the equation of the line that:
   (a) Is parallel to $y = -3x + 2$ and passes through $(1, 4)$
   (b) Is perpendicular to $2x - 5y = 10$ and has x-intercept 3
   (c) Passes through $(4, -2)$ and is parallel to the y-axis
   (d) Makes equal intercepts on both coordinate axes and passes through $(2, 3)$

9. Two lines have equations $L_1 : 3x + 2y - 12 = 0$ and $L_2 : x - 4y + 8 = 0$.
   (a) Find the point of intersection of L_1 and L_2
   (b) Calculate the obtuse angle between the lines
   (c) Find the equation of the line through the intersection point parallel to L_2
   (d) Find the distance from the origin to line L_1

10. A triangle has vertices at $A(1, 2)$, $B(5, 6)$, and $C(3, 0)$.
    (a) Find the equation of the perpendicular bisector of side AB
    (b) Find the equation of the altitude from C to side AB
    (c) Find the point where the altitude and perpendicular bisector intersect
    (d) Calculate the area of triangle ABC using coordinate geometry

Section C: Angle Between Lines

11. Calculate the acute angle between these pairs of lines:
    (a) $y = 3x - 2$ and $y = -\frac{1}{2}x + 4$
    (b) $2x + y = 5$ and $x - 3y = 6$
    (c) $4x - 3y + 5 = 0$ and $2x + 5y - 7 = 0$
    (d) $y = \tan 30^\circ \cdot x + 1$ and $y = \tan 75^\circ \cdot x - 2$

12. The line L passes through $(2, 3)$ and makes an angle of 45° with the positive x-axis.
    (a) Find the equation of line L
    (b) Find where L intersects the line $2x + y = 8$
    (c) Calculate the angle between L and the line $2x + y = 8$

13. Two lines L_1 and L_2 intersect at the point $(2, 4)$. If L_1 has gradient $\frac{3}{4}$ and the acute angle between the lines is $30°$:

    (a) Find the two possible gradients for L_2

    (b) Write the equations of both possible lines L_2

    (c) Determine which line makes the smaller angle with the x-axis

14. A line passes through $(1, 3)$ and makes an angle of $120°$ with another line having equation $y = 2x - 1$.

    (a) Find the gradient of the required line

    (b) Write the equation of the line

    (c) Find where this line intersects the coordinate axes

    (d) Verify the angle between the lines

Section D: Equation of a Circle

15. Write the equation of the circle with:

    (a) Center $(0, 0)$ and radius 7

    (b) Center $(4, -3)$ and radius 5

    (c) Center $(-2, 6)$ and passing through $(1, 2)$

    (d) Diameter with endpoints $(1, 3)$ and $(7, -1)$

16. Express these equations in standard form and identify the center and radius:

    (a) $x^2 + y^2 - 8x + 6y - 11 = 0$

    (b) $x^2 + y^2 + 4x - 10y + 13 = 0$

    (c) $x^2 + y^2 - 6x + 8y + 16 = 0$

    (d) $3x^2 + 3y^2 - 12x + 18y - 15 = 0$

17. A circle has center $(1, -2)$ and passes through the point $(4, 2)$.

    (a) Find the equation of the circle

    (b) Determine if the point $(-2, 1)$ lies inside, outside, or on the circle

    (c) Find the equation of the tangent to the circle at $(4, 2)$

    (d) Find the points where the circle intersects the x-axis

18. Two circles have equations $C_1 : x^2 + y^2 = 16$ and $C_2 : (x - 5)^2 + (y - 3)^2 = 9$.

    (a) Find the distance between the centers

    (b) Determine if the circles intersect, touch externally, touch internally, or are separate

    (c) Find the equation of the line joining the centers

    (d) If a third circle is tangent to both given circles, suggest where its center might be

19. A circle passes through the points $A(2, 1)$, $B(6, 3)$, and $C(4, 7)$.

    (a) Use the general form to find the equation of the circle

    (b) Convert to standard form to find the center and radius

    (c) Find the circumcenter of triangle ABC

    (d) Calculate the circumradius of triangle ABC

Section E: Parabolas

20. For the parabola $x^2 = 4py$:
    (a) Identify the focus and directrix when $p = 2$
    (b) If the focus is at $(0, 5)$, find the value of p and the equation of the directrix
    (c) Find the equation of the parabola with focus $(3, 0)$ and directrix $x = -3$
    (d) Sketch the parabola $x^2 = 16y$, showing the focus and directrix

21. A parabola has vertex at the origin and focus at $(4, 0)$.
    (a) Find the equation of the parabola
    (b) Find the equation of the directrix
    (c) If a point on the parabola has y-coordinate 9, find its x-coordinates
    (d) Find the length of the focal chord that is perpendicular to the axis

22. The parabola $y = px^2 + qx + r$ passes through $(1, 0)$, $(2, 3)$, and $(3, 8)$.
    (a) Find the values of p, q, and r
    (b) Express the equation in vertex form
    (c) Find the coordinates of the vertex
    (d) Find the axis of symmetry

23. A parabolic reflector has equation $y^2 = 16x$.
    (a) Find the focus of the parabola
    (b) If the reflector is 8 units wide at its opening, find its depth
    (c) Where should a light bulb be placed for a parallel beam?
    (d) Find the equation of the tangent at the point $(4, 8)$

Section F: Ellipses

24. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:
    (a) When $a = 6$ and $b = 4$, find the coordinates of the foci
    (b) Find the eccentricity when $a = 5$ and $b = 4$
    (c) If the foci are at $(0, \pm 3)$ and $b = 5$, find a
    (d) Sketch the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

25. An ellipse has center at the origin, major axis along the y-axis of length 12, and minor axis of length 8.
    (a) Write the equation of the ellipse
    (b) Find the coordinates of the foci
    (c) Calculate the eccentricity
    (d) Find the length of the latus rectum

26. The ellipse $\frac{(x+1)^2}{36} + \frac{(y-2)^2}{25} = 1$ has center at $(-1, 2)$.
    (a) Identify the lengths of the semi-major and semi-minor axes
    (b) Find the coordinates of the vertices
    (c) Calculate the coordinates of the foci

(d) Find the equations of the major and minor axes

27. An ellipse has vertices at $(\pm 4, 0)$ and passes through the point $(2, 3)$.

(a) Find the value of b

(b) Write the equation of the ellipse

(c) Calculate the eccentricity

(d) Find the coordinates of the foci

Section G: Hyperbolas

28. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

(a) When $a = 4$ and $b = 3$, find the coordinates of the foci

(b) Find the equations of the asymptotes when $a = 5$ and $b = 2$

(c) If the foci are at $(\pm 6, 0)$ and $a = 4$, find b

(d) Sketch the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$

29. A hyperbola has equation $\frac{y^2}{25} - \frac{x^2}{16} = 1$.

(a) Find the coordinates of the vertices

(b) Calculate the coordinates of the foci

(c) Write the equations of the asymptotes

(d) Find the eccentricity

30. The hyperbola $xy = c$ (rectangular hyperbola):

(a) When $c = 18$, find the coordinates where the hyperbola intersects the line $y = 6$

(b) Find the equation of the tangent to $xy = 16$ at the point $(4, 4)$

(c) Find the area of the triangle formed by the tangent at $(a, \frac{c}{a})$ and the coordinate axes

(d) If the hyperbola passes through $(3, 4)$, find the value of c

31. A hyperbola has center at $(2, 1)$, vertices at $(2, 4)$ and $(2, -2)$, and eccentricity $\frac{5}{3}$.

(a) Find the value of a (semi-transverse axis)

(b) Calculate the value of b (semi-conjugate axis)

(c) Write the equation of the hyperbola

(d) Find the coordinates of the foci

Section H: Mixed Conic Sections

32. Identify the type of conic section and find its key properties:

(a) $9x^2 + 4y^2 = 36$

(b) $4x^2 - y^2 = 16$

(c) $x^2 = 12y$

(d) $x^2 + y^2 + 8x - 6y + 9 = 0$

33. For the general second-degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$:

(a) When $B = 0$, state the discriminant conditions for each conic type

(b) What type of conic is represented by $x^2 + 4xy + 4y^2 - 6x - 12y + 9 = 0$?

(c) Identify the conic: $3x^2 - 2xy + 3y^2 - 8x - 8y + 8 = 0$

(d) What happens to the classification when $A = C$ and $B = 0$?

34. Find the points of intersection (if any) between:

(a) The line $y = 2x - 1$ and the circle $x^2 + y^2 = 10$

(b) The line $x = 3$ and the parabola $y^2 = 12x$

(c) The ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(d) The circle $(x - 2)^2 + y^2 = 8$ and the hyperbola $xy = 4$

35. Find the equations of tangents:

(a) To the circle $x^2 + y^2 = 16$ at the point $(2\sqrt{2}, 2\sqrt{2})$

(b) To the parabola $y^2 = 16x$ at the point $(4, 8)$

(c) To the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at the point $(3, \frac{16}{5})$

(d) To the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ at the point $(5, \frac{9}{4})$

Section I: Applications and Problem Solving

36. A tunnel has a semi-elliptical cross-section with width 20 meters and maximum height 8 meters.

(a) Set up coordinates and find the equation of the ellipse

(b) Find the height of the tunnel at distances 3, 6, and 9 meters from the center

(c) A truck is 2.5 meters wide and 6 meters tall. Can it pass through?

(d) Find the width of the tunnel at a height of 5 meters

37. A parabolic satellite dish has diameter 3 meters and depth 0.5 meters.

(a) Find the equation of the parabola representing the dish

(b) Calculate the focal length of the dish

(c) Where should the receiver be positioned?

(d) If the dish were twice as wide but the same depth, how would the focal length change?

38. A comet follows a hyperbolic path around the sun. At its closest approach, it is 2 AU from the sun, and the eccentricity of its orbit is 1.5. (1 AU = distance from Earth to Sun)

(a) Find the semi-major axis of the hyperbolic orbit

(b) Calculate the semi-minor axis

(c) Where is the sun located relative to the hyperbola?

(d) Find the equation of the comet's path

39. A whispering gallery has an elliptical ceiling with major axis 30 meters and minor axis 20 meters.

(a) Find the equation of the ellipse

(b) Calculate the distance between the foci

(c) If a person stands at one focus, where should another person stand to hear clearly?

(d) What is the eccentricity of this ellipse?

40. GPS navigation uses hyperbolic positioning. Two stations A and B are 600 km apart. A receiver gets signals from A and B with a time difference of 0.002 seconds.

(a) What is the difference in distances from the receiver to stations A and B? (Radio waves travel at 3×10^8 m/s)

(b) Set up coordinates with A and B as foci of a hyperbola

(c) Find the equation of the hyperbola on which the receiver lies

(d) If a third station C at $(0, 400)$ gives another time difference, how would you find the exact position?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

For more resources and practice materials, visit:
stepupmaths.co.uk